

Real-Time Optimization with Prior-selected Time Convergence for Economic Dispatch[★]

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Abstract: In this study, a prior-selected time convergence (PTC) optimization algorithm is proposed to obtain the optimal solution to RTEDPs in prior-selected time (PT) with mathematical convergence analysis. The prior-selected convergence time is independent of initial conditions and any problem parameter. To the best of the author's knowledge, this is the first time an PTC optimization algorithm is introduced, which obtains the optimal solution for the dual optimization problem of RTEDPs in PT. The implementation of the proposed solution is simple, and only one tuning parameter is required using the adaptive learning rate. The numerical simulations were conducted using an IEEE 118 bus grid under various situations such as constraint variations, load variations, and generator plug-off. The simulation results demonstrate that the proposed method always guarantee PTC in 10 iterative simulations with random initial conditions and different coefficients of local objective function while satisfying the demand-supply energy balance.

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1. INTRODUCTION

Economic dispatch problems (EDPs) are one of the primary energy management problems in power grid systems, which determine the energy level of all generators to meet the required load demands while minimizing operating cost under system constraints [Wang et al. (2019); Yun et al. (2019)]. In recent years, renewable energy sources (RES) have substantially increased as alternatives to conventional thermal energy sources to achieve decarbonization. In this context, micro-grids consisting of RES and energy storage systems have also drawn considerable attention owing to their efficient and flexible operation characteristics. However, inherent uncertainties in RES and rapid changes in load demand in micro-grids result in power grid uncertainties, which pose a significant challenge of power system operation and planning in EDPs [Hu et al. (2019); Li et al. (2022)].

To resolve the operational uncertainty problem, various real-time EDPs (RTEDP) have been proposed. The RT-EDP is solved at intervals between 5 min and 15 min according to the actual power grid operating price update

period [Reddy et al. (2015)]. The model-predictive control (MPC)-based RTEDP has been proposed to ensure a power-demand balance under the uncertainties of RES and flexible demand [Du et al. (2017)]. The sequential RTEDP has been developed using subintervals (1 min) by taking into account the minute-to-minute variability of RES [Reddy et al. (2015)]. The machine-learning (ML) based RTEDP have been investigated using a sample labeling process for the supervised training [Dong et al. (2021)]. However, high computational power may be required when algorithms such as ML and MPC are implemented to solve the RTEDP at an interval time. Although previous RTEDP algorithms assert that the optimal solution can be obtained at an interval time for real-time operation, there is no mathematical analysis for the case in which the optimal solution convergence time is guaranteed in a predefined interval time. Therefore, the convergence time of a solution changes according to the varying initial conditions of RTEDPs owing to variations of cost function coefficient, load, and constraint.

To guarantee the finite/fixed convergence of the optimal solution for convex optimization problem, various optimization algorithms, that guarantee the finite-time convergence of the solution to the optimal point have been developed. Normalized and signed gradient flow algorithms have been proposed for finite-time convergence [Chen et al. (2018)]. Additionally, continuous Lyapunov-based first-

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and second-order finite-time optimization designs have also been developed [Romero et al. (2020); Chen et al. (2021)]. Although previous methods successfully improve the convergence speed and obtain finite-time convergence, the convergence time is determined according to initial conditions. Therefore, the convergence time may be large when the initial value is unexpectedly large [Chen et al. (2021)]. To resolve this problem, fixed-time optimization algorithms, in which the convergence time is uniformly bounded for all initial conditions have been investigated [Sanchez-Torres et al. (2014)]. The FxT convex optimization problems with linear equality constraints have been solved in [Garg et al. (2021)]. The FxT optimization for EDP with distributed manner has been applied to address with frequently variable pricing policies and highly fluctuating loads [Baranwal et al. (2021)]. However, previous algorithms have a challenging point in implemented aspect. The tuning of at least four design parameters is necessary to obtain the desired Fxt convergence, in some cases, which may require considerable effort.

In this study, a prior-selected time convergence (PTC) optimization algorithm is proposed to obtain the optimal solution to RTEDPs in prior-selected time (PT). The PTC optimization solution is designed based on the special differential equation form, that the solution converges to zero in PT. The PT is independent of the initial conditions and any problem parameter. To the best of the author's knowledge, this is the first time an PTC optimization algorithm is introduced, which obtains the optimal solution for the dual optimization problem of RTEDPs in PT. The proposed solution is simple, and only one tuning parameter is required using adaptive learning rate. Because the RTEDP is periodically redefined and solved in a prior-selected short time, the proposed optimization method is robust under different varying situations. The effectiveness and reliability of the proposed method were verified through numerical simulations, which were conducted using an IEEE 118 bus grid. The simulation results demonstrate that the proposed method always guarantee PTC in 10 iterative simulations with random initial conditions and different coefficients of local objective function while satisfying the demand-supply energy balance.

2. PROBLEM FORMULATION AND PRELIMINARIES

Before introducing the PTC optimization algorithm, the basic economic dispatch problem for real-time energy management, definition and lemma for the optimization stability analysis are presented in this section. The RTEDP can be formulated as the following convex optimization problem with an equality and inequality constraint:

$$\min_{p_1, p_2, \dots, p_N} \sum_{i=1}^N J_i(p_i) = \sum_{i=1}^N (\xi_i p_i^2 + \zeta_i p_i + \eta_i) \quad (1)$$

s.t.

$$\sum_{i=1}^N p_i = \sum_{i=1}^N d_i \quad (2)$$

$$p_{i \min} \leq p_i \leq p_{i \max}, \quad \forall i = 1, \dots, N, \quad (3)$$

where p_i is the power generation in each node, and d_i is the local power demand in each node. $p_{i \min}$ and $p_{i \max}$

are the lower and upper limitations of the i th of the node energy generation, $J_i(p_i)$ is the cost of each power generation as the local objective function, which is continuously differentiable and strictly convex, η_i , ζ_i , and ξ_i are the positive coefficients of local objective function, and N is the number of nodes. The EDP is periodically redefined each constant period T_p . Furthermore, to obtain the feasibility of the above RTEDP, we assume that total demand, $\sum_{i=1}^N d_i$, satisfies conditions such as $\sum_{i=1}^N p_{i \min} \leq \sum_{i=1}^N d_i \leq \sum_{i=1}^N p_{i \max}$.

Lemma 1. (Pal et al. (2020)). : Let us assume that there exists a set that contains the origin such that $D \subset \mathbb{R}^n$, and continuous positive-definite functions α_1 and α_2 on D . Suppose there exists a real-valued piecewise continuously differentiable function $V(t, x) : \tau \times D \rightarrow \mathbb{R}_+ \cup 0$, where $\tau = [t_0, t_f)$. Consider a time t_f , which is the prior-selected convergence time and is independent of initial conditions and any problem parameter. Suppose that there exists a real constant $k > 1$ such that the following conditions hold:

$$\alpha_1 \leq V(t, x) \leq \alpha_2 \quad \forall t \in \tau \quad (4)$$

$$\dot{V}(t, x) \leq -\frac{k(e^{V(t, x)} - 1)\sigma(t)}{e^{V(t, x)}}, \quad \forall V(t, x) \neq 0, \quad \forall t \in \tau, \quad (5)$$

$\forall x \in D$, where $\sigma(t) = \frac{1}{t_f - t}$. Then, the origin is fixed-time globally asymptotically stable [Song et al. (2017)], and the regulation is achieved in PT t_f .

The proof of Lemma 1 is presented in Pal et al. (2020).

3. PRIOR-SELECTED TIME CONVERGENCE OPTIMIZATION SOLUTION

In this section, the PTC optimization algorithm for (1) is introduced to obtain the optimal energy generation solution considering the generation cost in PT. From economic dispatch problem (1), the Lagrangian can be represented as follows:

$$\mathcal{L}(p, \lambda) = \sum_{i=1}^N J_i(p_i) + \lambda \left(\sum_{i=1}^N d_i - \sum_{i=1}^N p_i \right), \quad (6)$$

where λ is the Lagrange multiplier to consider constraint (2). Although constraint (3) is not considered in (6), constraint (3) will be considered after the auxiliary variable. The Lagrangian dual function $g(\lambda)$ is defined as [Boyd et al. (2004)]:

$$g(\lambda) = \inf_{P \in \mathcal{P}_i} \sum_{i=1}^N \underbrace{(J_i(p_i) + \lambda(d_i - p_i))}_{:=g_i(\lambda)}, \quad (7)$$

where $P := [p_1, p_2, \dots, p_N]$, $\mathcal{P}_i := \{p_i \in \mathbb{R} : p_{i \min} \leq p_i \leq p_{i \max}\}$, and $g_i(\lambda)$ is a distributed dual function. The derivative of $g_i(\lambda)$ with respect to p_i can be obtained as follows:

$$\frac{\partial g_i(\lambda)}{\partial p_i} = \frac{\partial J_i(p_i)}{\partial p_i} - \lambda. \quad (8)$$

Considering constraint (3), if $\frac{\partial J_i}{\partial p_i}(p_{i \max}) - \lambda < 0$, the cost decreases on $[p_{i \min}, p_{i \max}]$, and converges to a minimum at $p_i = p_{i \max}$. Likewise, if $\frac{\partial J_i}{\partial p_i}(p_{i \min}) - \lambda > 0$, the cost increases on $[p_{i \min}, p_{i \max}]$, and converges to a minimum at $p_i = p_{i \min}$. And, if $\frac{\partial J_i}{\partial p_i}(p_{i \min}) \geq \lambda \geq \frac{\partial J_i}{\partial p_i}(p_{i \max})$,

the minimum of the cost is obtained on $[p_{i \min}, p_{i \max}]$ as follows:

$$\frac{\partial J_i}{\partial p_i}(p_i) = \lambda. \quad (9)$$

The solution of (9) is defined as $\nu_i(\lambda) = \frac{\lambda - \zeta_i}{2\xi_i}$, where $\nu_i(\cdot)$ is the inverse function of $(\partial J_i / \partial p_i)(\cdot)$. Therefore, the explicit form of g_i can be expressed by

$$g_i(\lambda) = J_i(\theta_i(\lambda)) + \lambda(d_i - \theta_i(\lambda)), \quad (10)$$

where θ_i is defined as

$$\theta_i(\lambda) = \begin{cases} p_{i \min}, & \lambda < \frac{\partial J_i}{\partial p_i}(p_{i \min}) \\ \nu_i(\lambda), & \frac{\partial J_i}{\partial p_i}(p_{i \min}) \leq \lambda \leq \frac{\partial J_i}{\partial p_i}(p_{i \max}) \\ p_{i \max}, & \frac{\partial J_i}{\partial p_i}(p_{i \max}) < \lambda \end{cases}. \quad (11)$$

Therefore, constraint (3) is considered in auxiliary variable (11). Then, the duality gap is always zero by Proposition 6.4.3 in [Bertsekas et al. (2003)], provided that one of the primal or dual problems is feasible. Finally, the Lagrange dual problem of (1) can be obtained as a form of an unconstrained optimization problem as

$$\max_{\lambda} g(\lambda) = \sum_{i=1}^N g_i(\lambda). \quad (12)$$

The gradient of the Lagrange dual problem can be obtained as follows:

$$\nabla g(\lambda) = \sum_{i=1}^N \frac{\partial g_i(\lambda)}{\partial \lambda} = \sum_{i=1}^N (d_i - \theta_i(\lambda)). \quad (13)$$

The explicit Lagrangian dual function in $\lambda \in [\frac{\partial J_i}{\partial p_i}(p_{i \min}), \frac{\partial J_i}{\partial p_i}(p_{i \max})] = S$ can be obtained as follows:

$$g(\lambda) = \sum_{i=1}^N \left(-\frac{\lambda^2}{4\xi_i} + \left(\frac{\zeta_i}{2\xi_i} + d_i \right) \lambda - \frac{\zeta_i^2}{4\xi_i} + \eta_i \right). \quad (14)$$

The dual function, $g(\lambda)$, is strictly concave, because the second partial derivative of $g(\lambda)$ with respect to λ is always negative such as

$$\frac{\partial^2 g(\lambda)}{\partial \lambda^2} = - \sum_{i=1}^N \left(\frac{1}{2\xi_i} \right) < 0, \quad \because \xi_i > 0. \quad (15)$$

Some helpful lemmas using the strictly concave property of $g(\lambda)$ are presented for the convergence analysis hereafter.

Lemma 2. Suppose that there is the optimal solution λ^* to (12). Define the error such that $\varepsilon = \lambda - \lambda^*$. The sign of ε is not equal to the sign of ∇g as follows:

$$\text{sign}(\varepsilon) \text{sign}(\nabla g) < 0, \quad \forall \varepsilon \neq 0. \quad (16)$$

Proof. As $g(\lambda)$ is concave and differentiable, the optimal value is bounded using the first-order Taylor approximation as follows:

$$g(\lambda^*) < g(\lambda) + \nabla g(\lambda^* - \lambda) = g(\lambda) - \nabla g \varepsilon, \quad (17)$$

where $g(\lambda^*)$ is the maximum value. Then, (17) can be rewritten as

$$\nabla g \varepsilon < g(\lambda) - g(\lambda^*) < 0, \quad \forall \varepsilon \neq 0. \quad (18)$$

◇

Lemma 3. The error between λ and any optimal value, λ^* , is defined such that $\varepsilon = \lambda - \lambda^*$, and ε is bounded in terms of $|\nabla g|$ as follows:

$$|\varepsilon| \leq \frac{2}{m} |\nabla g(\lambda)|, \quad \forall \lambda \in S, \quad (19)$$

where m is a positive constant as the upper bound of $\nabla^2 g(\lambda)$ such that $\nabla^2 g(\lambda) \leq -m$.

Proof. Because $g(\lambda) \in \mathbb{R}$ is strictly concave, there exists an $m > 0$ such that

$$\nabla^2 g(\lambda) \leq -m, \quad \forall \lambda \in S. \quad (20)$$

For $\lambda, \lambda^* \in S$, we can obtain the following equation [Ch.9.1.2, Boyd et al. (2004)]:

$$g(\lambda^*) \leq g(\lambda) + \nabla g(\lambda^* - \lambda) + \frac{1}{2}(\lambda^* - \lambda)^2 \nabla^2 g(z), \quad (21)$$

for some z on the line segment $[\lambda, \lambda^*]$. From (20), we can obtain the following inequality:

$$g(\lambda^*) \leq g(\lambda) + \nabla g(\lambda^* - \lambda) - \frac{m}{2}(\lambda^* - \lambda)^2. \quad (22)$$

As $g(\lambda^*) - g(\lambda) > 0$, we can rewrite (22) as follows:

$$\frac{m}{2}(\lambda^* - \lambda)^2 \leq \nabla g(\lambda^* - \lambda) \leq |\nabla g| |\lambda^* - \lambda|. \quad (23)$$

Therefore, (23) can be rearranged as follows:

$$|\varepsilon| \leq \frac{2}{m} |\nabla g|, \quad \because |\lambda - \lambda^*| = |\lambda^* - \lambda| = |\varepsilon|. \quad (24)$$

◇

The PTC optimal solution to (1) is proposed as the following theorem.

Theorem 1. Consider the feasible RTEDP, the PTC solution is designed to obtain the optimal solution in PT at finite time interval $t \in T = [0, t_f]$ as follows:

$$\dot{\lambda}(t) = \frac{\mu(t)(e^{\gamma|\nabla g(\lambda(t))|} - 1)}{e^{\gamma|\nabla g(\lambda(t))|}} \sigma(t) \text{sign}(\nabla g(\lambda(t))), \quad (25)$$

where $\gamma > \sqrt{2/m}$ is a positive design constant, $\mu(t) = \sqrt{((\gamma|\nabla g(\lambda(t))|)^2 + \gamma^2)}$ is the adaptive learning rate such that $\inf_t(\mu(t)) = \gamma$, and t_f is the solution convergence time independent of the initial conditions, and $\sigma(t) = \frac{1}{t_f - t}$. Suppose there is the optimal solution, λ^* . Define the error such that $\varepsilon = \lambda - \lambda^*$. If the Lyapunov function satisfies these conditions in finite time interval $T = [0, t_f]$ as follows:

$$\begin{aligned} V_o(t, \varepsilon) &= \frac{\gamma m}{2} |\varepsilon|, \quad \forall t \in T \\ \dot{V}_o(t, \varepsilon) &\leq \frac{k(e^{V_o(t, \varepsilon)} - 1)}{e^{V_o(t, \varepsilon)}} \sigma(t), \quad \forall V_o(t, \varepsilon) \neq 0, \quad \forall t \in T, \end{aligned} \quad (26)$$

where m is a positive constant as the upper bound of $\nabla^2 g(\lambda)$ such as $\nabla^2 g(\lambda) \leq -m$, the proposed solution (25) converges to equilibrium point $\varepsilon = 0$ within time t_f .

Proof. The Lyapunov candidate function is defined as

$$V_o(t, \varepsilon) = \frac{\gamma m}{2} |\varepsilon|, \quad \forall t \in T. \quad (27)$$

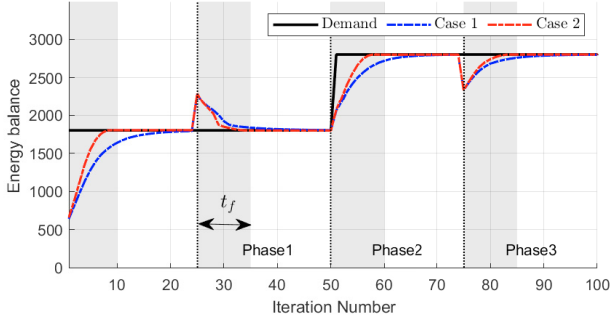
The derivative of $V(t, \varepsilon)$ can be expressed using proposed solution (25) as follows:

$$\dot{V}_o(t, \varepsilon) = \frac{m\gamma\mu}{2} \text{sign}(\varepsilon) \text{sign}(\nabla g(\lambda)) \frac{(e^{\gamma|\nabla g(\lambda)|} - 1)}{e^{\gamma|\nabla g(\lambda)|}} \sigma(t). \quad (28)$$

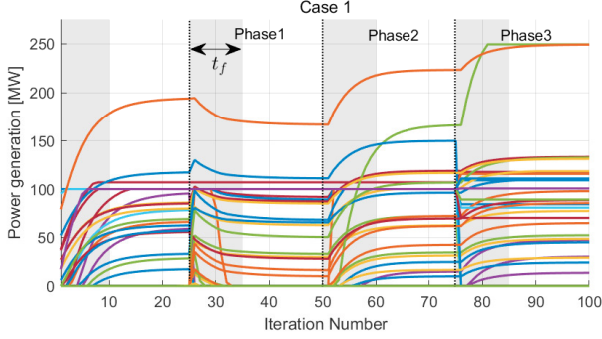
Therefore, (28) can be rewritten by Lemma 2 as follows:

$$\dot{V}_o(t, \varepsilon) \leq -\frac{k(e^{\gamma|\nabla g(\lambda)|} - 1)}{e^{\gamma|\nabla g(\lambda)|}} \sigma(t), \quad (29)$$

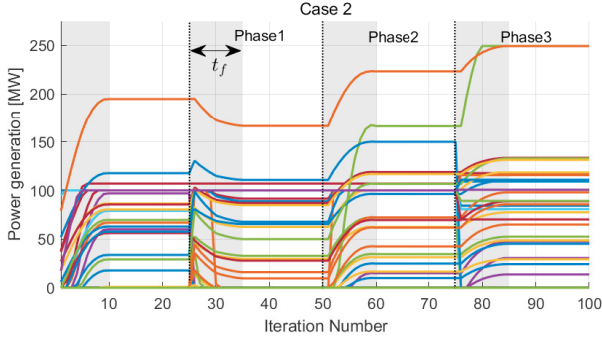
where k is a positive constant such that $k = \frac{m\gamma^2}{2}$, because $\inf_t(\mu(t)) = \gamma$. From Lemma 3, (29) can be obtained as follows:



(a) Demand-supply energy balance



(b) Power generation of 118 nodes: Case 1



(c) Power generation of 118 nodes: Case 2 (proposed method)

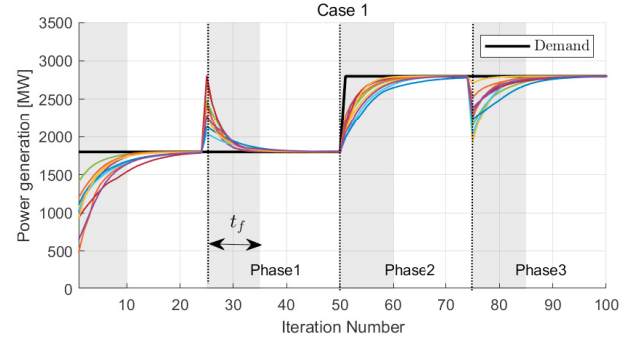
Fig. 1. Comparison results of IEEE 118 bus simulation

$$\dot{V}_o(t, \varepsilon) \leq -\frac{k(e^{V_o(t, \varepsilon)} - 1)}{e^{V_o(t, \varepsilon)}} \sigma(t). \quad (30)$$

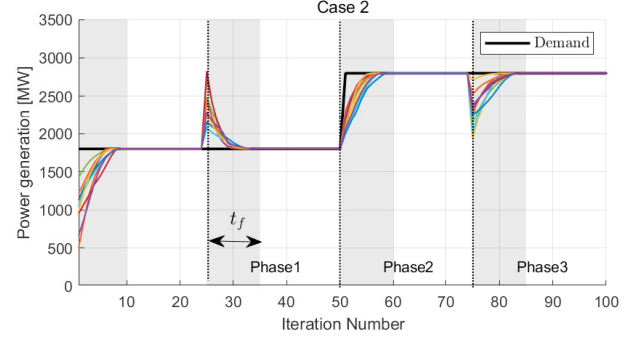
If $\gamma > \sqrt{2/m}$, then $k > 1$. As (30) satisfies the condition of the PTC dynamics from Lemma 1, $V_o(t, \varepsilon)$ becomes zero at some time in $t \leq t_f$. Therefore, it is evident that $V_o(t, \varepsilon) = 0$ implies $\lambda = \lambda^*$, and the proposed solution converges to the optimal solution in prior-selected convergence time t_f . $\diamond$

4. NUMERICAL SIMULATION RESULTS : IEEE 118 BUS SYSTEM

In this section, the RTEDP is considered to validate the proposed method using an IEEE 118 bus with 54 generators, and 91 loads. The Local objective function for each generator is represented as $J_i(p_i) = \xi_i p_i^2 + \zeta_i p_i + \eta_i$, whose coefficients are $\eta_i \in [6.78, 74.33]$, $\zeta_i \in [8.3391, 37.6968]$, and $\xi_i \in [0.0024, 0.0679]$. The initial value of solution $\lambda(0)$ is selected considering feasible generation range such that $\lambda(0) = 2\eta_i(p_{\max} + p_{\min}) + b_i \in [\lambda_{\min}, \lambda_{\max}]$. The sampling time is $T_s = 0.001$ s. The RTEDP redefined



(a) Energy balance: Case 1



(b) Energy balance: Case 2

Fig. 2. Comparison results of the 10 iterative simulations for a random cost function coefficient and initial condition

period is $T_p = 0.025$ s. The prior-selected convergence time is selected as $t_f = 0.01$ s. Therefore, the optimal solution should be obtained in 10 iterations in a discrete implementation environment because $t_f/T_s = 10$. The time t to obtain the solution is periodically reset to zero at each T_p . The scenarios consisted to show that the proposed algorithm successfully obtains the optimal solution in PT under online variations, including load demands, generation cost coefficients, max constraints, and plug-off of generators through the following phases:

- Phase 1 ($t = 0.025s$): Generation costs coefficient variation
Generation cost coefficients are randomly changed: ξ_i by $[-10 \ 10]\%$, ζ_i by $[-40 \ 40]\%$, η_i by $[-30 \ 30]\%$;
- Phase 2 ($t = 0.05s$): Load demand increase
Total power demands increase $1800\text{MW} \rightarrow 2800\text{MW}$;
- Phase 3 ($t = 0.075s$): Generator capacity variation and plug-off of generators
The upper power limits of each generator are randomly changed by $[-20 \ 20]\%$. Generators 6, 7, 12, 13, 29, 30, 31, 38, 41, and 46 are plugged off.

The following two cases were investigated to evaluate the performance of the proposed method:

- **Case 1: Adaptive gradient-descent optimization algorithm [Dong et al. (2021)]**
The adaptive gradient-descent optimization solution was used in this case as follows:

$$\begin{aligned} \dot{\lambda} &= \frac{\eta_1}{\sqrt{b} + \epsilon} \nabla g(\lambda) \\ \dot{b} &= \gamma_1 (\nabla g(\lambda))^2, \end{aligned} \quad (31)$$

where $\eta_1 = 1$, $\gamma_1 = 0.1$, and $\epsilon = 1$ are the design constant parameters, which are well tuned for fair comparison.

• Case 2: PTC optimization algorithm

The design constant parameter $\gamma = 0.1$ was set to satisfy the design condition such that $\gamma > 0.071 = \sqrt{2/m}$. Additionally, the prior-selected convergence time was set to $t_f = 0.01$ s.

The numerical simulation results using the IEEE 118 bus system are shown in Fig. 1. In both cases, the demand-supply balance was well satisfied. In the case 2 (proposed method), we demonstrate that the solution was successfully solved in a predefined convergence $t_f = 0.01$ s, which was set to $N = 10$ iterations by simulation sampling time $T_s = 0.001$ s. When RTEDP was periodically redefined each constant interval time $T_p = 0.025$ s, which was set 25 iterations by simulation sampling time $T_s = 0.001$ s. The each optimal generated power level was also obtained in $N = 10$ iterations under different online varying situations, and converged to same power level in both cases, as shown in Fig. 1 (c). Therefore, the proposed method demonstrates that it can handle the rapid variation for uncertainties of grid in the predefined brief convergence time. The comparison results of the 10 iterative simulations for the random cost function coefficient and initial conditions are shown in Fig. 2. We can observe that the convergence time changed according to the random initial conditions and phase at for each simulation in Fig. 2 (a). On the other hand, the proposed method successfully achieved the optimal solution in 10 iteration numbers regardless of the random cost function coefficient and initial condition for all 10 iterative simulations, as shown in Fig. 2 (b).

5. CONCLUSIONS

In this paper, we proposed a PTC optimization algorithm for RTEDP operation. The PTC optimization solution was designed to obtain the optimal solution to RTEDPs in PT, which is independent of initial conditions and any problem parameter. In addition, the proposed solution was simply designed using only one tuning parameter in adaptive learning rate. From the simulation results using an IEEE 118 bus grid, the proposed solution successfully obtained the optimal generation level in PT under various situations such as constraint variations, load variations, and generator plug-off, and also achieved the optimal solution in PT regardless of the random cost function coefficient and initial conditions in 10 iterative simulations.

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