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Nonlinear Disturbance Observer-Based Adaptive Anti-Lock Braking Control of Electro-Hydraulic Brake Systems with Unknown Tire–Road-Friction Coefficient

Soon Gu Kwon  and Sung Jin Yoo * 

School of Electrical and Electronics Engineering, Chung-Ang University, 84 Heukseok-Ro, Seoul 06974, Republic of Korea; tnsrn1061@naver.com

* Correspondence: sjyoo@cau.ac.kr

Abstract

This paper addresses a recursive adaptive anti-lock braking (AB) control design problem for electro-hydraulic brake (EHB) systems subject to unknown tire–road-friction coefficients and disturbances. Compared with the relevant literature, the primary contributions are (i) the development of a novel nonlinear AB model integrated with a bond-graph-based EHB (BGEHB) system, and (ii) the proposal of an adaptive neural AB control approach incorporating a nonlinear disturbance observer (NDO). The AB and BGEHB models are unified into a single nonlinear braking model, with the wheel speed as the system output and the duty ratios of the BGEHB inlet and outlet valves as control inputs. To maintain an optimal slip ratio during braking, we design the NDO-based adaptive AB controller to regulate the wheel speed in a recursive manner. The designed controller incorporates a delay-compensation term to address the time-delay characteristics of the hydraulic system, employs a neural-network function approximator in the NDO and controller to compensate for the unknown tire–road-friction coefficient, and applies NDOs to compensate for disturbances due to the vehicle motion and BGEHB dynamics. The stability of the proposed control scheme is established via the Lyapunov theory, and a simulation comparison is presented to demonstrate the effectiveness of the proposed design approach.

Keywords: adaptive control; anti-lock braking (AB) control; bond-graph-based electro-hydraulic brake (BGEHB) systems; nonlinear disturbance observer (NDO)

1. Introduction

The electro-hydraulic brake (EHB) system is an advanced brake-by-wire technology that replaces conventional mechanical connections with electronically controlled hydraulic actuation, enabling fast and precise brake pressure modulation for high-performance safety functions. Prior studies have focused on converting the braking input into accurate wheel-cylinder pressure [1], leading to the development of various architectures, such as integrated EHB [2] and e-Booster systems [3], and the adoption of control strategies, including sliding mode [4], dynamic surface [5], and model predictive control [6]. To account for the damping and inertia effects of the caliper and the capacitive effect of the hydraulic fluid, a bond-graph-based EHB (BGEHB) model that realizes brake pressure modulation solely via inlet and outlet valve control was proposed in [7]. However, most existing works have addressed only normal braking conditions and rarely investigated EHB integration with anti-lock braking (AB) systems in realistic emergency scenarios, where



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rapid pressure modulation, nonlinear hydraulic dynamics, and uncertain tire–road-friction coefficients significantly affect braking performance. Furthermore, disturbances due to vehicle motion and EHB dynamics, combined with the challenge of ensuring robust control under varying road conditions, have achieved limited progress in this area.

The AB system is a crucial safety feature to prevent wheel lock-up during emergency braking, allowing the driver to maintain steering control and vehicle stability. The primary objective of AB control is to regulate the slip ratio, defined as the ratio between vehicle speed and wheel circumferential speed, to an optimal value that maximizes braking force under varied road conditions. Contemporary nonlinear control approaches offer robust adaptive strategies that can handle complex, uncertain dynamic systems [8–11]. In particular, observer-based methods have increasingly been employed to estimate and compensate for external disturbances, enhancing control robustness [12–14]. Several AB control strategies have been proposed, including sliding mode control [15,16], sliding mode proportional–integral control [17], and model predictive control [18]. Although effective in theory, these approaches typically require precise knowledge of the tire–road-friction coefficient, which is challenging to measure in practice. In the context of transport safety, tire–road interaction constitutes a fundamental aspect of safety systems because the achievable braking force and stability margin are constrained due to tire-contact characteristics. Moreover, variations in tire-related parameters (e.g., vertical load, inflation pressure, and structural stiffness) can directly alter the effective contact condition and friction utilization, influencing the braking efficiency and vehicle stability. Accordingly, recent studies have demonstrated that detailed tire modeling and road–surface interaction analyses are associated with transport safety and accident prevention [19]. Approximation methods for the unknown friction coefficient have been investigated to address these uncertainties [20]. Various approximation-based AB control strategies employing neural networks (NNs) and fuzzy logic systems have been introduced in [21–27], and disturbance-observer-based sliding-mode controllers have been developed to improve the slip-ratio accuracy under uncertainty [28]. Despite these advances, existing studies have predominantly neglected the dynamic properties of brake-pressure modulation. In practical applications, AB control performance is substantially influenced by brake actuation dynamics, yet most studies have treated the pressure-modulation process as static or ideal. An AB control scheme that incorporates electro-mechanical brake dynamics has been proposed to partially address this issue [29]. Moreover, EHB and AB systems are typically studied in isolation [30,31], neglecting their coupling in real vehicles. To the best of our knowledge, no recursive AB control frameworks have been developed for a unified nonlinear model that integrates EHB and AB systems. Formulating a novel nonlinear AB model embedded in the BGEHB system that accounts for disturbances from vehicle motion and EHB dynamics poses a considerable challenge in recursive controller design. Addressing these challenges is critical to bridging the gap between theoretical AB control methods and their realistic implementation in advanced brake-by-wire systems.

Based on these observations, this study addresses a recursive adaptive control design problem for unified AB-BGEHB systems with unknown tire–road-friction coefficients and disturbances. To this end, we propose a nonlinear disturbance observer (NDO)-based adaptive AB control strategy that integrates hydraulic-delay compensation and an NN approximator. The NN is employed to estimate and compensate for the unknown tire–road-friction coefficient, while the NDO suppresses disturbances due to vehicle motion and BGEHB dynamics in real time. The primary contributions of this study are as follows:

- (i) In contrast to the existing AB and EHB control approaches [1–31], this paper develops a unified nonlinear dynamic model of the AB-BGEHB system in strict-feedback form with a time delay. The model employs the wheel speed as the system out-

put and the duty ratios of the inlet and outlet valves of the BGEHB as the control input. Unlike the hierarchical AB-EHB modeling strategy [31], which requires the linearization of the brake dynamics to derive the braking torque, the proposed model retains the nonlinearities of the EHB system to represent the practical braking torque more accurately.

- (ii) We design a recursive adaptive AB controller that incorporates an NDO, NN, and a time-delay compensation mechanism. The NN-based approximator addresses the unknown tire–road-friction coefficient in the adaptive NDO and controller, and the NDO compensates for disturbances caused by vehicle motion and BGEHB hydraulic dynamics. The closed-loop stability of the proposed scheme is established via Lyapunov theory. A simulation comparison is presented to demonstrate the effectiveness and robustness of the proposed design.

The remainder of this paper is organized as follows. Section 2 formulates the mathematical model of the integrated AB-BGEHB system. Section 3 presents the proposed adaptive AB control design, which combines an NN, NDO, and time-delay compensation, followed by a Lyapunov-based stability analysis of the closed-loop system. Then, Section 4 provides the simulation results to validate the proposed method. Finally, Section 5 concludes this work with critical findings.

2. Preliminaries

2.1. Modeling of AB Dynamics

The AB dynamics during braking are commonly represented by a quarter-car vehicle model [32], which provides a simplified yet effective representation of the wheel–road interaction under braking conditions. The model consists of two coupled state equations: one governing the rotational wheel dynamics and the other capturing the longitudinal dynamics of the vehicle body, as shown in Figure 1.

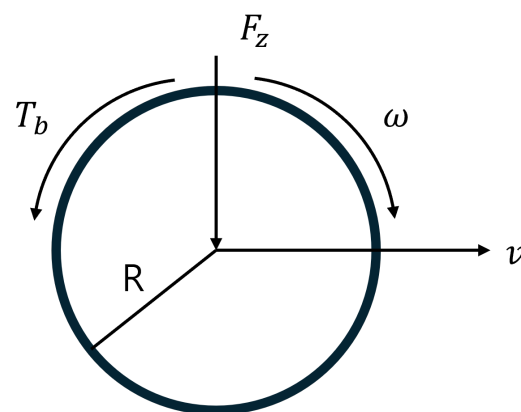


Figure 1. Quarter-car tire model.

The rotational and longitudinal dynamics of the wheel can be described as

$$J\dot{\omega} = \mu(\lambda)F_z\Phi - T_b + d_1(t) \quad (1)$$

$$m\dot{v} = -\mu(\lambda)F_z, \quad (2)$$

where J is the wheel moment of inertia, ω denotes the angular velocity, Φ is the effective wheel radius, T_b is the braking torque, and $d_1(t)$ represents unmodeled torques such as rolling resistance and braking torque fluctuations. The parameter m is the equivalent mass of the quarter-car model, v is the longitudinal vehicle velocity, and F_z is the vertical tire load, given by $F_z = mg$ with gravitational acceleration g . The slip ratio during braking is

defined as $\lambda = (v - \omega\Phi)/v$. Since the braking process ensures $\omega\Phi \leq v$, the slip ratio is bounded in $0 \leq \lambda \leq 1$.

The tire–road-friction coefficient $\mu(\lambda)$ is modeled using Pacejka’s magic formula [33]:

$$\mu(\lambda) = D \sin(C \arctan(B\lambda - E(B\lambda - \arctan(B\lambda))))$$

where B , C , D , and E denote the stiffness factor, shape factor, peak value, and curvature factor, respectively, which vary with road conditions.

2.2. Modeling of BGEHB Dynamics

The dynamics of the BGEHB system are defined as follows [7]:

$$\begin{aligned} \dot{V}_f &= C_{d1}A_1D_{v1}\sqrt{\frac{2}{\rho}(p_0 - \frac{V_f}{C_f})} \\ &\quad - C_{d2}A_2D_{v2}\sqrt{\frac{2}{\rho}\frac{V_f}{C_f} - \frac{A}{m_c}P_c} + d_2(t) \\ \dot{P}_c &= A\frac{V_f}{C_f} - \frac{R_d}{m_c}P_c - k_eX_c \\ \dot{X}_c &= \frac{P_c}{m_c} \\ p_w(t) &= \frac{V_f(t-T)}{C_f}, \end{aligned} \quad (3)$$

where p_0 is the supply pressure, ρ is the brake fluid density, and C_f denotes its volumetric compliance. The inlet and outlet valves are characterized by the maximum flow coefficients C_{d1} and C_{d2} , along with the maximum orifice areas A_1 and A_2 . Their opening levels are regulated via the duty ratios $D_{v1}, D_{v2} \in [0, 1]$, which change during the pressure build-up, holding, and release phases. The piston has an effective cross-sectional area, A , and the caliper dynamics are defined by the momentum P_c , mass m_c , displacement X_c , damping coefficient R_d , and stiffness k_e . The disturbance term $d_2(t)$ represents the unmodeled effects, such as hydraulic hysteresis, parameter uncertainties, and the wear of components. Finally, the wheel cylinder pressure p_w is expressed as the chamber pressure subject to a pure time delay, T , accounting for the lag induced via the hydraulic process.

Assumption 1 ([28,34]). Disturbances d_1 and d_2 and their time derivative are unknown and bounded.

Remark 1. Disturbances d_1 and d_2 represent the lumped disturbances of the system. Specifically, d_1 serves as a lumped disturbance representing the unmodeled driving torques, such as rolling resistance, and external disturbances acting on the wheel. In addition, d_2 captures lumped disturbances from parameter uncertainties due to thermal effects or component wear and the hysteresis characteristics of the hydraulic system. Assumption 1, which states that disturbances d_1 and d_2 are bounded, is a fundamental requirement for maintaining the stability of the control system. The wheel disturbance d_1 is physically limited to less than 5–10% of the maximum available braking torque. Similarly, the hydraulic disturbance d_2 is realistically bounded within 10–15% of the dynamic capacity of the nominal hydraulic system.

2.3. Radial Basis Function Neural Network

Let $f(\psi)$ be an unknown continuous function defined over a compact domain, Ω_ψ . With the utilization of the universal approximation capability of radial-basis-function neural networks (RBFNNs), $f(\psi)$ can be approximated as

$$f(\psi) = \theta^\top R(\psi) + \epsilon(\psi) \quad (4)$$

where $\psi \in \Omega_\psi$ denotes the network input, $\theta \in \mathbb{R}^M$ is the ideal weight vector satisfying $\|\theta\| \leq \bar{\theta}$ for some constant, $\bar{\theta} > 0$, $M \in \mathbb{N}$ denotes the number of nodes, $R(\psi) \in \mathbb{R}^M$ is the vector of Gaussian basis functions with $\|R(\psi)\| \leq \bar{R}$ for some $\bar{R} > 0$, and $\epsilon(\psi)$ is the approximation error. Here, $\epsilon(\psi)$ and its time derivative are bounded.

3. NDO-Based Adaptive AB Control for Unified AB-BGEHB Systems

3.1. Unified State-Space Model of AB-BGEHB Systems and Control Objective

To facilitate controller synthesis, we derive a unified state-space model by integrating the AB wheel dynamics with the BGEHB hydraulics. We exploit the fact that the EHB wheel-cylinder pressure directly determines the braking torque necessary for the AB subsystem. The braking torque produced via the wheel-cylinder pressure p_w is modeled as follows:

$$T_b = k_p p_w \quad (5)$$

where $k_p > 0$ is the gain relating the wheel-cylinder pressure to the brake torque.

Let the state and input variables be defined as $x_1 = \omega$, $x_2 = V_f/C_f$, $x_3 = P_c$, $x_4 = X_c$, and let the control inputs be denoted as $u_1 = 1 - D_{v1}$ and $u_2 = D_{v2}$. Since the hydraulic compliance induces a delay from fluid volume to caliper pressure, $p_w(t)$ is defined as $p_w(t) = x_2(t - T(t))$, where $T(t) \geq 0$ represents the time-varying delay due to temperature-dependent fluid viscosity changes. Then, the unified AB-BGEHB model using (1) and (3) is represented as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1 x_2(t - T) + d_1(t) \\ \dot{x}_2 &= f_2(x_3) + g_2(x_2)(1 - u_1) - g_3(x_2)u_2 + d_2(t) \\ \dot{x}_3 &= f_3(x_2, x_3, x_4) \\ \dot{x}_4 &= f_4(x_3) \end{aligned} \quad (6)$$

where $f_1(x_1) = \frac{1}{J}\mu(\lambda(x_1))F_z\Phi$, $f_2(x_3) = -\frac{A}{m_c C_f}x_3$, $f_3(x_2, x_3, x_4) = Ax_2 - \frac{R_d}{m_c}x_3 - k_e x_4$, $f_4(x_3) = \frac{1}{m_c}x_3$, $g_1 = -\frac{k_p}{J}$, $g_2(x_2) = \frac{C_{d1}A_1}{C_f}\sqrt{\frac{2}{\rho}(p_0 - x_2)}$, and $g_3(x_2) = \frac{C_{d2}A_2}{C_f}\sqrt{\frac{2x_2}{\rho}}$.

The objective of the paper is to design NDO-based adaptive valve commands, $u_1, u_2 \in [0, 1]$, to ensure that the wheel angular velocity $x_1 = \omega$ tracks a desired optimal profile, $\omega_{\text{opt}}(t)$, despite the unknown friction map $\mu(\cdot)$ and disturbances d_1 and d_2 .

Remark 2. The proposed unified model (6) is structurally distinct from the hierarchical architecture in [31] and is more suitable for AB control. Instead of relying on a separate EHB controller to track a desired torque, the proposed approach enables direct wheel speed regulation via the valve inputs. This unified recursive design avoids the linearization and torque-interface simplifications of the existing hierarchical approach [31], allowing a more immediate and tightly coupled response to the wheel slip, which is vital in emergency braking scenarios.

3.2. NDO-Based Adaptive AB Control Design

This section details the design of the NDO-based adaptive AB controller for the system (6). The desired wheel angular velocity ω_{opt} is provided via the desired slip ratio λ_{opt} and the current vehicle velocity v as follows:

$$\omega_{\text{opt}} = \frac{1 - \lambda_{\text{opt}}}{\Phi} v \quad (7)$$

where the desired slip ratio λ_{opt} represents a constant determined by the road conditions.

Remark 3. This study focuses on robustness against external disturbances and adaptive compensation for the tire–road-friction coefficient; therefore, the estimation of λ_{opt} is not explicitly addressed. However, since online estimation algorithms for λ_{opt} have been investigated in the literature [35], the proposed controller has the potential to be integrated with such methods.

In the EHB system, the time delay T of the hydraulic pressure response is a known parameter, enabling the design of a specific term to compensate for the state delay. Thus, the following compensator is introduced:

$$\dot{\zeta} = \frac{-n_1 \tanh(\zeta) + g_1(x_2(t-T) - x_2)}{n_2 \text{sech}^2(\zeta)} \quad (8)$$

where $n_1, n_2 > 0$ are design constants.

For the dynamic surface design method, the error surfaces z_1 and z_2 , and the boundary error e_1 are defined as follows:

$$z_1 = x_1 - \omega_{\text{opt}} - n_2 \tanh(\zeta) \quad (9)$$

$$z_2 = x_2 - \bar{\alpha}_1 \quad (10)$$

$$e_1 = \bar{\alpha}_1 - \alpha_1, \quad (11)$$

where α_1 represents the virtual control law, and the filtered signal $\bar{\alpha}_1$ is derived via the low-pass filter given by $\dot{\bar{\alpha}}_1 = (\alpha_1 - \bar{\alpha}_1)/\tau$ with $\bar{\alpha}_1(0) = \alpha_1(0)$.

The error dynamics using (6)–(8) are expressed as

$$\dot{z}_1 = f_1(x_1) + \mu(\lambda)F_z \frac{(1 - \lambda_{\text{opt}})}{m\Phi} + g_1x_2 + d_1 + n_1 \tanh(\zeta) \quad (12)$$

$$\dot{z}_2 = f_2(x_3) + g_2(x_2)(1 - u_1) - g_3(x_2)u_2 + d_2 - \dot{\bar{\alpha}}, \quad (13)$$

where $\dot{\omega}_{\text{opt}} = \frac{1 - \lambda_{\text{opt}}}{\Phi} (-\frac{\mu F_z}{m})$ is used.

The RBFNN is employed to approximate the model uncertainties, including the unknown friction coefficient μ , as follows:

$$f_1(x_1) + \mu(\lambda(x_1))F_z \frac{(1 - \lambda_{\text{opt}})}{m\Phi} = \theta^\top R(x_1) + \epsilon. \quad (14)$$

With the use of (14), the error dynamics are represented as

$$\dot{z}_1 = \theta^\top R(x_1) + g_1x_2 + \gamma_1 D_1 + n_1 \tanh(\zeta) \quad (15)$$

$$\begin{aligned} \dot{z}_2 = & f_2(x_3) + g_2(x_2)(1 - u_1) - g_3(x_2)u_2 \\ & + \gamma_2 D_2 - \frac{\alpha_1 - \bar{\alpha}_1}{\tau}, \end{aligned} \quad (16)$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$ are design constants, and $D_1 = (d_1 + \epsilon)/\gamma_1$ and $D_2 = d_2/\gamma_2$ are disturbance terms to be estimated by NDO.

Step 1: Consider the first Lyapunov function, $V_1 = z_1^2/2 + \tilde{\theta}^\top \tilde{\theta}/(2\eta) + \tilde{D}_1^2/(2\varsigma_1)$, where η and ς_1 are positive constants, $\tilde{\theta} = \theta - \hat{\theta}$, and $\tilde{D}_1 = D_1 - \hat{D}_1$. Here, $\hat{\theta}$ and $\hat{D}_1$ are estimates of θ and D_1 , respectively.

With the use of (10) and (15), the time derivative of V_1 is obtained as

$$\begin{aligned} \dot{V}_1 = & z_1(\theta^\top R(x_1) + g_1z_2 + g_1\alpha_1 + g_1e_1 + \gamma_1 D_1 \\ & + n_1 \tanh(\zeta)) - \frac{1}{\eta} \tilde{\theta}^\top \dot{\tilde{\theta}} + \frac{1}{\varsigma_1} \tilde{D}_1(\dot{D}_1 - \dot{\hat{D}}_1). \end{aligned} \quad (17)$$

The virtual control law α_1 is designed as

$$\alpha_1 = -\frac{1}{g_1}(k_1 z_1 + \hat{\theta}^\top R(x_1) + \gamma_1 \hat{D}_1 + n_1 \tanh(\zeta)), \quad (18)$$

where $k_1 > 0$ is a design constant, and $\hat{D}_1$ is provided via the following NDO:

$$\hat{D}_1 = \hat{r}_1 + \varsigma_1 z_1 \quad (19)$$

$$\hat{r}_1 = -\varsigma_1(\hat{\theta}^\top R(x_1) + g_1 x_2 + \gamma_1 \hat{D}_1 + n_1 \tanh(\zeta)), \quad (20)$$

and the adaptive law for $\hat{\theta}$ is given by

$$\dot{\hat{\theta}} = \eta(R(x_1)z_1 - c\hat{\theta}) \quad (21)$$

with constants $\eta > 0$ and $c > 0$.

Remark 4. The delay compensator (8) is designed to compensate for the delayed state $x_2(t - T)$ in the control design. The adjustment term proportional to $\tanh(\zeta)$ ensures gradual compensation, while the second term involving $x_2(t - T)$ corrects discrepancies caused by the delayed hydraulic response. The $\text{sech}^2(\zeta)$ term is incorporated to cancel the derivative of $\tanh(\zeta)$ during the design. The virtual controller (18) functions as an internal command that indirectly regulates the wheel-braking torque so that the angular wheel velocity tracks the desired reference. This term is crucial to the recursive backstepping procedure because it stabilizes the upper subsystem (i.e., the wheel dynamics) and guides the design of the valve inputs (i.e., u_1 and u_2) in the following steps.

Substituting (18)–(21) into (17) yields

$$\begin{aligned} \dot{V}_1 &= z_1(-k_1 z_1 + \hat{\theta}^\top R(x_1) + \gamma_1 \hat{D}_1 + g_1 z_2 + g_1 e_1) - \hat{\theta}^\top (R(x_1)z_1 - c\hat{\theta}) \\ &\quad + \frac{1}{\varsigma_1} \tilde{D}_1(\dot{D}_1 - \varsigma_1(\hat{\theta}^\top R(x_1) + \gamma_1 \hat{D}_1)) \\ &= -k_1 z_1^2 + \gamma_1 z_1 \tilde{D}_1 + c\hat{\theta}^\top \hat{\theta} + \frac{1}{\varsigma_1} \tilde{D}_1 \dot{D}_1 - \gamma_1 \tilde{D}_1^2 - \tilde{D}_1 \hat{\theta}^\top R(x_1) + g_1 z_1 z_2 + g_1 z_1 e_1. \end{aligned} \quad (22)$$

Step 2: Consider the second Lyapunov function, $V_2 = z_2^2/2 + \tilde{D}_2^2/(2\varsigma_2)$, where $\tilde{D}_2 = D_2 - \hat{D}_2$; $\hat{D}_2$ is an estimate of D_2 , and ς_2 is a positive constant.

From (16), the time derivative of V_2 is given by

$$\dot{V}_2 = z_2 \left(f_2(x_3) + u_t + \gamma_2 D_2 - \frac{\alpha_1 - \bar{\alpha}_1}{\tau} \right) + \frac{1}{\varsigma_2} \tilde{D}_2(\dot{D}_2 - \dot{\hat{D}}_2), \quad (23)$$

where $u_t = g_2(x_2)(1 - u_1) - g_3(x_2)u_2$. The total input u_t is designed as

$$u_t = -k_2 z_2 - f_2(x_3) - \gamma_2 \hat{D}_2 - g_1 z_1 + \frac{\alpha_1 - \bar{\alpha}_1}{\tau}, \quad (24)$$

where $k_2 > 0$ is a constant, and $\hat{D}_2$ is obtained by the following NDO:

$$\hat{D}_2 = \hat{r}_2 + \varsigma_2 z_2 \quad (25)$$

$$\hat{r}_2 = -\varsigma_2(f_2(x_3) + u_t + \gamma_2 \hat{D}_2 - \dot{\alpha}_1). \quad (26)$$

Substituting (24)–(26) into (23), we have

$$\dot{V}_2 = -k_2 z_2^2 + \gamma_2 z_2 \tilde{D}_2 - g_1 z_1 z_2 + \frac{1}{\varsigma_2} \tilde{D}_2 \dot{D}_2 - \gamma_2 \tilde{D}_2^2. \quad (27)$$

The control inputs u_1 and u_2 correspond to the inlet and outlet valves, respectively. To achieve coordinated control, the inlet valve u_1 is activated to increase the brake pressure, whereas the outlet valve u_2 is activated to decrease it. This logic is implemented by defining u_1 and u_2 based on the total control input u_t , as follows:

$$\begin{cases} u_1 = 1 - \frac{u_t}{g_2(x_2)}, u_2 = 0; & \text{if } z_2 < 0 \\ u_2 = -\frac{u_t}{g_3(x_2)}, u_1 = 1; & \text{if } z_2 > 0 \\ u_1 = 1, u_2 = 0; & \text{if } z_2 = 0. \end{cases} \quad (28)$$

The block diagram of the proposed NDO-based adaptive AB control system is given in Figure 2. Table 1 summarizes the key variables used in the control design.

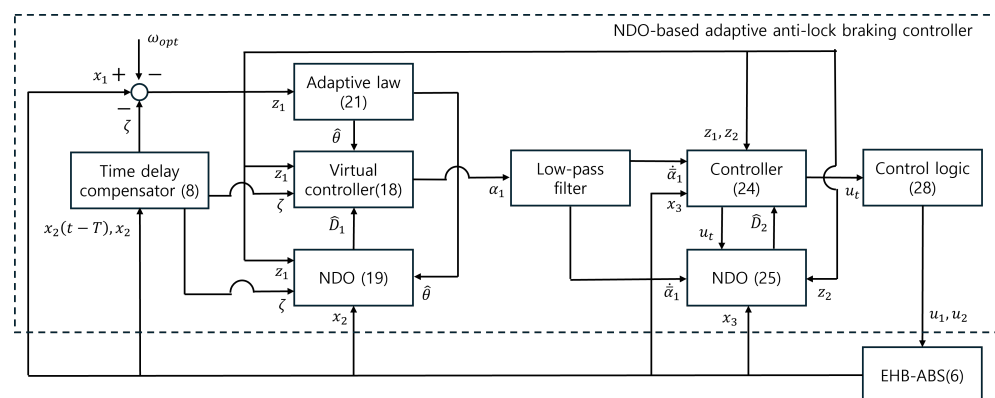


Figure 2. Block diagram of the proposed NDO-based adaptive AB control system.

Table 1. List of symbols and abbreviations.

Symbol/Abbreviation	Description
z_1, z_2	Tracking errors
e_1	Filter boundary error
α_1	Virtual control law
$\tilde{\alpha}_1$	Filtered virtual control signal
ζ	Delay compensation state
$\hat{\theta}$	Estimated NN weights
$\hat{D}_1, \hat{D}_2$	Estimated lumped disturbances
u_1, u_2	Control inputs (inlet/outlet valves)
AB	Anti-lock braking
EHB	Electro-hydraulic brake
BGEHB	Bond-graph-based EHB
NDO	Nonlinear disturbance observer
NN	Neural network
RBFNN	Radial-basis-function neural network

Remark 5. A potential singularity may occur if $g_2(x_2)$ or $g_3(x_2)$ becomes zero. This concern, however, can be avoided in the proposed controller. The term $g_2(x_2)$ involves $\sqrt{p_0 - x_2}$, which cannot vanish because the wheel cylinder pressure x_2 always remains lower than the source pressure p_0 during operation. Likewise, $g_3(x_2)$ depends on x_2 , which is strictly positive in the operating range. Therefore, the proposed control allocation is free of singularities.

Remark 6. Compared with the BGEHB controller in [7], this study develops a unified AB-BGEHB model that accounts for internal disturbances in the BGEHB system. In addition, an NDO-based adaptive AB control strategy is proposed in the presence of unknown tire–road-friction coefficients.

3.3. Stability Analysis

For the stability analysis, the comprehensive Lyapunov function candidate is defined as $V = V_1 + V_2 + e_1^2/2$.

Theorem 1. Consider the closed-loop system formed by the unified AB-BGEHB systems (6), time delay compensator (8), NDOs (19), (20), (25) and (26), and adaptive law (21). Given the bounded initial conditions, such that $V(0) \leq \Psi$ with some constant $\Psi > 0$, all closed-loop signals are semiglobally, uniformly, and ultimately bounded.

Proof. Based on (22) and (27), the time derivative of V is represented as

$$\begin{aligned}\dot{V} = & -k_1 z_1^2 - k_2 z_2^2 + \gamma_1 z_1 \tilde{D}_1 + \gamma_2 z_2 \tilde{D}_2 + c \tilde{\theta}^\top \hat{\theta} \\ & - \gamma_1 \tilde{D}_1^2 - \gamma_2 \tilde{D}_2^2 + \frac{1}{\varsigma_1} \tilde{D}_1 \dot{D}_1 + \frac{1}{\varsigma_2} \tilde{D}_2 \dot{D}_2 \\ & - \tilde{D}_1 \tilde{\theta}^\top R(x_1) - \frac{e_1^2}{\tau} + e_1 \Gamma,\end{aligned}\quad (29)$$

where $\Gamma = -\dot{a}_1 + g_1 z_1$.

Due to the boundedness of $\dot{d}_i$ and $\dot{e}$, there exists a positive constant, $\bar{D}_i$, such that $|\dot{D}_i| \leq \bar{D}_i$, where $i = 1, 2$. Let us define the compact sets $\Lambda_1 = \{z_1^2 + z_2^2 + \tilde{\theta}^\top \tilde{\theta}/\eta + \tilde{D}_1^2/\varsigma_1 + \tilde{D}_2^2/\varsigma_2 \leq 2\varrho\}$ and $\Lambda_2 = \{\omega_{opt}^2 + \dot{\omega}_{opt}^2 \leq \bar{\omega}\}$. $|\Gamma| < \bar{\Gamma}$ is ensured on $\Lambda_1 \times \Lambda_2$, where $\bar{\Gamma}$ is a constant. Using Young's inequality, we obtain

$$\gamma_i z_i \tilde{D}_i \leq \frac{\gamma_i z_i^2}{2} + \frac{\gamma_i \tilde{D}_i^2}{2} \quad (30)$$

$$c \tilde{\theta}^\top (\theta - \tilde{\theta}) \leq -\frac{c \|\tilde{\theta}\|^2}{2} + \frac{c \bar{\theta}^2}{2} \quad (31)$$

$$\frac{1}{\varsigma_i} \tilde{D}_i \dot{D}_i \leq \frac{\tilde{D}_i^2}{2\varsigma_i} + \frac{\bar{D}_i^2}{2\varsigma_i} \quad (32)$$

$$-\tilde{D}_1 \tilde{\theta}^\top R(x_1) \leq \frac{\tilde{D}_1^2}{2} + \frac{\bar{R}^2 \|\tilde{\theta}\|^2}{2} \quad (33)$$

$$e_1 \Gamma \leq \frac{e_1^2}{2} + \frac{\bar{\Gamma}^2}{2}, \quad (34)$$

where $i = 1, 2$.

Substituting these inequalities into (29) yields

$$\begin{aligned}\dot{V} \leq & -\left(k_1 - \frac{\gamma_1}{2}\right) z_1^2 - \left(k_2 - \frac{\gamma_2}{2}\right) z_2^2 - \frac{1}{2}(c - \bar{R}^2) \|\tilde{\theta}\|^2 \\ & - \left(\gamma_1 - \frac{1}{2\varsigma_2}\right) \tilde{D}_1^2 - \left(\gamma_2 - \frac{1}{2\varsigma_2}\right) \tilde{D}_2^2 \\ & - \left(\frac{1}{\tau} - \frac{1}{2}\right) e_1^2 + \frac{c \bar{\theta}^2}{2} + \frac{\bar{D}_1^2}{2\varsigma_1} + \frac{\bar{D}_2^2}{2\varsigma_2} + \frac{\bar{\Gamma}^2}{2}.\end{aligned}\quad (35)$$

Through the selection of the design parameters as $k_i = \gamma_i/2 + k_i^*$, $i = 1, 2$, $c = \bar{R} + c^*$, $\gamma_i = 1/(2\varsigma_i) + \gamma_i^*$, $i = 1, 2$, and $1/\tau = 1/2 + \tau^*$, inequality (35) becomes $\dot{V} \leq -\Xi_1 V + \Xi_2$, where $i = 1, 2$, $\Xi_1 = \min\{2k_1^*, 2k_2^*, c^*, 2\gamma_1^*, 2\gamma_2^*, 2\tau^*\}$, and $\Xi_2 = c\bar{\theta}^2/2 + \bar{D}_1^2/(2\varsigma_1) + \bar{D}_2^2/(2\varsigma_2) + \bar{\Gamma}^2/2$. When $\Xi_1 > \Xi_2/\varrho$, it follows that $\dot{V} < 0$ on $V = \varrho$. Hence, the error signals z_i , $\tilde{\theta}$, $\tilde{D}_i$, and e_1 ($i = 1, 2$) are all bounded. Since ω_{opt} and $\tanh(\zeta)$ are also bounded signals, x_1 is bounded as well. Moreover, the boundedness of θ and D_1 guarantees that their estimates, $\hat{\theta}$ and $\hat{D}_1$, remain bounded. This further ensures the boundedness of α_1 and $\bar{\alpha}_1$, which in turn implies that x_2 is also bounded. Therefore, the set $\{V \leq \varrho\}$

is invariant, and all signals in the closed-loop system are semiglobally, uniformly, and ultimately bounded.

By integrating the differential inequality $\dot{V} \leq -\Xi_1 V + \Xi_2$, we obtain the following solution for $V(t)$:

$$V(t) \leq \left(V(0) - \frac{\Xi_2}{\Xi_1} \right) e^{-\Xi_1 t} + \frac{\Xi_2}{\Xi_1}. \quad (36)$$

Therefore, $V(t) \leq \max\{V(0), \Xi_2/\Xi_1\}$ is ensured.

From (36), the ultimate bounds of the error signals are obtained as follows:

$$|z_i| \leq \sqrt{\frac{2\Xi_2}{\Xi_1}}, \quad |\tilde{D}_i| \leq \sqrt{\frac{2\zeta_i\Xi_2}{\Xi_1}}, \quad \|\tilde{\theta}\| \leq \sqrt{\frac{2\eta\Xi_2}{\Xi_1}}, \quad |e_1| \leq \sqrt{\frac{2\Xi_2}{\Xi_1}}$$

where $i = 1, 2$. These inequalities explicitly show that all error signals converge to a compact set around the origin. It is important to note that the size of these ultimate bounds can be systematically adjusted by decreasing Ξ_1 (i.e., by selecting the design parameters).

Then, we check the stability of the system's internal dynamics. The internal dynamics are represented as the states that are not directly involved in the output tracking, namely x_3 and x_4 , as follows:

$$\begin{cases} \dot{x}_3 = Ax_2 - \frac{R_d}{m_c}x_3 - k_e x_4 \\ \dot{x}_4 = \frac{1}{m_c}x_3. \end{cases} \quad (37)$$

From the boundedness of all signals in the closed-loop system, the input x_2 is bounded. With the bounded input x_2 , the dynamics can be written in the following state-space form with the state vector $x_{inner} = [x_3, x_4]^\top$:

$$\dot{x}_{inner} = \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = H \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} A \\ 0 \end{bmatrix} x_2 \quad (38)$$

where $H = \begin{bmatrix} -\frac{R_d}{m_c} & -k_e \\ \frac{1}{m_c} & 0 \end{bmatrix}$. Owing to $R_d > 0$, $m_c > 0$, and $k_e > 0$, the matrix H is a Hurwitz matrix. Therefore, the internal dynamics are stable, and the internal states x_3 and x_4 remain bounded. $\square$

Remark 7. The selection of the controller gains is guided by the Lyapunov stability analysis, which establishes that the system states are uniformly ultimately bounded. (36) reveals the ultimate bound $V(t)$ is determined by the ratio of Π_2 to Π_1 . Thus, a smaller bound can be realized by selecting larger control and adaptation gains $k_1, k_2, c, \gamma_1, \gamma_2$ to increase the magnitude of Π_1 or by selecting larger observer gains ζ_1, ζ_2 to decrease the magnitude of Π_2 . A larger adaptive gain, η , and observer gains, ζ_1 and ζ_2 , improve the adaptation speed of the NN and NDO, respectively, but this can also increase the sensitivity of the model to disturbances and model uncertainties.

Remark 8. The proposed NDO-based adaptive AB controller is suitable for real-time implementation. The online computations consist of low-order NDO-based controller updates for the unified AB-BGEHB model with the fourth-order dynamics and an RBFNN function approximator with a small number of nodes, M , where the Gaussian centers/width are fixed, and only the weights are updated online. Thus, the per-update computational load is dominated by the RBFNN evaluation and weight adaptation and scales linearly with M (i.e., $\mathcal{O}(M)$), while the remaining controller computations add only a small constant overhead. Since the controller is formulated in continuous

time, practical ECU implementation can be achieved via standard digital realization (e.g., fixed-step discretization). Accordingly, the computational and memory requirements remain modest.

4. Simulation

In this section, simulation results are provided to validate the theoretical properties of the proposed method. To demonstrate its effectiveness, the proposed controller is compared with the baseline approaches presented in [16,31]. The method in [16] is adopted as a benchmark for nonlinear control under time-delay conditions. Although this approach utilizes a sliding-mode controller without the EHB structure, it addresses the input delay, facilitating a valid comparison of the delay-compensation capabilities. In addition, the baseline linear–quadratic regulator approach is presented in [31]. The method in [31] was applied to an EHB–ABS model formulated as a hierarchical structure, in which the high-level slip-ratio dynamics were coupled with the low-level EHB pressure dynamics. The baseline controller relies on a linearized representation of this hierarchical model and assumes that the tire–road-friction coefficient is precisely known. In contrast, the proposed adaptive scheme is developed to handle the nonlinearities of the coupled system and accounts for parameter uncertainties. The physical parameters of the unified model are summarized in Table 2. The controller gains are set to $k_1 = 110$, $k_2 = 80$, $\gamma_1 = 3$, $\gamma_2 = 1$, $\varsigma_1 = 2$, $\varsigma_2 = 5$, $\eta = 600$, $c = 0.001$, $n_1 = 70$, $n_2 = 70$, and $\tau = 0.0001$. The disturbances are defined as $d_1 = 10 \sin(6\pi t)$ and a composite sinusoidal signal, d_2 , as shown in Figure 3. To ensure a fair comparison, the proposed controller and baseline are subjected to these disturbances. The RBFNN utilizes five nodes initialized with zero weights. The Gaussian centers are spaced evenly across $[-1, 1]$ without input normalization, and the width parameter is set to 0.707. The initial state variables are set to $x_1(0) = 134.2$, $x_2(0) = 0.01$, and $x_3(0) = x_4(0) = 0$.

Table 2. Model parameters [7,32].

Definition	Symbol	Value	Unit
Wheel's moment of inertia	J	0.6	$\text{kg} \cdot \text{m}^2$
Effective wheel radius	Φ	0.3	m
Equivalent mass of quarter-car model	m	400	kg
Braking torque coefficient	k_p	3×10^{-4}	$\text{N} \cdot \text{m}/\text{Pa}$
Maximum flow coefficient of inlet valve	C_{d1}	0.7	—
Maximum flow coefficient of outlet valve	C_{d2}	0.7	—
Cross-sectional area at maximum opening of inlet valve	A_1	8×10^{-7}	m^2
Cross-sectional area at maximum opening of outlet valve	A_2	8×10^{-7}	m^2
Volumetric compliance effect of brake fluid	C_f	3.4×10^{-13}	m^5/N
Density of brake fluid	ρ	850	kg/m^3
Piston area	A	1.963×10^{-3}	m^2
Caliper mass	m_c	0.75	kg
Elastic coefficient of elastic element in brake	k_e	3.37×10^7	N/m
Damping coefficient of damping element in brake	R_d	10,000	$\text{N}/(\text{m}/\text{s})$
Maximum pressure of master cylinder	p_0	1.5×10^7	Pa
Hydraulic time delay	T	$0.02 + 0.005 \sin(0.5t)$	s

The simulation is conducted to verify that the proposed control scheme can achieve effective wheel-speed tracking in the presence of varying road-friction coefficients and external disturbances. The scenario considers time-varying road conditions, where the surface changes sequentially from snow ($0 \text{ s} \leq t < 2 \text{ s}$) to dry asphalt ($2 \text{ s} \leq t < 4 \text{ s}$) and, finally, to wet asphalt ($t \geq 4 \text{ s}$), with the corresponding Pacejka coefficients listed in Table 3. Figure 4 illustrates the wheel-speed-tracking performance of the proposed

approach. Although transient degradations occur at the instants of surface change, the controller rapidly recovers due to its real-time compensation capability. For comparison with both previous methods, the wheel-speed-tracking result is converted into the slip-ratio control result through the dynamic relation $\lambda = (v - \omega\Phi)/v$. The slip-ratio control performance is shown in Figure 5. Compared with the previous methods, the proposed control scheme demonstrates superior tracking performance, particularly in the low-speed wet asphalt region after 4 s. Specifically, the proposed controller proves to be more robust against chattering than the sliding-mode control used in [16]. Additionally, when the road surface changes from dry to wet asphalt at 4 s, transient deviations are induced via the pressure modulation lag of the hydraulic system; nevertheless, the proposed approach enables a faster recovery of tracking performance compared to the technique in [31], owing to its NDO and NN mechanism. The mean absolute errors (MAEs) are compared in Table 4. The results indicate that the proposed scheme reduces the tracking error by 68.0% and 61.2% compared to the methods in [16] and [31], respectively. The adaptive responses of the NDO and NN weights to changes in road conditions and disturbances are presented in Figures 6 and 7, respectively. Finally, Figure 8 depicts the duty ratios of the BGEHB inlet and outlet valves, which are modulated to generate the required brake-pressure variations for effective control.

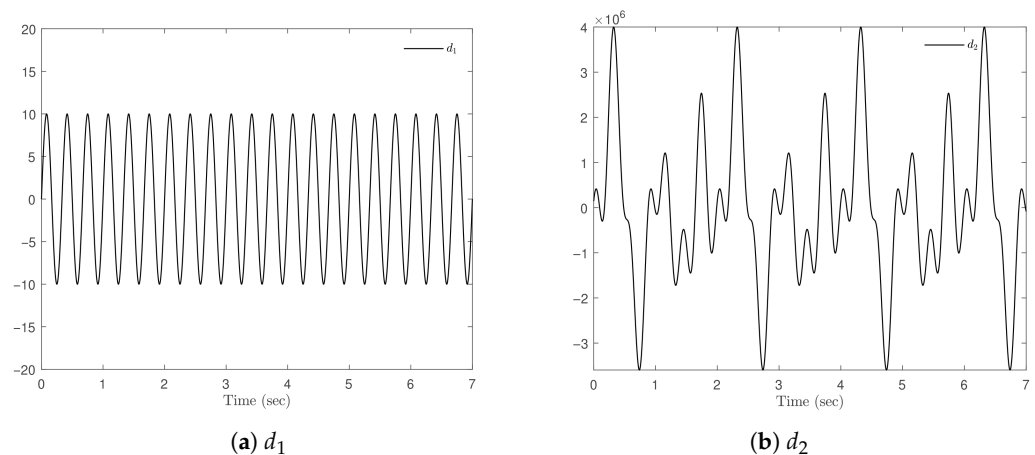


Figure 3. Disturbance d_2 .

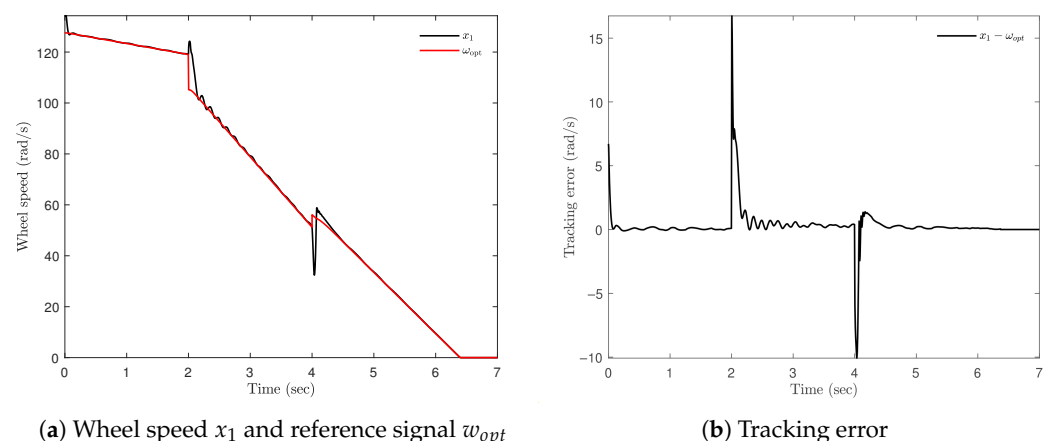


Figure 4. Wheel-speed-tracking results of the proposed approach.

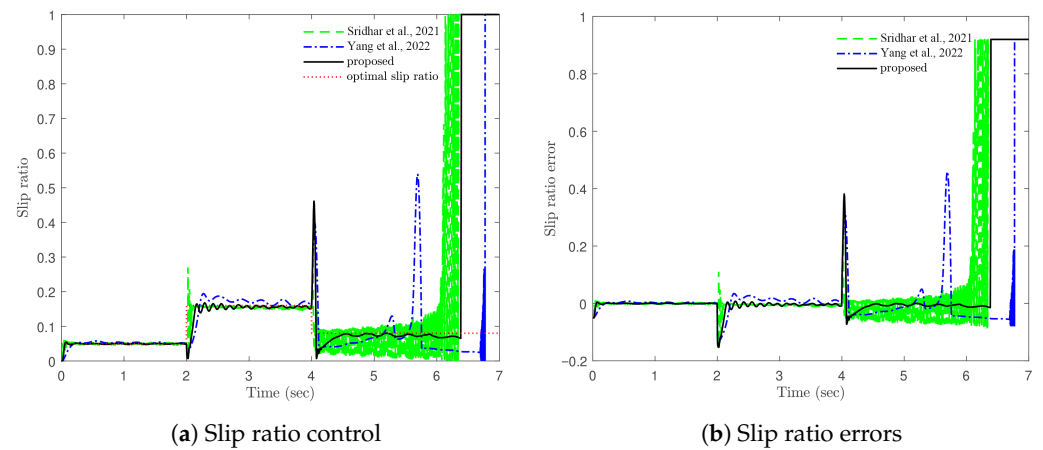


Figure 5. Comparison of tracking results [16,31].

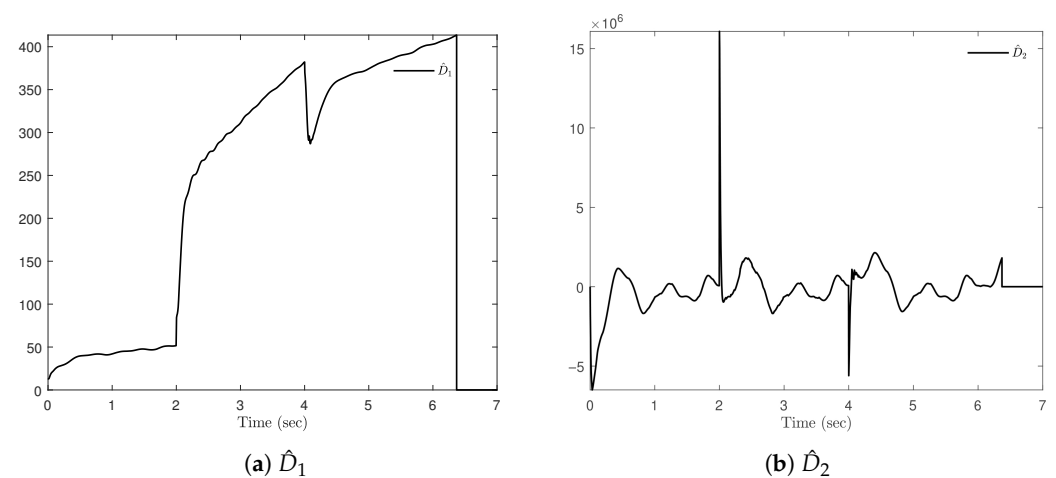


Figure 6. Estimated disturbances of the proposed approach.

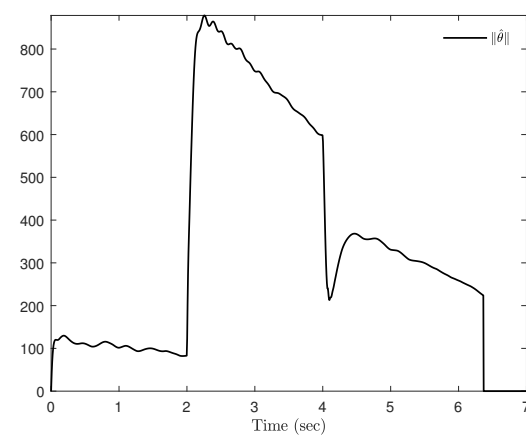


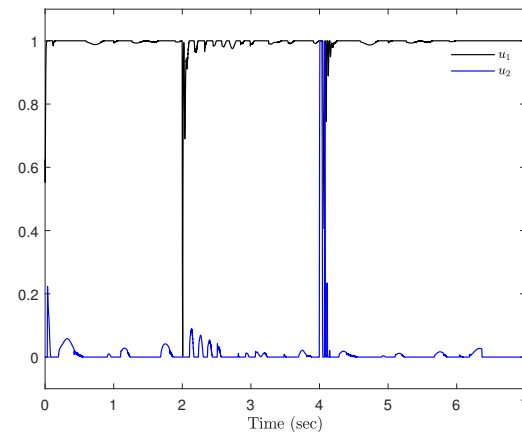
Figure 7. Adaptive estimate $\|\hat{\theta}\|$ of the proposed approach.

Table 3. Pacejka coefficients [33].

Road Condition	B	C	D	E	λ_{opt}
Snow	5	2	0.3	1.0	0.05
Wet asphalt	12	2.3	0.82	1	0.08
Dry asphalt	10	1.9	1	0.97	0.16

Table 4. MAE of slip ratio.

	Proposed	[16]	[31]
Slip ratio error	0.0112	0.0350	0.0289

**Figure 8.** Control inputs u_1 and u_2 of the proposed approach.

5. Conclusions

In this paper, a recursive, adaptive AB control strategy has been presented for unified AB-BGEHB systems subject to unknown tire–road-friction coefficients and disturbances. By designing a nonlinear AB-BGEHB dynamic model in strict-feedback form, the proposed framework has enabled an accurate representation of the braking torque while preserving the nonlinearities of the hydraulic system. The adaptive controller has achieved robust wheel-speed regulation and optimal slip-ratio tracking under practical braking conditions by incorporating an NDO, an NN approximator, and a delay-compensation mechanism. The NDO has effectively compensated for disturbances due to vehicle motion and BGEHB dynamics, while the NN has estimated and compensated for the unknown tire–road-friction coefficient. The stability of the closed-loop system has been confirmed using Lyapunov analysis. The simulation results have demonstrated that the proposed control design has enhanced robustness and braking performance compared with the existing approaches. As future research directions, we recommend extending the proposed NDO-based adaptive AB framework to EHB systems with unknown nonlinear parameters by incorporating NN-based approximation for the uncertain EHB nonlinearities, with a particular emphasis on singularity-free learning and the control of the unknown input-gain functions g_2 and g_3 . In addition, the proposed approach can be further developed within a prescribed performance control framework [36,37] to guarantee transient performance.

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