

# Semi-Analytical Model for Predicting Roll Forces in the Finishing Stage of H-Beam Rolling and Its Application to Determine Setup Roll Gap

Seong-Gi KIM,<sup>1)</sup> Seok-Eui LEE,<sup>1)</sup> Chang-Hyun WEE,<sup>2)</sup> Hyun-Seok JEONG,<sup>2)</sup> Jong-Eun LEE,<sup>2)</sup> Kyung-Ho SEO,<sup>2)</sup> Sang-Min BYON<sup>3)\*</sup> and Youngseog LEE<sup>1)\*</sup>

1) Department of Mechanical Engineering, Chung-Ang University, Seoul, 06974 Korea.

2) Hyundai Steel, Chungnam, 167-32 Korea.

3) Department of Mechanical Engineering, Dong-A University, Busan, 49315 Korea.

(Received June 24, 2025; Accepted November 11, 2025; Advance online published November 20, 2025; Published January 15, 2026)

This study presents a semi-analytical model to accurately predict roll force and setup roll gap ( $G_{set}$ ) in the finishing stage of H-beam rolling. The model modifies Sims' classical strip rolling formula to capture the unique geometric and deformation characteristics of H-beam sections. The cross-section is divided into web and flange regions using a rectangular approximation. A geometry-dependent correction function is then introduced to account for the effects of reduction ratio, reduction balance, and contact geometry. This function is calibrated through finite element (FE) simulations under varying rolling conditions. Parameters such as web and flange thickness reduction, junction radius, and flange inclination are systematically adjusted, and the resulting roll forces are used to optimize the function via the Nelder–Mead method.

The model is validated using two H-beam specifications from Hyundai Steel. Compared to the company's empirical model, the proposed model reduces web and flange thickness deviations by up to 71%, demonstrating improved prediction accuracy. Statistical analysis also shows substantial reductions in standard deviation and mean error, confirming better dimensional control. This semi-analytical model provides a fast, reliable alternative to full FE simulations and manual adjustment procedures, improving both efficiency and product quality in industrial H-beam rolling.

KEY WORDS: H-beam; finishing rolling mill; roll force; semi-analytical model; setup roll gap.

## 1. Introduction

The H-beam rolling process is used to manufacture H-shaped structural beams with flanges and a web. These beams are widely used in construction and engineering because of their high strength-to-weight ratio, as discussed by Davison and Owens.<sup>1)</sup> This process involves multiple stages of rolling, utilizing universal rolling mills equipped with both horizontal and vertical rolls to shape the beam's cross-section.

The process starts with a beam blank, a semi-finished steel product produced via continuous casting with a near-net-shape cross section approximating the H-beam profile.<sup>2)</sup> Unlike a bloom or billet, which is nearly rectangular, the beam blank's shape is specifically designed to reduce rolling passes and improve dimensional accuracy in H-beam production. The beam blank first undergoes breakdown

rolling through horizontal rolls to roughly form the web and flanges into an H-like shape. It then passes through the universal rolling mill, where the horizontal rolls refine the web thickness and set the overall height, while the vertical rolls adjust the flange width and maintain the desired cross-sectional profile, as reported by Saiki and Matsuda.<sup>3)</sup>

Due to the relatively short length of H-beam products (about 60–90 m) compared to other rolled products like strip products (about 800 m–2 500 m), feedforward control, which relies on a predetermined setup roll gap ( $G_{set}$ ), is essential to meet the dimensional tolerance requirements of H-beams. Accurate determination of  $G_{set}$  in advance is crucial for engineers managing the rolling process because there isn't enough time for feedback control to adjust web and flange thicknesses effectively. The setup roll gap is adjusted from the design roll gap to compensate for mill elastic deformation under load. Accurate prediction of the roll force is therefore crucial for maintaining the dimensional accuracy of H-beam cross-sections.

\* Corresponding author: E-mail: smbyon@dau.ac.kr (S.-M. Byon)  
ysl@cau.ac.kr (Y. Lee)



The dimensional quality of H-beams is primarily determined by two factors: the straightness<sup>4)</sup> along the longitudinal direction and the accuracy of the cross-sectional dimensions. While the former is often affected by heat treatment<sup>5)</sup> or asymmetric rolling,<sup>6)</sup> the latter—cross-sectional dimensional accuracy, which is the main focus of this study—is strongly governed by the roll force during the rolling process. Few studies have been conducted on the prediction of the roll force applicable to H-beam rolling. Zhu *et al.*<sup>7)</sup> simulated the rolling process of a large H-beam at the Laiwu Iron & Steel Group in China by using a commercial finite element (FE) analysis program called LS-Dyna. They analyzed the metal flow, web cracking, cooling waves, residual stress, microstructure evolution, and roll force. He *et al.*<sup>8)</sup> developed mechanism-based constitutive models to investigate the effects of the rolling parameters on the microstructural evolution of Q235 steel in multiple passes of H-shaped rolling. They used the commercial FE code Abaqus and validated their models by comparing the calculated and measured roll forces. However, these studies have limitations in their application because they are based on FE analysis, which requires considerable time for modeling and running.

Zhang *et al.*<sup>9)</sup> used the upper-bound method to calculate the roll forces on horizontal and vertical rolls as well as the roll torque and energy consumption during rolling. They compared these predictions with the measurements from multiple passes but did not specify the experimental data source. Furthermore, the procedure for deriving the equations to calculate the roll forces is unclear, making it difficult to apply their equations to H-beam rolling processes. A survey of the literature reveals that most existing models are either FEM-based<sup>10–12)</sup> or derived from the upper-bound method,<sup>13,14)</sup> however, no analytical model has yet been developed that is practically applicable to actual rolling mills. This need for a more practical approach forms the foundation of the present work.

This study introduces a semi-analytical model that enables efficient and accurate prediction of roll forces and setup roll gaps ( $G_{set}$ ) in H-beam rolling. The rolling force prediction formula for strip rolling, first proposed by Sims,<sup>15)</sup> was modified for application to H-beam rolling. For this purpose, we divided the H-beam cross-section into the web and flange based on rectangular approximation, as proposed by Shinokura and Takai.<sup>16,17)</sup> In this framework, a geometry-dependent correct function  $Q_p$  was introduced to identify the effects of rolling geometry on roll force in H-beam rolling.

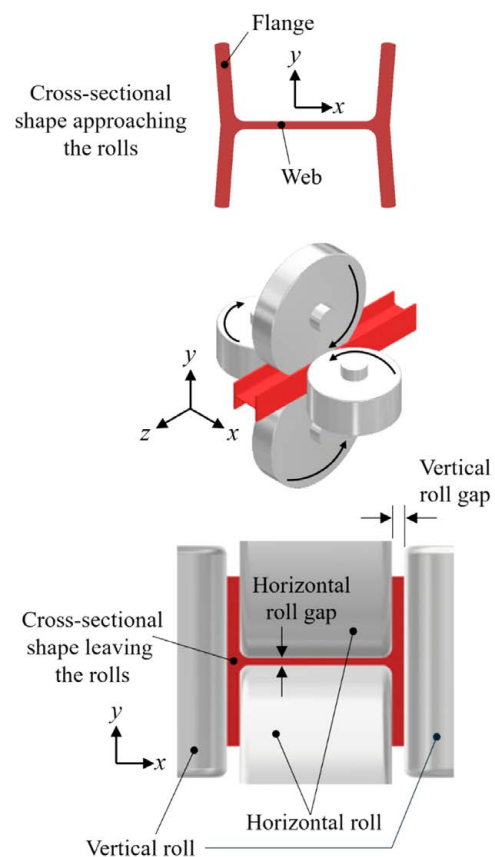
To determine the constants in the  $Q_p$ , Nelder–Mead (NM) method<sup>18)</sup> was employed through iterative optimization to minimize the loss function  $L$ , defined as the mean absolute percentage error. Based on the optimization results, the most appropriate form of  $Q_p$  was selected, thereby completing the development of a semi-analytical model for predicting roll forces in H-beam rolling.

The proposed semi-analytical model was applied to the finishing rolling mill of an H-beam production line at Hyundai Steel Company, Korea. Using predicted roll forces and the measured mill modulus,  $G_{set}$  values were calculated and compared with those from the company's empirical model to assess accuracy and practical utility.

## 2. H-beam Rolling Process

The H-beam rolling process consists of three main stages: breakdown, intermediate, and finishing. The breakdown-rolling stage consists of a pair of horizontal rolls, where the beam blank is shaped into a preliminary H-profile by passing through the engraved grooves on the rolls. By contrast, universal rolling (serving as the intermediate and finishing stages) utilizes both horizontal and vertical rolls. The pair of horizontal rolls reduces the web thickness of the material, whereas the pair of vertical rolls reduces the flange thickness, thus shaping the cross-section of the material. While the intermediate rolling mill primarily adjusts the web height and flange width to approach their target values,<sup>19,20)</sup> the finishing rolling mill straightens the bent flanges to a perpendicular orientation and rolls the web and flange to their final thicknesses. **Figure 1** illustrates the finishing rolling along with the cross-sectional shapes of the material at the entry and exit. This step ultimately shapes the dimensions and profile of the H-beam product, as required by the customer.

In this study, which focuses on the finishing rolling mill, roll forces measured for two specifications (A and B) at Hyundai Steel Company, Korea, were employed to validate the proposed model. Specification A corresponds to a section with 350 mm web height and 350 mm flange width, whereas Specification B corresponds to a section with 700 mm web height and 300 mm flange width. These two representative cases were selected to examine the predictive capability and robustness of the semi-analytical model across different cross-sectional aspect ratios.



**Fig. 1.** Schematic of horizontal and vertical rolls in the finishing rolling mill. (Online version in color.)

### 3. Semi-analytical Model for Predicting the Roll Force in H-beam Rolling

#### 3.1. Adaptation of Sims' Model for H-Beam Rolling

A commonly used model for computing the roll forces ( $F_{flat}$ ) in hot strip rolling is due to Sims.<sup>15)</sup>

$$F_{flat} = w \cdot k_m \cdot l_m \cdot Q_p \quad (1)$$

where  $w$  is the strip width and  $k_m$  is the average flow effective stress of the material (strip) being plastically deformed and it is typically obtained through hot compression or torsion tests using material specimens representative of the rolling conditions.<sup>21)</sup>  $l_m$  is the projected contact length between the roll and material, and  $Q_p$  is a geometry-dependent correction function expressed as follows:

$$Q_p = \frac{\pi}{2} \sqrt{\frac{1-r}{r}} \tan^{-1} \left( \sqrt{\frac{r}{1-r}} \right) - \frac{\pi}{4} - \sqrt{\frac{1-r}{r}} \sqrt{\frac{R}{t_{out}}} \ln \left( \frac{t_n}{t_{out}} \right) + \frac{1}{2} \sqrt{\frac{1-r}{r}} \sqrt{\frac{R}{t_{out}}} \ln \left( \frac{1}{1-r} \right) \quad (2)$$

$r = (t_{in} - t_{out}) / t_{in}$  represents the thickness reduction ratio, where  $t_{in}$  and  $t_{out}$  denote the incoming and outgoing thickness of material, respectively.  $R$  is the roll radius, and  $t_n$  represents the strip thickness at the neutral point.

Considering the difference between the thickness reduction in strip rolling and that in H-beam rolling, where two connected regions (web and flange) deform simultaneously, this study adapts the Sims' model by dividing the H-beam cross-section into web and flange regions using a rectangular approximation and by modifying  $Q_p$  to account for the concurrent thickness reductions in both regions. In the following sections, the H-beam cross-section is divided into web and flange regions using a rectangular approximation, and the form of  $Q_p$  and its parameters are determined to make the model applicable to the finishing stage of the H-beam rolling process.

#### 3.2. Partition of H-shaped Cross-section Based on Rectangular Approximation

Figure 2(a) shows the overall cross-sectional shape of the H-beam. Because the H-shaped cross-section is symmetrical both vertically and horizontally, only one-quarter of the section was used for the analysis. Figure 2(b) enlarges this one-quarter section and represents its geometric features using symbols. The meanings of the symbols are listed in Table 1.

Figure 3 shows the cross-sectional features obtained by dividing the web and flange. First, the coordinates of point A were determined to facilitate the division between the web and flange. Point A serves as the reference point for this division, and its coordinates are expressed by Eqs. (3) and (4). It is important that point A is located below the center of the junction radius ( $R_y > A_y$ ), as shown in Fig. 2(b).

$$A_x = H_{in} / 2 \quad (3)$$

$$A_y = R_y - \sqrt{R_{juc}^2 - (A_x - R_x)^2} \quad (4)$$

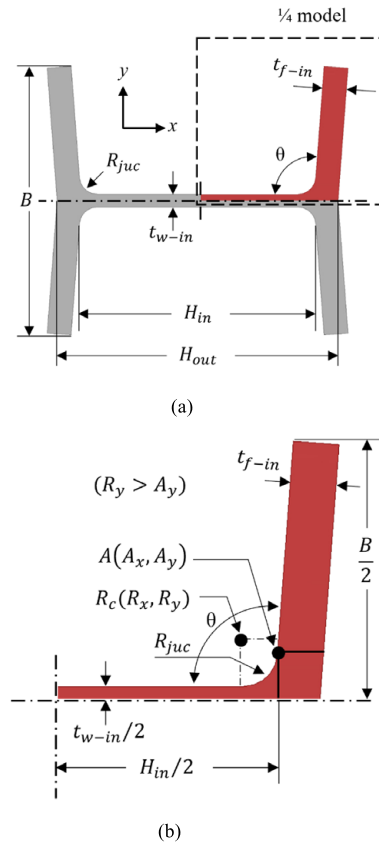


Fig. 2. Geometrical specifications of the H-beam approaching the finishing rolling. (a) Overall cross-sectional shape and (b) enlarged view of 1/4 cross-section. Symbolic parameters are described in Table 1. (Online version in color.)

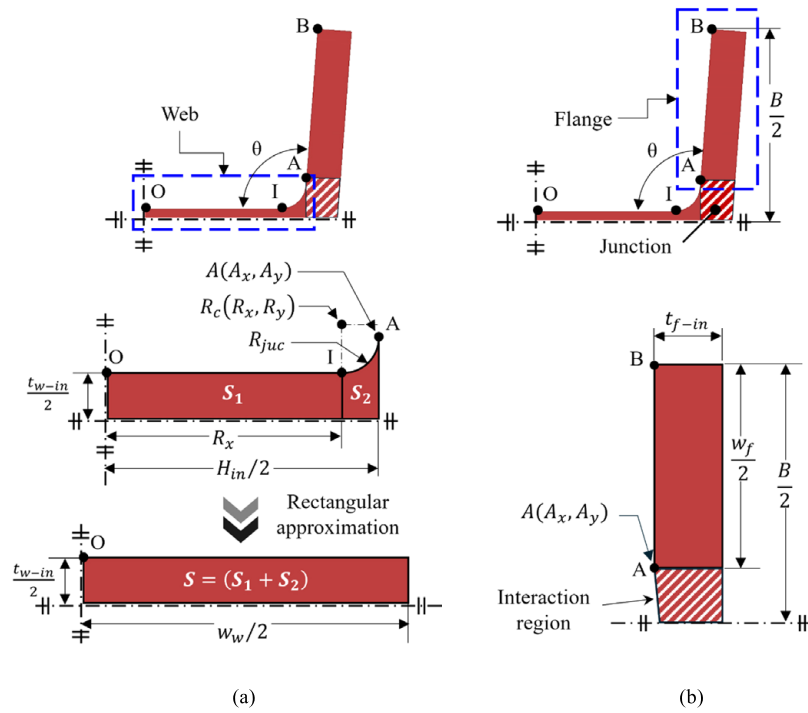
Table 1. Description of symbols used to define the geometry of the H-beam cross-section.

| Symbol     | Meaning                                                      |
|------------|--------------------------------------------------------------|
| $t_{w-in}$ | Incoming web thickness                                       |
| $t_{f-in}$ | Incoming flange thickness                                    |
| $R_{juc}$  | Radius at junction round between web and flange              |
| $R_x, R_y$ | Center coordinates of junction round                         |
| $A_x, A_y$ | Coordinates of the intersection point between web and flange |
| $\theta$   | Inclination angle                                            |
| $H_{in}$   | Web inner height                                             |
| $H_{out}$  | Web outer height                                             |
| $B$        | Flange width                                                 |

$$R_x = \frac{H_{in}}{2} - \frac{R_{juc}^2 \tan^2 \left( \pi - \frac{\pi\theta}{180} \right)}{\tan^2 \left( \pi - \frac{\pi\theta}{180} \right) + 1} \quad (5)$$

$$R_y = t_{w-in} / 2 + R_{juc} \quad (6)$$

The web section rolled by horizontal rolls can be defined as the O-I-A region, as shown in Fig. 3(a). The irregular shape below the O-I-A region can be transformed into a rectangular area. In other words, the sum of areas  $S_1$  and  $S_2$  is converted into a single rectangular area ( $=S_1+S_2$ ). The areas



**Fig. 3.** Partitions of cross-section of H-beam by rectangular approximation. (a) Transformed width of the web and (b) transformed width of the flange. (Online version in color.)

$S_1$  and  $S_2$  are expressed as Eqs. (7) and (8), respectively.

$$S_1 = R_x (t_{w-in} / 2) \dots\dots\dots (7)$$

$$S_2 = \int_{R_x}^{H_{in}/2} R_y - \sqrt{R_{juc}^2 - (x - R_x)^2} dx \dots\dots\dots (8)$$

Therefore, the transformed width of the web ( $w_w$ ) can be calculated by dividing the sum of the areas  $S_1$  and  $S_2$  by  $t_{w-in}/2$ , as shown in Eq. (9).

$$w_w = 4(S_1 + S_2) / t_{w-in} \dots\dots\dots (9)$$

In the finishing stage of H-beam rolling,  $w_w$  is typically about 10% larger than  $H_{in}$ , due to the geometric transformation from the web inner height to the equivalent transformed web width.

The flange, rolled by the vertical rolls, is defined as the rectangular region A-B, as shown in Fig. 3(b). Therefore, the width of the flange ( $w_f$ ) is determined by the difference between the width  $B$  and vertical coordinate of point A ( $A_y$ ), as expressed in Eq. (10).

$$w_f = 2(B / 2 - A_y) \dots\dots\dots (10)$$

The hatched area represents the interaction region where the effects of both the horizontal and vertical rolls are applied simultaneously. The size of this intersection region varies depending on the relative difference in thickness reduction ratios between the web and flange. Therefore, a parameter composed of the two reduction ratios (*i.e.*, the thickness reduction ratios of the web and flange) was included in the geometry-dependent correction function to ensure that the effect of this area is reflected in the roll force estimation.

### 3.3. Geometry-dependent Correction Function $Q_p$ Suitable for H-beam Rolling

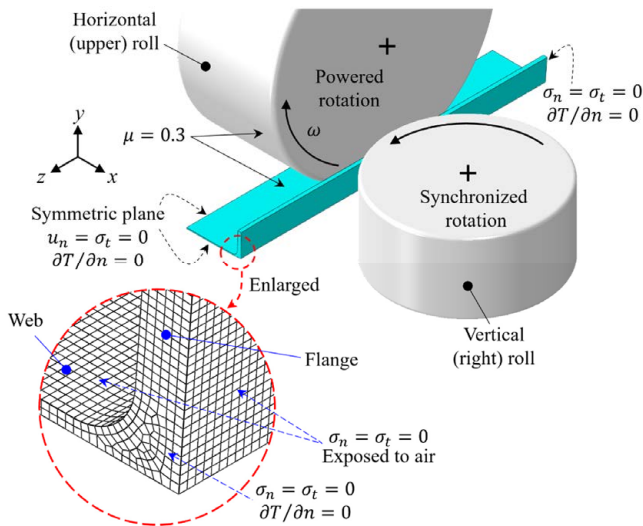
To develop the  $Q_p$  suitable for H-beam rolling, *i.e.* extended version of  $Q_p$  for H-beam rolling, we identified the reduction ratios of the web and flange thicknesses, the

junction radius, and the flange inclination angle as principal geometric parameters governing deformation behavior. To evaluate their influence on roll force and to calibrate the proposed function, a series of finite element (FE) analyses were performed to simulate H-beam rolling under various conditions, with the parameters systematically varied to assess their individual and combined effects.

**Figure 4** shows the boundary conditions and mesh used in the FE analysis of H-beam rolling. The meanings of the symbols are listed in **Table 2**. Because the material was symmetrical both vertically and horizontally, a 1/4 model was employed by assuming the horizontal and vertical rolls as rigid bodies. The boundary conditions for the symmetrical planes of the material were set as  $u_n=0$  for normal displacement and  $\sigma_t=0$  for tangential traction. Additionally, a no-load condition was applied at both ends of the material, as no external traction ( $\sigma_n=\sigma_t=0$ ) was applied.

At the boundary where the rolls and material come in contact, Coulomb's law of friction was applied with a friction coefficient of  $\mu=0.3$ . For the H-beam rolling conditions considered in this study, an average value at the roll bite was adopted, resulting in a friction coefficient of 0.3. This value reflects typical hot rolling conditions and aligns with previous findings in the literature.<sup>22)</sup> The horizontal rolls acted as the driving rolls and were given a rotational angular velocity. The vertical rolls were driven by friction as the material passed through the stand. The initial temperature of the material was set to 900°C and it was assumed to be uniform throughout the material. The flow stress  $\bar{\sigma}$  was determined using the formula proposed by Shida.<sup>23)</sup> The mesh type used for the analysis was C3D8RT (8-node trilinear displacement and temperature, reduced integration) with 204 000 elements. The commercial software Abaqus (2017) was used to perform the FE analysis.

**Figure 5(a)** shows the roll forces values measured by two



**Fig. 4.** Finite element mesh and mechanical/thermal boundary conditions assigned to the H-beam. Symbolic parameters are described in Table 2. (Online version in color.)

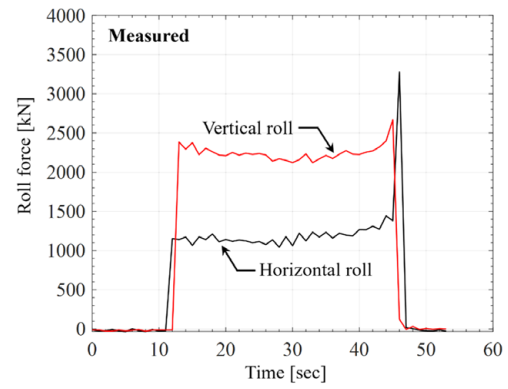
**Table 2.** Description of symbols used in finite element analysis.

| Symbol                    | Meaning                                        |
|---------------------------|------------------------------------------------|
| $u_n$                     | Normal displacement                            |
| $\sigma_n$                | Normal surface traction                        |
| $\sigma_t$                | Tangential surface traction                    |
| $\partial T / \partial n$ | Temperature gradient                           |
| $\omega$                  | Angular velocity of horizontal roll            |
| $\mu$                     | Friction coefficient between roll and material |

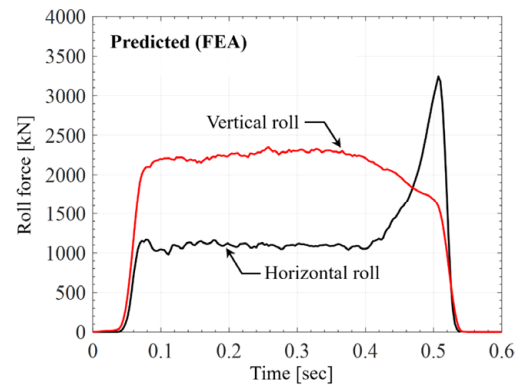
load cells installed on both sides of the chocks in the H-beam rolling mill. The roll force data were acquired at a sampling interval of approximately 1.0 sec. During the steady-state interval (approximately from 12 sec to 44 sec), the vertical roll force is about twice as large as the horizontal roll force. At the point where rolling ends (around 44 sec), the vertical roll force increases, while the horizontal roll force experiences a sharp rise. This increase is likely due to a sudden surge in the plastic deformation caused by the end effect.

FE analysis for predicting the roll force over the entire material length (74 m) could not be performed because of computing power limitations. Instead, a significantly shortened material length of 1 m, corresponding to a rolling duration of approximately 0.5 sec, was analyzed. In this way, the variations in the measured roll force in Fig. 5(a) were compared with the FE predictions in Fig. 5(b). In Fig. 5(b), the sudden increase in vertical roll force due to the end effect, as seen in Fig. 5(a), was not observed. The difference between the measured and predicted average roll forces during the steady-state interval was approximately 5% for the web (horizontal roll) and 4% for the flange (vertical roll). Therefore, the FE model and analysis for H-beam rolling were verified based on this comparison. We then conducted a series of FE analysis of H-beam rolling under different rolling conditions by varying rolling parameters and examined their impact on the roll force.

The rolling conditions used in the FE analysis of H-beam



(a)



(b)

**Fig. 5.** Comparison between measured and predicted roll forces during H-beam rolling. (a) Roll forces measured by two load cells installed on both sides of the chocks in the H-beam rolling mill. (b) FE-predicted roll force variations using a shortened material length of 1 m due to computational limitations. (Online version in color.)

rolling are listed in **Table 3**. A-1 represents the standard rolling condition and serves as the reference condition. From A-2 to A-9, each parameter was varied by  $\pm 1.5\%$  to the reference condition (A-1). Specifically,

- A-2 and A-3 relate to changes in the web thickness reduction ratio.
- A-4 and A-5 relate to changes in the flange thickness reduction ratio.
- A-6 and A-7 relate to changes in the junction radius reduction ratio.
- A-8 and A-9 relate to changes in the inclination angle reduction ratio.

**Figure 6** illustrates the variation in the roll force when the parameter values in Table 3 were adjusted to the reference condition (A-1). In Fig. 6(a), the changes in the roll forces of the web and flange are shown when the web thickness reduction ratio is varied by  $\pm 1.5\%$  from the reference condition. A key observation is that as the web thickness reduction ratio increases, the web roll force increases whereas the flange roll force decreases. Conversely, when the web thickness reduction ratio decreases, the roll force of the web decreases and roll force of the flange increases.

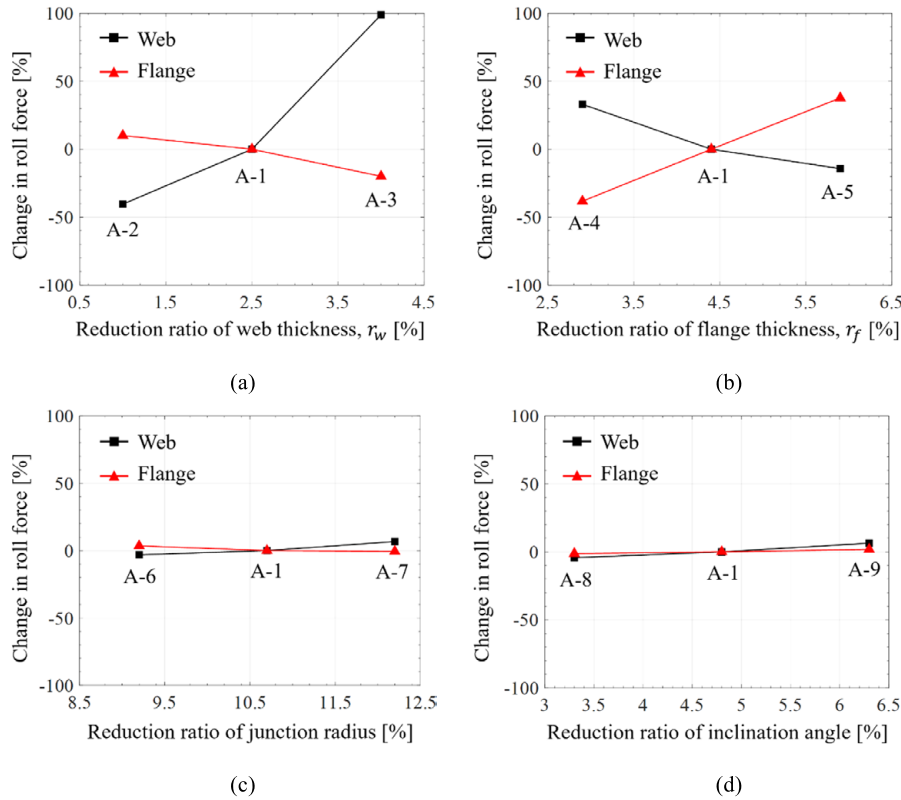
This phenomenon is similar to the changes in the flange thickness reduction ratio. As shown in Fig. 6(b), when the

**Table 3.** Rolling conditions and parameters used in FEA to investigate their effects on the roll force. The bold text indicates parameters and rolling conditions that have changed from the reference condition (A-1).

| Rolling condition | Dimension of incoming material section |                                     |                                   |                                   | Parameters                                 |                                               |                                        |                                          |
|-------------------|----------------------------------------|-------------------------------------|-----------------------------------|-----------------------------------|--------------------------------------------|-----------------------------------------------|----------------------------------------|------------------------------------------|
|                   | Web thickness<br>$t_{w-in}$ [mm]       | Flange thickness<br>$t_{f-in}$ [mm] | Junction radius<br>$R_{juc}$ [mm] | Inclination angle $\theta$ [deg.] | Reduction ratio of web thickness $r_w$ [%] | Reduction ratio of flange thickness $r_f$ [%] | Reduction ratio of junction radius [%] | Reduction ratio of inclination angle [%] |
| A-1 (Ref.)        | 13.35                                  | 24.12                               | 31                                | 94.3                              | 2.5                                        | 4.4                                           | 10.7                                   | 4.8                                      |
| A-2               | <b>13.15</b>                           | 24.12                               | 31                                | 94.3                              | <b>1.0</b>                                 | 4.4                                           | 10.7                                   | 4.8                                      |
| A-3               | <b>13.56</b>                           | 24.12                               | 31                                | 94.3                              | <b>4.0</b>                                 | 4.4                                           | 10.7                                   | 4.8                                      |
| A-4               | 13.35                                  | <b>23.79</b>                        | 31                                | 94.3                              | 2.5                                        | <b>2.9</b>                                    | 10.7                                   | 4.8                                      |
| A-5               | 13.35                                  | <b>24.55</b>                        | 31                                | 94.3                              | 2.5                                        | <b>5.9</b>                                    | 10.7                                   | 4.8                                      |
| A-6               | 13.35                                  | 24.12                               | <b>30.8</b>                       | 94.3                              | 2.5                                        | 4.4                                           | <b>9.2</b>                             | 4.8                                      |
| A-7               | 13.35                                  | 24.12                               | <b>31.9</b>                       | 94.3                              | 2.5                                        | 4.4                                           | <b>12.2</b>                            | 4.8                                      |
| A-8               | 13.35                                  | 24.12                               | 31                                | <b>93.0</b>                       | 2.5                                        | 4.4                                           | 10.7                                   | <b>3.3</b>                               |
| A-9               | 13.35                                  | 24.12                               | 31                                | <b>96.0</b>                       | 2.5                                        | 4.4                                           | 10.7                                   | <b>6.3</b>                               |

a) Web and flange thicknesses of outgoing material are fixed at 13.02 mm and 23.1 mm, respectively.

b) Junction radius and inclination angle of outgoing material are fixed at 28 mm and 90°, respectively.

**Fig. 6.** Variation in roll forces in the web and flange for each rolling condition under variation in the rolling parameters. (a) Reduction ratio of web thickness, (b) reduction ratio of flange thickness, (c) reduction ratio of junction radius, and (d) reduction ratio of inclination angle. (Online version in color.)

flange thickness reduction ratio increases or decreases from the reference condition, the roll force of the flange increases or decreases accordingly, whereas the web roll force exhibits the opposite behavior. This interaction is attributed to the crossflow in the plastic deformation between the web and flange.<sup>24,25)</sup>

Crossflow refers to the movement of plastic flow that is not confined to either the web or flange but instead flows between them. The direction of this flow is primarily determined by the dimensions of the web and flange and their respective thickness reduction ratios. If the reduction ratios are not set properly during the roll groove design stage,

wave-shaped instabilities may occur in the web.<sup>26,27)</sup>

From this, it becomes evident that an additional parameter is required to account for the interaction between the web and flange. To address this, we incorporated the reduction balance ( $\Delta\lambda$ ) proposed by Takashima and Yanagimoto,<sup>24)</sup> which represents the difference in thickness strain between the flange and web, as shown in Eq. (11). This variable reflects the interaction between the two sections.

$$\Delta\lambda = \ln\left(\frac{t_{f-in}}{t_{f-out}}\right) - \ln\left(\frac{t_{w-in}}{t_{w-out}}\right) = \ln\left(\frac{1-r_w}{1-r_f}\right) \dots\dots (11)$$

Here,  $r_w$  and  $r_f$  represent the thickness reduction ratios of

the web and flange, respectively. Compared with the reference rolling condition A-1, the roll force deviation between the web and flange increased by an average of 84% when the reduction balance was increased by approximately 79% (conditions A-2 and A-5). Conversely, when the reduction balance was decreased by about 79% (conditions A-3 and A-4), the roll force deviation was reduced by about 58%. These results indicate that a larger reduction imbalance leads to greater roll force deviation, which in turn exerts a stronger influence on achieving the required thickness quality.

Figures 6(c) and 6(d) illustrate the changes in the roll force resulting from variations in the junction radius and inclination angle reduction ratios. It was observed that these two parameters had a smaller influence on the roll force than those of the web and flange thickness reduction ratios. Consequently, the parameters  $r_w$ ,  $r_f$ , and  $\Delta\lambda$  were selected for inclusion in the geometry-dependent correction function  $Q_p$ .

As a result, the semi-analytical model for predicting the roll force,  $F_{H-beam}$  of the web or flange can be expressed as follows:

$$F_{H-beam} = w \cdot k_m \cdot l_m \cdot Q_p(r, \Delta\lambda) \quad (12-a)$$

where  $w$  represents the transformed width of the web or flange (calculated using Eq. (9) or (10)),  $k_m$  is the average flow stress of the material, and  $l_m$  is the projected contact length between the material and the rolls—horizontal rolls for the web and vertical rolls for the flange. These definitions are adapted from those in strip rolling to account for the specific geometry of H-beam rolling. The effect of roll radius,  $R$  on the roll force was not considered in the proposed semi-analytical model, as the model is specifically confined to the finishing rolling mill. Equation (12-a) can be consistently applied to H-beam rolling, providing horizontal roll force estimates through web-related parameters and vertical roll force estimates through flange-related parameters. In this equation,  $r$  should be set to  $r_w$  to predict the web roll force and to  $r_f$  to do the flange roll force.

To address the limitations of directly applying flat rolling models to H-beam rolling, an extended version of  $Q_p$  for H-beam rolling was introduced. This function enables the semi-analytical model to account for flange-to-web dimensional ratios, non-uniform strain distributions across the section, and additional frictional forces arising from inclined surfaces of the horizontal rolls. Consequently, Eq. (12-a) remains valid for estimating both vertical and horizontal roll forces in H-beam rolling. The explicit form of  $Q_p$  can be obtained by rewriting Eq. (12-a) as:

$$Q_p(r, \Delta\lambda) = \frac{F_{H-beam}}{w \cdot k_m \cdot l_m} \quad (12-b)$$

The right-hand side of Eq. (12-b) denotes the non-dimensional roll force. This indicates that if the roll force,  $F_{H-beam}$ , can be either measured or numerically estimated using FE analysis under various rolling conditions, the functional form of  $Q_p$  can be determined. This approach enables the development of a geometry-dependent correction function for  $Q_p$  that can be calibrated using measured or FE-simulated data and applied to different H-beam rolling scenarios.

### 3.4. Determining the form of $Q_p$ Suitable for H-beam Rolling

The non-dimensional roll force (right-hand side of Eq. (12-b)) was evaluated for various reduction ratios and reduction balances, and several functions exhibiting similar trends were selected to represent the behavior of  $Q_p$  as described below.

$$Q_p^I = (c_1 + c_2 \cdot r) \cot^{-1}(c_3 + c_4 \cdot r + c_5 \cdot \Delta\lambda) \quad (13)$$

$$Q_p^{II} = c_1 \cdot r^{c_2} + c_3 \cdot \Delta\lambda \quad (14)$$

$$Q_p^{III} = c_1 \cdot \exp\left(c_2 \sqrt{\frac{1-r}{r}} + c_3 \cdot \Delta\lambda\right) \quad (15)$$

Here,  $c_1, c_2, \dots, c_5$  are constants that can be determined using the roll forces calculated through FEA of H-beam rolling or from the roll forces measured from actual H-beam rolling mills. **Table 4** presents the rolling conditions used to determine these constants through FEA.

To select the most appropriate form of  $Q_p$  on a mathematical basis, we introduced a non-dimensional rolling load ( $Load_{nd}$ ) corresponding to Eq. (12-b) in the followings:

$$Load_{nd} = \frac{F_{fem}}{w \cdot k_m \cdot l_m} \quad (16)$$

where  $F_{fem}$  is the roll force obtained from the FEA. If the measured roll force ( $F_{measure}$ ) is used,  $F_{fem}$  in Eq. (16) is replaced by  $F_{measure}$ .

A comparison between Eqs. (12-b) and (16) reveals that the denominators  $w$ ,  $k_m$ , and  $l_m$  are identical. Therefore, by determining the constants for  $Q_p^I, Q_p^{II}, Q_p^{III}$  and ensuring  $Load_{nd}$  closely matches  $Q_p^{I,II,III}$ , we can identify the most suitable form of  $Q_p$  for H-beam rolling. To achieve this, we used the Nelder–Mead (NM) method,<sup>18)</sup> which determines the optimal constant values through iterative calculations that minimize the loss function  $L$ , defined as the mean absolute percentage error (MAPE):

$$L = \frac{1}{n} \sum_{i=1}^n \left| \frac{Load_{nd} - Q_p^{I,II,III}}{Load_{nd}} \right| \times 100 \quad (17)$$

where  $n$  represents the number of rolling conditions. The determined constants in this way are listed in **Table 5**.

We then selected the most suitable  $Q_p$  from the three candidates ( $Q_p^I, Q_p^{II}, Q_p^{III}$ ). **Figure 7** illustrates the change in the loss function  $L$  over the number of iterations. As the iterations progressed,  $L$  gradually decreases and eventually converges to a stable value. For the web, the loss function value for  $Q_p^{III}$  was the lowest at 19% (Fig. 7(a)). Similarly, for the flange, the loss function value for  $Q_p^{III}$  was the lowest at 9% (Fig. 7(b)). Thus,  $Q_p^{III}$  (Eq. (15)) was selected for further analysis. Given the wide parameter range (Table 4), an exact fit of the non-dimensional roll force with a single-form expression (Eq. (15)) is difficult; however, a deviation of 9–19% is considered acceptable for practical engineering use in hot H-beam rolling. The semi-analytical model for predicting the roll force in the finishing rolling stage of the H-beam rolling process is given by:

$$F_{H-beam} = w \cdot k_m \cdot l_m \cdot \left\{ c_1 \cdot \exp\left(c_2 \sqrt{\frac{1-r}{r}} + c_3 \cdot \Delta\lambda\right) \right\} \quad (18)$$

**Table 4.** Rolling conditions and parameters used in FEA to determine the constants for the geometry-dependent correction function  $Q_p$ .

| Rolling condition | Dimension of incoming mat'l      |                                     | Parameters                                |                                              |                                  |
|-------------------|----------------------------------|-------------------------------------|-------------------------------------------|----------------------------------------------|----------------------------------|
|                   | Web thickness<br>$t_{w-in}$ [mm] | Flange thickness<br>$t_{f-in}$ [mm] | Redc. ratio of web thickness<br>$r_w$ [%] | Redc. ratio of flange thickness<br>$r_f$ [%] | Redc. balance<br>$\Delta\lambda$ |
| B-1               | 13.15                            | 23.15                               | 1                                         | 0.2                                          | -0.008                           |
| B-2               |                                  | 23.63                               |                                           | 2.2                                          | 0.013                            |
| B-3               |                                  | 23.87                               |                                           | 3.2                                          | 0.023                            |
| B-4               |                                  | 24.12                               |                                           | 4.2                                          | 0.033                            |
| B-5               |                                  | 24.63                               |                                           | 6.2                                          | 0.054                            |
| B-6               | 13.35                            | 23.15                               | 2.5                                       | 0.2                                          | -0.023                           |
| B-7               |                                  | 23.63                               |                                           | 2.2                                          | -0.003                           |
| B-8               |                                  | 23.87                               |                                           | 3.2                                          | 0.008                            |
| B-9               |                                  | 24.12                               |                                           | 4.2                                          | 0.018                            |
| B-10              |                                  | 24.63                               |                                           | 6.2                                          | 0.039                            |
| B-11              | 13.56                            | 23.15                               | 4                                         | 0.2                                          | -0.039                           |
| B-12              |                                  | 23.63                               |                                           | 2.2                                          | -0.019                           |
| B-13              |                                  | 23.87                               |                                           | 3.2                                          | -0.008                           |
| B-14              |                                  | 24.12                               |                                           | 4.2                                          | 0.002                            |
| B-15              |                                  | 24.63                               |                                           | 6.2                                          | 0.023                            |
| B-16              | 13.78                            | 23.15                               | 5.5                                       | 0.2                                          | -0.055                           |
| B-17              |                                  | 23.63                               |                                           | 2.2                                          | -0.034                           |
| B-18              |                                  | 23.87                               |                                           | 3.2                                          | -0.024                           |
| B-19              |                                  | 24.12                               |                                           | 4.2                                          | -0.014                           |
| B-20              |                                  | 24.63                               |                                           | 6.2                                          | 0.007                            |

**Table 5.** Constants for the three types of functions ( $Q_p^I$ ,  $Q_p^{II}$ , and  $Q_p^{III}$ ) determined through FEA of H-beam rolling.

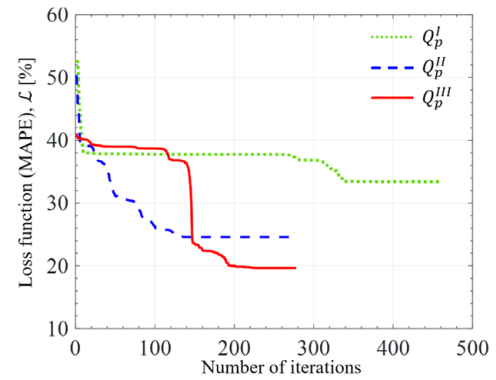
| Type of $Q_p$ | Constant |       |       |       |       |        |       |       |       |       |
|---------------|----------|-------|-------|-------|-------|--------|-------|-------|-------|-------|
|               | Web      |       |       |       |       | Flange |       |       |       |       |
|               | $c_1$    | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_1$  | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
| $Q_p^I$       | 2.0      | -11.3 | 0.5   | 12.9  | 22.9  | 3.3    | 0.1   | 0.5   | 4.8   | -7.2  |
| $Q_p^{II}$    | 1.5      | -0.1  | -41   | -     | -     | 1.8    | -0.2  | 3.9   | -     | -     |
| $Q_p^{III}$   | 1.7      | 0.02  | -21.3 | -     | -     | 1.2    | 0.1   | 17.7  | -     | -     |

## 4. Results and Discussion

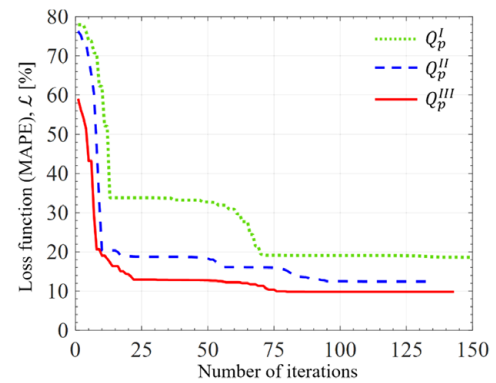
### 4.1. Validation of the Semi-analytical Model for Predicting the Roll Force, $F_{H-beam}$

We first determined the constants  $c_1$ ,  $c_2$ , and  $c_3$  in Eq. (18) using the measured roll force ( $F_{measure}$ ) and the NM method. The values of the determined constants are listed in Table 6. Next, we compared  $F_{measure}$  with  $F_{H-beam}$  (the predicted roll force) as shown in Figs. 8 and 9. The empty black circles represent  $F_{measure}$ , whereas the surface mesh illustrates  $F_{H-beam}$  calculated from Eq. (18).

In Fig. 8,  $F_{measure}$  for Specification A (350 mm web height and 350 mm flange width) agrees closely with the predicted surface mesh. This confirms that the proposed  $Q_p$  form and the NM-derived constants ( $c_1$ ,  $c_2$ ,  $c_3$ ) are valid for the finishing stage of H-beam rolling. The horizontal roll force applied to the web increases gradually as  $r_w$  increases and  $\Delta\lambda$  decreases, while the vertical roll force applied to the flange



(a)

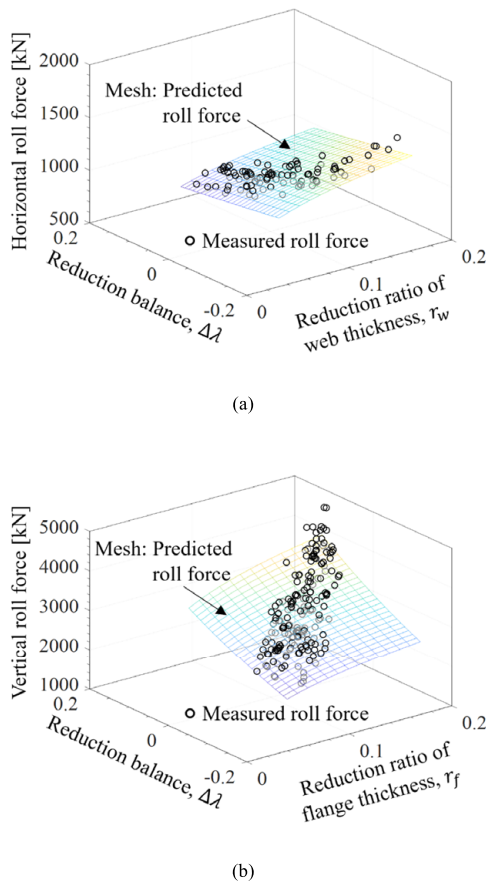


(b)

**Fig. 7.** Convergence of loss function for three types of geometry-dependent correction functions ( $Q_p^I$ ,  $Q_p^{II}$ , and  $Q_p^{III}$ ) as a function of the number of iterations. (a) Web and (b) flange. (Online version in color.)

**Table 6.** Constants of the geometry-dependent correction function  $Q_p$  for two H-beam product specifications, which were determined based on the measured roll forces from H-beam rolling mill.

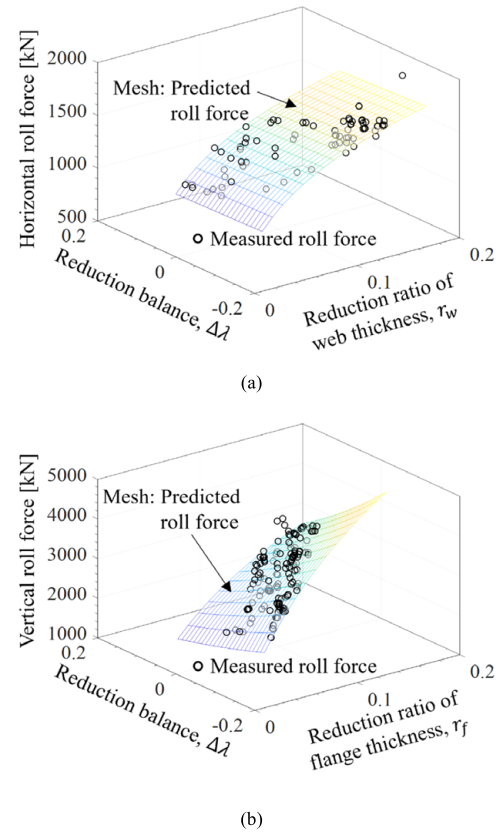
| Specification | Constant |       |       |        |       |       |
|---------------|----------|-------|-------|--------|-------|-------|
|               | Web      |       |       | Flange |       |       |
|               | $c_1$    | $c_2$ | $c_3$ | $c_1$  | $c_2$ | $c_3$ |
| A             | 0.12     | 0.41  | -0.19 | 0.45   | 0.37  | 1.71  |
| B             | 0.13     | 0.26  | 0.34  | 1.49   | 0.09  | -2.27 |



**Fig. 8.** Comparison of  $F_{measure}$  (measured roll force) and  $F_{H-beam}$  (roll force predicted from the proposed model) under varying parameters for specification A. (a) Web and (b) flange. (Online version in color.)

increases significantly as both  $r_f$  and  $\Delta\lambda$  increase. The effect of the reduction ratio ( $r_w$  or  $r_f$ ) on roll force was found to be similar for the web and flange, but the dependency on the reduction balance ( $\Delta\lambda$ ) was significantly higher for the flange.

In Fig. 9,  $F_{measure}$  also closely follows the surface mesh, confirming that the form of  $Q_p$  is appropriate for Specification B (700 mm web height and 300 mm flange width) as well. The horizontal roll force increases sharply with small increases in  $r_w$  but decreases slightly as  $\Delta\lambda$  increases. Similar trends are observed for the vertical roll force. When the web height and flange width are nearly equal, as in Specification A, the reduction balance more strongly affects the flange rolling force than in Specification B, where the web height greatly exceeds the flange width. This difference occurs because the crossflow ratio rises as the flange width approaches the web height.<sup>24)</sup>



**Fig. 9.** Comparison of  $F_{measure}$  (measured roll force) and  $F_{H-beam}$  (roll force predicted from the proposed model) under varying parameters for specification B. (a) Web and (b) flange. (Online version in color.)

#### 4.2. Setup Roll Gaps ( $G_{set}$ )

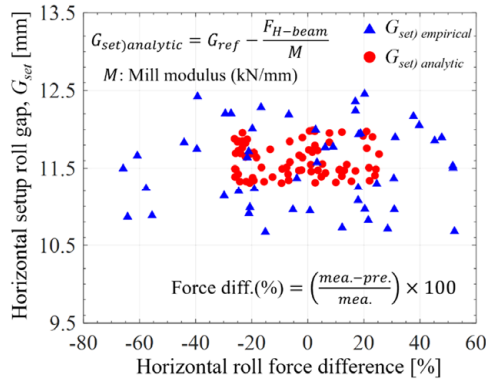
The setup roll gap ( $G_{set}$ ) can be calculated as follows:

$$G_{set} = G_{ref} - \frac{F_{H-beam}}{M} \quad (19)$$

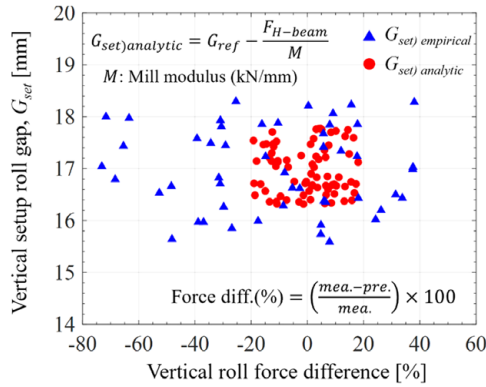
where  $M$  is the mill modulus, representing the load required to produce a unit displacement of the rolls.<sup>28)</sup>  $F_{H-beam}$  is the roll force in the finishing rolling mill, which can be calculated using Eq. (18) and  $F_{H-beam}/M$  is referred to as the mill stretch.<sup>29–31)</sup>  $G_{ref}$  denotes the reference roll gap, also referred to as the design roll gap.

Figures 10 and 11 present scatter plots comparing the setup roll gap values predicted by the analytical model proposed in this study with those predicted from the empirical model currently in use in H-beam rolling mill, as a function of roll force difference (%). Here,  $G_{set}^{empirical}$  represents the setup roll gap values determined using Hyundai Steel's empirical formula (model), whereas  $G_{set}^{analytic}$  refers to the setup roll gap values calculated using the semi-analytical model proposed in this study.

For both Specifications (A and B), the use of empirical model resulted in larger discrepancies in the roll force differences and a greater spread of the  $G_{set}$  values (indicated by the blue triangles). However, when the semi-analytical model was applied, the range of roll force deviations was narrowed significantly, falling approximately between -25% and 25%, and the scatter in the  $G_{set}$  values was reduced (represented by red circles). This improved results from the use of a geometry-dependent correction function  $Q_p$  tailored to H-beam deformation behavior, which plays a

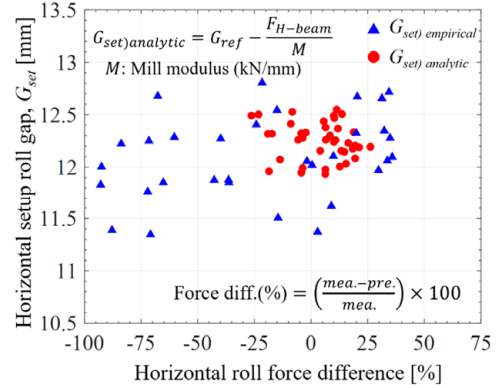


(a)

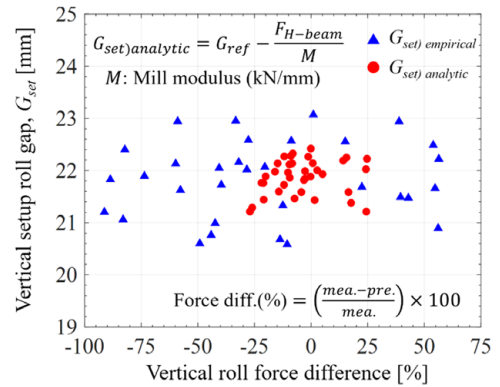


(b)

**Fig. 10.** Calculation of setup roll gaps using the semi-analytical model proposed in this study and the empirical model for manufacturing of product conforming to Specification A. (a) Horizontal roll gap (web) and (b) vertical roll gap (flange). (Online version in color.)



(a)



(b)

**Fig. 11.** Calculation of setup roll gaps using the semi-analytical model proposed in this study and the empirical model for manufacturing of product conforming to Specification B. (a) Horizontal roll gap (web) and (b) vertical roll gap (flange). (Online version in color.)

critical role in accurately predicting roll forces.

#### 4.3. Difference between the Target and Actual Thickness

H-beam rolling was carried out using setup roll gap values obtained from both the proposed semi-analytical model and Hyundai Steel's empirical model. The measured roll forces during rolling and the web and flange thicknesses after rolling were compared with the predicted values, and the results are shown in **Figs. 12** and **13**. The x-axis represents the difference, defined as the percentage difference between the measured and predicted roll forces. The y-axis represents the thickness difference (%), which is the difference between the target and actual thickness of the web or flange. More specifically, it indicates the deviation ratio of the actual thickness ( $t_{act}$ ) from the target thickness ( $t_{target}$ ). When expressed in terms of roll gap, this corresponds to  $(F_{H-beam}/M)/G_{ref}$ . The target thickness corresponds to the reference roll gap ( $G_{ref}$ ), which is designed the roll gap between the upper and lower horizontal rolls and the left and right vertical rolls. The actual thickness reflects the actual roll gap values during rolling.

Figure 12(a) illustrates the difference in thickness relative to the horizontal roll force difference for Specification A. When using the empirical model, the horizontal roll forces ranged from  $-62\%$  to  $52\%$ , and the web thickness difference ranged from  $-1.5\%$  to  $2.5\%$ . In contrast, applying the

semi-analytical model reduced the roll force difference to within  $\pm 25\%$ , and the web thickness difference decreased to within  $\pm 1\%$ . Figure 12(b) illustrates that the vertical roll force difference ranged from  $-74\%$  to  $39\%$ , and the flange thickness difference ranged from  $-2\%$  to  $3.2\%$  with the empirical model. However, the semi-analytical model reduced these differences to within  $\pm 20\%$  for roll force and within  $\pm 1\%$  for flange thickness. This indicates that the roll force prediction and corresponding  $G_{set)analytic}$  values are more accurate than those from the empirical model.

Figure 13(a) displays the thickness difference relative to the horizontal roll force difference for Specification B. Using the empirical model, the horizontal roll force difference ranged between  $-93\%$  and  $36\%$ , and the web thickness difference ranged from  $-3\%$  to  $1.5\%$ . The semi-analytical model reduced the roll force difference to within  $\pm 25\%$  and the web thickness difference to within  $\pm 1\%$ . Figure 13(b) shows that, with the empirical model, the vertical roll force difference ranged from  $-92\%$  to  $62\%$ , and the flange thickness difference ranged from  $-2.7\%$  to  $2.8\%$ . Applying the semi-analytical model reduced the roll force difference to between  $-28\%$  and  $25\%$  and the flange thickness difference to between  $-1\%$  and  $0.9\%$ .

For Specifications A and B, there appears to be an almost positive linear relationship between the roll force differences and thickness differences. When the predicted roll force

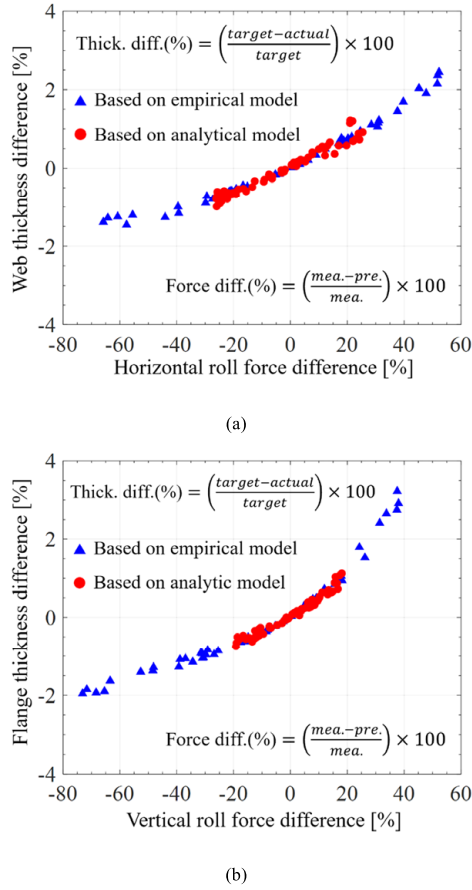


Fig. 12. Thickness difference for Specification A. (a) Web thickness and (b) flange thickness. (Online version in color.)

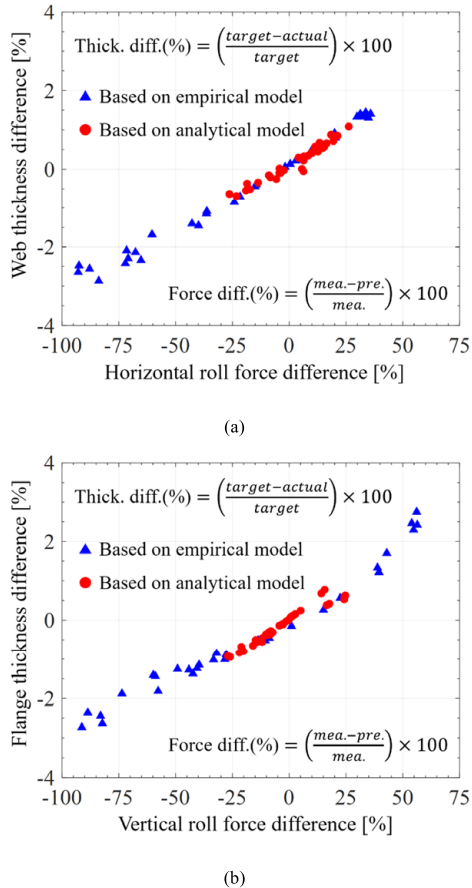


Fig. 13. Thickness difference for Specification B. (a) Web thickness and (b) flange thickness. (Online version in color.)

exceeded the measured force (*i.e.*, a negative roll force difference), the difference in thickness was negative, indicating that the over-prediction led to a smaller  $G_{set}$  than the standard value. Conversely, when the predicted roll force was less than the measured force (*i.e.*, a positive roll force difference), the difference in thickness was positive, suggesting that the under-prediction resulted in a larger  $G_{set}$ . This demonstrates that the roll force difference directly influences  $G_{set}$ , which ultimately affects the product thickness.

Figure 14 shows a histogram for the statistical analysis of the thickness variations in the web and flange of the H-beam cross-section; the results before and after the application of the semi-analytical model are compared. The blue histogram and dashed line represent the thickness difference distribution using the empirical model, whereas the red histogram and solid line represent the distribution after applying the semi-analytical model.

For Specification A (Fig. 14(a)), the standard deviation (SD) decreases from 1.2% to 0.5%, and the mean drops from 1.0% to 0.4%, reflecting an improvement of approximately 60%. For Specification B (Fig. 14(b)), the SD decreases from 1.5% to 0.5%, and the mean decreases from 1.4% to 0.4%, indicating an improvement of approximately 71%. The improvement in Specification B is more

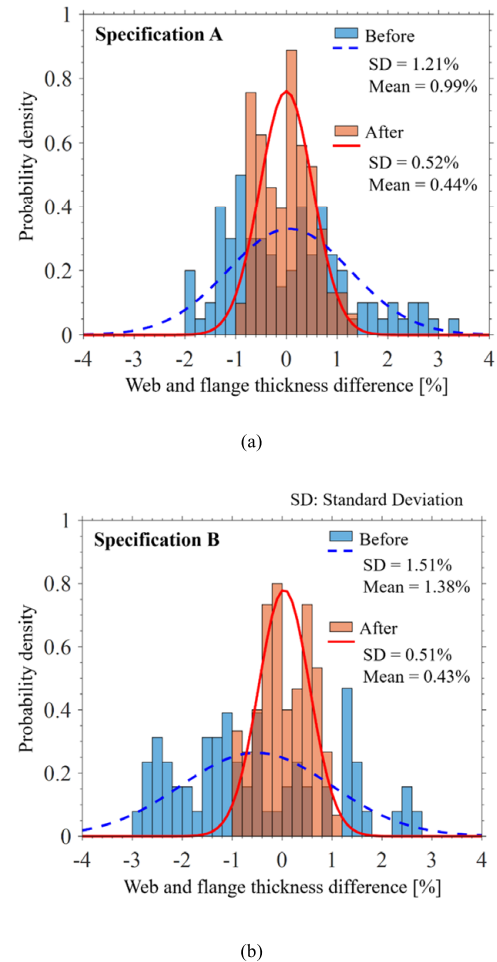


Fig. 14. Statistical analysis of thickness difference. “Before” indicates the application of the empirical model. “After” indicates the application of the semi-analytical model. (a) Probability density of Specification A and (b) probability density of Specification B. (Online version in color.)

pronounced than that in Specification A.

## 5. Conclusions

This study proposed a semi-analytical model for quickly predicting the roll forces and setup roll gap ( $G_{set}$ ) in the finishing mill of H-beam rolling process. The main idea was to approximate the H-beam cross-section as two rectangular parts—the web and the flange—and to create a geometry-dependent correction function  $Q_p$ , that efficiently represents the effect of rolling geometry on roll force. The proposed model was applied to the finishing mill of actual H-beam rolling process to calculate  $G_{set}$  for two Specifications (A and B). The web and flange thicknesses of H-beams rolled with  $G_{set}^{analytic}$  (from the proposed model) were compared to those rolled with  $G_{set}^{empirical}$  (from the empirical model). The following conclusions were drawn:

(1) The exponential form of the correction function,  $Q_p$  expressed in terms of reduction balance and the web/flange reduction ratios, proved effective.

(2) Applying the semi-analytical model resulted in significant improvements in the thickness accuracy, reducing the variation by 60% for Specification A and 71% for Specification B.

(3) The proposed model, combined with the developed  $Q_p$  function, has practical applicability and serves as a reliable tool for determining  $G_{set}$  in the finishing mill of H-beam rolling process.

## Statement for Conflict of Interest

The authors declare that there is no conflict of interest.

## Acknowledgment

This work was financially supported by the Hyundai Steel Company.

## Nomenclature

|                          |                                                              |
|--------------------------|--------------------------------------------------------------|
| $\delta$ :               | Mill stretch (mm)                                            |
| $L$ :                    | Loss function (%)                                            |
| $\theta$ :               | Inclination angle (degree)                                   |
| $\Delta\lambda$ :        | Reduction balance                                            |
| $\mu$ :                  | Friction coefficient                                         |
| $\omega$ :               | Angular velocity of roll (radian/s)                          |
| $A_x, A_y$ :             | Coordinates of the intersection point between web and flange |
| $B$ :                    | Flange width (mm)                                            |
| $c_1, c_2, \dots, c_5$ : | Constants                                                    |
| $F_{H-beam}$ :           | Roll force of the web or flange (kN)                         |
| $G_{set}$ :              | Setup roll gap (mm)                                          |
| $G_{ref}$ :              | Reference roll gap (mm)                                      |
| $H$ :                    | Web height (mm)                                              |
| $l_m$ :                  | Projected contact length between roll and material (mm)      |
| $Load_{nd}$ :            | Non-dimensional rolling load                                 |
| $k_m$ :                  | Average flow stress of the material (MPa)                    |
| $M$ :                    | Mill modulus (kN/mm)                                         |
| $Q_p$ :                  | Geometry-dependent correction function                       |
| $r$ :                    | Reduction ratio of thickness of the web or flange            |
| $R$ :                    | Roll radius (mm)                                             |
| $R_{juc}$ :              | Radius at junction round between web and                     |

flange (mm)

$S$ : Converted area using the rectangular approximation method (mm<sup>2</sup>)

SD: Standard deviation (%)

$t_{in}$ : Incoming thickness (mm)

$t_n$ : Strip thickness at the neutral point (mm)

$t_{out}$ : Outgoing thickness (mm)

$w_f$ : Flange width (mm)

$w_w$ : Transformed web width (mm)

## REFERENCES

- 1) B. Davison and G. W. Owens: *Steel Designers' Manual*, 7th ed., Wiley-Blackwell, UK, (2012), 157. ISBN978-1119249863
- 2) J. E. Lee, J. K. Yoon and H. N. Han: *ISIJ Int.*, **38** (1998), 132. <https://doi.org/10.2355/isijinternational.38.132>
- 3) E. Saiki and K. Matsuda: *Nippon Steel & Sumitomo Metal, Tech. Rep.*, **111** (2016), 59. <https://www.nipponsteel.com/en/tech/report/nssmc/pdf/111-08.pdf>
- 4) Y. Liu, X. Li, W. Du, S. Liu, J. Ji, Y. Li, Z. Wang and T. Wang: *Optics & Laser Technology*, **184** (2025), 112571. <https://doi.org/10.1016/j.optlastec.2025.112571>
- 5) B. S. Koo: *Eng. Fail. Anal.*, **133** (2022), 106009. <https://doi.org/10.1016/j.engfailanal.2021.106009>
- 6) B. S. Koo and D. J. Park: *Constr. Build. Mater.*, **458** (2025), 139649. <https://doi.org/10.1016/j.conbuildmat.2024.139649>
- 7) G. M. Zhu, Y. L. Kang, W. Chen and G. T. Ma: *Mater. Sci. Forum*, **575** (2008), 532. <https://doi.org/10.4028/www.scientific.net/MSF.575-578.532>
- 8) Q. He, J. Sun, C. Yan, J. Zhao and Z. Zhang: *Finite Elem. Anal. Des.*, **76** (2013), 13. <https://doi.org/10.1016/j.finela.2013.08.005>
- 9) W. Zhang, C. Zhu and G. E. O. Wiedera: *J. Mater. Process. Technol.*, **56** (1996), 820. [https://doi.org/10.1016/0924-0136\(95\)01895-6](https://doi.org/10.1016/0924-0136(95)01895-6)
- 10) K. Komori and K. Koumura: *J. Mater. Process. Technol.*, **105** (2000), 24. [https://doi.org/10.1016/S0924-0136\(00\)00537-9](https://doi.org/10.1016/S0924-0136(00)00537-9)
- 11) H. Yu and X. Liu: *Adv. Mater. Res.*, **148–149** (2011), 1. <https://doi.org/10.4028/www.scientific.net/AMR.148-149.1>
- 12) K. Li, P. Wang, G. Liu, P. Yuan and Q. Zhang: *Int. J. Adv. Manuf. Technol.*, **85** (2016), 1649. <https://doi.org/10.1007/s00170-015-8015-0>
- 13) Y. Dong, W. Zhang and J. Song: *J. Iron Steel Res. Int.*, **17** (2010), 27. [https://doi.org/10.1016/S1006-706X\(10\)60040-4](https://doi.org/10.1016/S1006-706X(10)60040-4)
- 14) D. L. Shvarts, S. O. Nepryakhin and V. A. Shilov: *Metallurgist*, **60** (2017), 1256. <https://doi.org/10.1007/s11015-017-0437-5>
- 15) R. B. Sims: *Proc. Inst. Mech. Eng.*, **168** (1954), 191. [https://doi.org/10.1243/PIME\\_PROC\\_1954\\_168\\_023\\_02](https://doi.org/10.1243/PIME_PROC_1954_168_023_02)
- 16) T. Shinokura and K. Takai: *J. app. Metalwork.*, **2** (1982), 94. <https://doi.org/10.1007/BF02834206>
- 17) Y. Lee: *ISIJ Int.*, **42** (2002), 726. <https://doi.org/10.2355/isijinternational.42.726>
- 18) J. A. Nelder and R. Mead: *Comput. J.*, **7** (1965), 308. <https://doi.org/10.1093/comjnl/7.4.308>
- 19) H. Hayashi, S. Saito, K. Kataoka, E. Nagayama and K. Takahashi: *Tetsu-to-Hagane*, **79** (1993), 1338 (in Japanese). [https://doi.org/10.2355/tetsutohagane1955.79.12\\_1338](https://doi.org/10.2355/tetsutohagane1955.79.12_1338)
- 20) T. Ariizumi, T. Hirasawa, I. Nakauchi, K. Morioka and N. Wakimoto: *Tetsu-to-Hagane*, **79** (1993), 409 (in Japanese). [https://doi.org/10.2355/tetsutohagane1955.79.3\\_409](https://doi.org/10.2355/tetsutohagane1955.79.3_409)
- 21) S. I. Kim, Y. Lee and S. M. Byon: *J. Mater. Process. Technol.*, **140** (2003), 84. [https://doi.org/10.1016/S0924-0136\(03\)00742-8](https://doi.org/10.1016/S0924-0136(03)00742-8)
- 22) S. M. Byon, S. I. Kim and Y. Lee: *Proc. Inst. Mech. Eng., Part B: J. Eng. Manuf.*, **220** (2006), 1059. <https://doi.org/10.1243/09544054JEM307>
- 23) S. Shida: *J. Jpn. Soc. Technol. Plast.*, **10** (1969), 610 (in Japanese). <https://ndlsearch.ndl.go.jp/books/R000000004-I8287175#ndl>
- 24) Y. Takashima and J. Yanagimoto: *Steel Res. Int.*, **82** (2011), 1240. <https://doi.org/10.1002/srin.201100078>
- 25) M. Kiuchi and J. Yanagimoto: *ISIJ Int.*, **30** (1990), 142. <https://doi.org/10.2355/isijinternational.30.142>
- 26) S. Saito, E. Yamanaka, K. Aso, T. Seto and H. Hayashi: *Tetsu-to-Hagane*, **79** (1993), 402 (in Japanese). [https://doi.org/10.2355/tetsutohagane1955.79.3\\_402](https://doi.org/10.2355/tetsutohagane1955.79.3_402)
- 27) X. Y. Guo, G. M. Zhu and Y. L. Kang: *Mater. Sci. Forum*, **749** (2013), 504. <https://doi.org/10.4028/www.scientific.net/MSF.749.504>
- 28) Theory and Practice of Flat Rolling ISIJ, Tokyo, (1984), 201 (in Japanese).
- 29) P. Kucsera and Z. Béres: *Acta Polytech Hung.*, **12** (2015), 93. <https://doi.org/10.12700/APH.12.6.2015.6.6>
- 30) F. Zhang, Y. Zhang and H. Chen: *Int. J. Control Autom.*, **9** (2016), 143. <https://doi.org/10.14257/ijca.2016.9.2.14>
- 31) A. A. Radionov, V. R. Gasiyarov, A. S. Karandaev, B. M. Loginov and V. R. Khramshin: *Appl. Sci.*, **11** (2021), 8865. <https://doi.org/10.3390/app11198865>