



Self-resonant dark matter with Z_4 gauged symmetry

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Abstract We present a new model for two-component scalar dark matter (DM), consisting of two complex scalar fields. In this model, both the DM components are stable due to the remaining Z_4 gauge symmetry, which is the remnant of the $U(1)'$ local symmetry. When the resonance condition for DM masses is fulfilled, we show that the elastic co-scattering processes between two components of dark matter (u -channel processes) are enhanced due to the Yukawa potential with a small effective mass for the lighter DM mediator, so we can use such co-scattering processes for dark matter to explain the small-scale problems at galaxies. Moreover, there are also semi-annihilation processes that two components of dark matter annihilate into one dark matter particle and a dark photon/Higgs, which are enhanced by the u -channel Sommerfeld factor. Focusing on some benchmark models for two-component dark matter satisfying the observed relic density, we obtain the bounds for the dark photon portal couplings from the direct detection for boosted dark matter, which is produced from the semi-annihilation processes at the galactic center.

1 Introduction

Dark matter (DM) is ubiquitous in the universe, from galaxies, large scale structures to cosmological scales, and it is measured precisely by Cosmic Microwave Background (CMB) data from Planck [1]. However, there is no clue for identifying the nature of dark matter, although there have been a plenty of models for dark matter and a lot of efforts to test them by direct and indirect experiments. Weakly Interacting Massive Particles (WIMP) paradigm has been one

of the interesting avenues in searching for dark matter, but the bounds from the direct detection experiments such as XENONnT [2], PandaX-4T [3], LZ [4], etc, have ruled out most of the parameter space for weak-scale dark matter.

Dark matter has been assumed to be composed of a single species of particles in the extension of the Standard Model (SM), so there has been an interesting interplay between the DM annihilation, the DM-nucleon scattering and the DM production, providing a guideline for constructing a consistent model for dark matter. However, dark matter can be composed of more than one species of particles in the so called multi-component dark matter scenarios, so new interactions between dark matter particles can play a crucial role not only in the evolution of dark matter abundance, large scale structures and galaxies, but also in the detection prospects for dark matter.

The extended dark sector is welcoming to make the N-body DM simulation compatible with the observed rotation velocities in galaxies at small scales. For instance, light mediators for dark matter (t -channel mechanism) [5] or a multi-component dark matter with resonant mass conditions (u -channel mechanism) [6–8] can make the dark matter self-interactions enhanced and velocity-dependent [9]. Then, self-interacting dark matter can provide a solution to the small-scale problems such as core-cusp problem, too-big-to-fail problem, diversity problem, etc [5, 10–14], although we need a more precise understanding of the baryonic feedback on the DM simulation [15, 16]. In particular, in the latter u -channel mechanism where there are more than one species for dark matter candidates with comparable masses, the dark matter self-scattering can be enhanced to solve the small-scale problems for a small velocity of dark matter at dwarf galaxies without a light mediator, and there are interesting signals for direct and indirect detections, depending on the relative fractions of dark matter abundances [6, 7].

In this article, we consider a model with Z_4 discrete gauge symmetry for two-component dark matter. The $U(1)'$ local

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symmetry is spontaneously broken to Z_4 such that a complex scalar field ϕ_1 and the lighter component of another complex scalar field ϕ_2 can be both candidates for stable dark matter. In comparison, in the Z_2 model considered in Refs. [6, 7], ϕ_1 was taken as being a complex singlet scalar field charged under the $U(1)'$, while ϕ_2 was considered as being a neutral singlet scalar under the $U(1)'$. In this case, ϕ_2 would not be stable due to the loop-induced decays with Z' interactions unless a twin partner of ϕ_1 with a similar direct coupling to ϕ_2 is introduced [8]. In the current paper, the Z_4 symmetry provides a consistent framework for two-component dark matter because it makes both ϕ_2 (or the lighter of scalar fields belong to ϕ_2) and ϕ_1 absolutely stable.

Focusing on the case with degenerate masses for two real components of ϕ_2 , we just call the second dark matter ϕ_2 . The Z_4 -invariant self-interactions between ϕ_1 and ϕ_2 are responsible for the u -channel mechanism for enhancing the DM self-scattering cross sections under the resonant mass condition as well as determining the relic density for multi-component dark matter. We call the dark matter in our model “self-resonant dark matter”. We remark that the qualitative discussion of the current work is still valid even for the general case with split masses for ϕ_2 , after the resonant mass condition is introduced between ϕ_1 and the lighter component of ϕ_2 .

In the presence of the resonant mass condition for ϕ_1 and ϕ_2 , namely, $m_2 \simeq 2m_1$, the tree level co-scattering processes between ϕ_1 and ϕ_2 (or $\phi_2^\dagger$) become divergent in the limit of a small momentum transfer for dark matter. Thus, we develop the Bethe–Salpeter formalism for getting the enhanced co-scattering cross sections following closely the method taken by some of the authors in Refs. [6, 7]. Moreover, we show how the semi-annihilation processes, $\phi_1\phi_2^{(\dagger)} \rightarrow \phi_1^\dagger Y$, with $Y = X, h_X$, and their complex conjugates, where X is the dark photon and h_X is the dark Higgs, can be also enhanced by the u -channel Sommerfeld factor. The boosted dark matter ϕ_1 produced from the semi-annihilation processes carries a sizable kinetic energy such that it can be probed by direct detection experiments above the detector threshold. We discuss the compatibility of the effective self-scattering cross section for dark matter with the observed rotation velocities in galaxies and galaxy clusters. We also choose some benchmark models for dark matter with weak-scale or sub-GeV scale masses and show how the parameter space for dark photon portal couplings is constrained by the relic density, indirect detection such as CMB bound, and the direct detection for boosted dark matter coming from the galactic center.

The paper is organized as follows. We begin with the model setup consisting of two complex scalar fields for dark matter and a complex scalar field for the dark Higgs mechanism and identify the dark sector interactions as well as the DM interactions with the SM. We apply the Bethe–Salpeter formalism for the non-relativistic co-scattering processes

Table 1 The $U(1)'$ charges and Z_4 transformations for dark scalars

	ϕ_1	ϕ_2	χ
$U(1)'$	+1	+2	+4
$U(1)' \supset Z_4$	i	−1	+1

between ϕ_1 and ϕ_2 and obtain the effective Yukawa-type potential only for one linear combination of the initial states, $|\phi_1\phi_2\rangle$ and $|\phi_1\phi_2^\dagger\rangle$. Then, we obtain the velocity-averaged cross section for the co-scattering between two different species for dark matter and the one for the self-scattering for ϕ_1 only and compare between them. Next we provide the Boltzmann equations for the relic abundances for two-component dark matter and present the direct detection cross sections for halo dark matter, the bound from CMB recombination, direct detection for boosted dark matter. Finally conclusions are drawn. There are three appendices containing the DM annihilation cross sections in the model, the direct detection cross sections for Higgs and dark photon portal couplings as well as the kinematics for the semi-annihilation processes for multi-component dark matter.

2 The setup

For a working model for self-resonant dark matter [6–8], we introduce a complex scalar χ for the symmetry breaking of $U(1)'$ to Z_4 , two additional complex scalars, and ϕ_1, ϕ_2 , for multi-component dark matter. The extra gauge boson associated with $U(1)'$ is denoted by X_μ . We assume that the SM particles including the Higgs doublet are neutral under the $U(1)'$. We specify the charges for a local $U(1)'$ symmetry and the Z_4 transformations for the dark scalars, as summarized in Table 1.

The $U(1)'$ invariant Lagrangian for the SM Higgs and dark scalars is given by

$$\mathcal{L} = |D_\mu \chi|^2 + |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2 - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} \sin \xi X_{\mu\nu} B^{\mu\nu} - V(\chi, \phi_1, \phi_2) + \mathcal{L}_{H,\text{portal}} \quad (1)$$

where $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ is the field strength tensor for the extra gauge boson, the covariant derivatives are $D_\mu \chi = (\partial_\mu - 4ig_X X_\mu)\chi$, $D_\mu \phi_1 = (\partial_\mu - ig_X X_\mu)\phi_1$, $D_\mu \phi_2 = (\partial_\mu - 2ig_X X_\mu)\phi_2$, and the scalar potential is

$$\begin{aligned} V(\chi, \phi_1, \phi_2) = & m_\chi^2 |\chi|^2 + \lambda_\chi |\chi|^4 + m_1^2 |\phi_1|^2 + \lambda_1 |\phi_1|^4 \\ & + m_2^2 |\phi_2|^2 + \lambda_2 |\phi_2|^4 \\ & + \sum_{i=1,2} \lambda_{\chi i} |\chi|^2 |\phi_i|^2 + \lambda_{12} |\phi_1|^2 |\phi_2|^2 + (g_1 m_1 \phi_2^\dagger \phi_1^2 + \text{h.c.}) \\ & + (\kappa_1 \chi^\dagger \phi_2^2 + \kappa_2 \chi^\dagger \phi_2 \phi_1^2 + \text{h.c.}). \end{aligned} \quad (2)$$

Moreover, $\sin \xi$ is a gauge kinetic mixing term between the extra gauge boson and the $U(1)_Y$ gauge boson. We can also introduce Higgs-portal couplings like $|H|^2 \mathcal{O}_{\text{dark}}$ with $\mathcal{O}_{\text{dark}} = |\chi|^2, |\phi_1|^2, |\phi_2|^2$. That is, the Lagrangian for the SM Higgs doublet is extended with the Higgs-portal interactions to the dark scalars,

$$\mathcal{L}_{H,\text{portal}} = |D_\mu H|^2 - \lambda_H |H|^4 - m_H^2 |H|^2 - \lambda_{\chi H} |H|^2 |\chi|^2 - \lambda_{H1} |H|^2 |\phi_1|^2 - \lambda_{H2} |H|^2 |\phi_2|^2. \quad (3)$$

2.1 $U(1)'$ symmetry breaking vacuum

We assume that the SM Higgs H and the dark Higgs χ get nonzero VEVs by $H = (0, v_H)^T / \sqrt{2}$ and $\langle \chi \rangle = \frac{1}{\sqrt{2}} v_\chi$, respectively, but ϕ_1, ϕ_2 do not get VEVs. By setting the VEVs for ϕ_1 and ϕ_2 to zero, we find that the minimization conditions for the potential with respect to the dark Higgs and the SM Higgs are given by

$$0 = m_\chi^2 + \lambda_\chi v_\chi^2 + \frac{1}{2} \lambda_{\chi H} v_H^2, \quad (4)$$

$$0 = m_H^2 + \lambda_H v_H^2 + \frac{1}{2} \lambda_{\chi H} v_\chi^2, \quad (5)$$

or the VEVs are determined by

$$v_H^2 = \frac{2(\lambda_{\chi H} m_\chi^2 - 2\lambda_\chi m_H^2)}{4\lambda_H \lambda_\chi - \lambda_{\chi H}^2},$$

$$v_\chi^2 = \frac{2(\lambda_{\chi H} m_H^2 - 2\lambda_H m_\chi^2)}{4\lambda_H \lambda_\chi - \lambda_{\chi H}^2}. \quad (6)$$

Furthermore, from the mass matrix for the SM Higgs and the dark Higgs, it turns out that the conditions for a local minimum are given [17] by

$$4\lambda_H \lambda_\chi - \lambda_{\chi H}^2 > 0, \quad (7)$$

$$\lambda_{\chi H} m_\chi^2 - 2\lambda_\chi m_H^2 > 0, \quad (8)$$

$$\lambda_{\chi H} m_H^2 - 2\lambda_H m_\chi^2 > 0. \quad (9)$$

We note that there are other important constraints on the quartic couplings in our model, such as unitarity, perturbativity, vacuum stability [17], when the fields are far away from the local minimum or the energy increases far above the VEVs [18]. However, we don't consider the extra constraints on the quartic couplings explicitly in the following discussion, as energies or field ranges for the typical processes are near the VEVs.

Then, after χ gets a VEV, the $U(1)'$ gauge symmetry is broken spontaneously to Z_4 and the $U(1)'$ gauge boson gets a nonzero mass as $m_\chi = 4g_\chi v_\chi$. Moreover, the effective mass terms and the dimensionful interaction terms for ϕ_1, ϕ_2 ,

which are relevant for the u -channel processes, are shifted, as follows,

$$\mathcal{L}_{\text{eff}} = -m_{1,\text{eff}}^2 |\phi_1|^2 - m_{2,\text{eff}}^2 |\phi_2|^2 - (m_3^2 \phi_2^2 + g_1 m_1 \phi_2^\dagger \phi_1^2 + g_2 m_1 \phi_2 \phi_1^2 + \text{h.c.}) \quad (10)$$

with

$$m_{1,\text{eff}}^2 \equiv m_1^2 + \frac{1}{2} \lambda_{\chi 1} v_\chi^2 + \frac{1}{2} \lambda_{H1} v_H^2, \quad (11)$$

$$m_{2,\text{eff}}^2 \equiv m_2^2 + \frac{1}{2} \lambda_{\chi 2} v_\chi^2 + \frac{1}{2} \lambda_{H2} v_H^2, \quad (12)$$

$$m_3^2 \equiv \frac{1}{\sqrt{2}} \kappa_1 v_\chi, \quad (13)$$

$$g_2 \equiv \frac{\kappa_2 v_\chi}{\sqrt{2} m_1}. \quad (14)$$

In particular, the scalars in ϕ_2 , namely, $\phi_2 = \frac{1}{\sqrt{2}}(s + ia)$, have split masses, due to the effective ϕ_2^2 term, as

$$m_s^2 = m_{2,\text{eff}}^2 + 2m_3^2, \quad (15)$$

$$m_a^2 = m_{2,\text{eff}}^2 - 2m_3^2. \quad (16)$$

As a result, the direction for ϕ_1 and the scalars in ϕ_2 are stable as far as $m_{1,\text{eff}}^2 > 0$, $m_s^2 > 0$ and $m_a^2 > 0$, which constrain the bare mass parameters and the quartic couplings for the DM scalars, as follows,

$$m_1^2 + \frac{1}{2} \lambda_{\chi 1} v_\chi^2 + \frac{1}{2} \lambda_{H1} v_H^2 > 0, \quad (17)$$

$$m_2^2 + \frac{1}{2} \lambda_{\chi 2} v_\chi^2 + \frac{1}{2} \lambda_{H2} v_H^2 \pm \sqrt{2} \kappa_1 v_\chi > 0. \quad (18)$$

We note that two real scalar fields of ϕ_2 are degenerate in mass, i.e. $m_s = m_a = m_{2,\text{eff}}$, for $m_3 = 0$ or $\kappa_1 = 0$. In this case, for the later discussion, we use the notations, m_2 for $m_{2,\text{eff}}$, for simplicity. If $m_3 \neq 0$ or $\kappa_1 \neq 0$, $m_s \neq m_a$: $a(s)$ is the second dark matter candidate for $m_3^2 > 0$ ($m_3^2 < 0$). In the latter case, the resonant mass condition changes from $m_2 = 2m_1$ (for $m_3 = 0$) to $m_{a(s)} = 2m_1$ (for $m_3 \neq 0$).

A nonzero κ_1 splits the masses between s and a , but it is sufficient to take the mass relation between ϕ_1 and one of s and a for the u -channel enhancement. So, the existence of the u -channel enhancement does not depend on the mass splitting between s and a . So, for simplicity, we can take the same masses for s and a .

In general, the dark matter abundance at present depends on the mass splitting between s and a , $|m_s - m_a|$, and the lifetime of the heavier one. As far as $|m_s - m_a| \lesssim T_{\text{f.o.}}$, where $T_{\text{f.o.}}$ is the freeze-out temperature of about $0.2 \min(m_s, m_a)$, the relic density calculations would depend little on the mass splitting. Moreover, the u -channel condition would be satisfied for both s and a , if $|m_s - m_a| \lesssim M = m_{s(a)} \sqrt{2 - \frac{m_{s(a)}}{m_1}}$ where M is the effective mass for the u -channel resonance, as will be shown in the later section. Henceforth, for simplicity,

we focus on the former case with degenerate masses for ϕ_2 , namely, $m_s = m_a$.

2.2 DM self-interactions

First, from Eq. (10), we collect the self-interaction terms for dark scalars, ϕ_1, s, a , as follows,

$$\begin{aligned} \mathcal{L}_{\phi_1-s,a} = & -\frac{1}{\sqrt{2}}(g_1 + g_2)m_1s(\phi_1^2 + \phi_1^{\dagger 2}) \\ & + \frac{1}{\sqrt{2}}i(g_1 - g_2)m_1a(\phi_1^2 - \phi_1^{\dagger 2}) \\ & - \frac{1}{2}\lambda_{12}(s^2 + a^2)|\phi_1|^2 \\ & - \frac{1}{4}\lambda_2(s^2 + a^2)^2 - \lambda_1|\phi_1|^4. \end{aligned} \quad (19)$$

Here, we assumed that all the couplings for dark scalars are real. After the $U(1)'$ is broken, the stability for ϕ_1 and the lighter particle among s and a are guaranteed because of the remaining Z_4 symmetry:

$$Z_4 : \phi_1 \rightarrow i\phi_1, \quad s \rightarrow -s, \quad a \rightarrow -a, \quad (20)$$

as far as $m_s, m_a < 2m_1$. As a result, the multi-component dark matter can be realized in our model.

2.3 DM interactions with the dark sector

Expanding the dark Higgs χ around the VEV by $\chi = \frac{1}{\sqrt{2}}(v_\chi + h_\chi)$ in unitary gauge, we can also get the interaction terms for the dark Higgs h_χ and the dark scalars, from Eq. (1), as follows,

$$\begin{aligned} \mathcal{L}_{h_\chi-\phi_1,s,a} = & -\lambda_\chi v_\chi h_\chi^3 - \frac{1}{4}\lambda_\chi h_\chi^4 \\ & - \frac{1}{2}\lambda_{\chi 1}(2v_\chi h_\chi + h_\chi^2)|\phi_1|^2 \\ & - \frac{1}{4}\lambda_{\chi 2}(2v_\chi h_\chi + h_\chi^2)(s^2 + a^2) \\ & - \frac{1}{\sqrt{2}}\kappa_1 h_\chi(s^2 - a^2) - \frac{1}{2}\kappa_2 h_\chi s(\phi_1^2 + \phi_1^{\dagger 2}) \\ & - \frac{1}{2}i\kappa_2 h_\chi a(\phi_1^2 - \phi_1^{\dagger 2}). \end{aligned} \quad (21)$$

We note that the Higgs-portal interaction for χ , namely, $|H|^2|\chi|^2$, gives rise to a Higgs mixing after electroweak symmetry is also broken.

Moreover, the $U(1)'$ gauge interactions for the dark Higgs h_χ and dark scalars, ϕ_1, s, a , are given by

$$\begin{aligned} \mathcal{L}_X = & \frac{m_X^2}{v_\chi} h_X X_\mu X^\mu + 8g_X^2 h_X^2 X_\mu X^\mu \\ & + ig_X X_\mu (\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + g_X^2 X_\mu X^\mu \phi_1^\dagger \phi_1 \\ & + 2ig_X X_\mu (\phi_2^\dagger \partial^\mu \phi_2 - \phi_2 \partial^\mu \phi_2^\dagger) + 4g_X^2 X_\mu X^\mu \phi_2^\dagger \phi_2 \end{aligned}$$

$$\begin{aligned} = & \frac{m_X^2}{v_\chi} h_X X_\mu X^\mu + 8g_X^2 h_X^2 X_\mu X^\mu \\ & + ig_X X_\mu (\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + g_X^2 X_\mu X^\mu \phi_1^\dagger \phi_1 \\ & - 2g_X X_\mu (s \partial^\mu a - a \partial^\mu s) + 2g_X^2 X_\mu X^\mu (s^2 + a^2). \end{aligned} \quad (22)$$

Due to the gauge interactions in Eq. (22), s decays into $a + X$ for $m_s > m_a + m_X$ while a decays into $s + X$ for $m_a > m_s + m_X$. Even if the X boson produced from the decays of s or a is off-shell, either s or a is unstable, as X can decay into the light fermions in the SM through the gauge kinetic mixing, so we can regard only the lighter particle among s and a as being a stable dark matter, whereas the relic abundance of the heavier particle depends on the mass splitting, $|m_s - m_a|$, and the lifetime of the heavier particle.

2.4 DM interactions with the SM sector

For $|\xi| \ll 1$, there is a small mixing between the dark gauge boson and the Z boson, for which the neutral current interactions [27, 28] become

$$\begin{aligned} \mathcal{L}_{\text{NC}} \simeq & A_\mu J_{\text{EM}}^\mu + Z_\mu \left(J_Z^\mu + \varepsilon t_W J_X^\mu \right) \\ & + X_\mu \left(-\varepsilon J_{\text{EM}}^\mu + J_X^\mu \right), \end{aligned} \quad (23)$$

with $\varepsilon \equiv c_W \xi$, and J_{EM}^μ , J_Z^μ and J_X^μ are electromagnetic, neutral and dark currents, given by

$$J_{\text{EM}}^\mu = e \bar{f} \gamma^\mu Q_f f, \quad (24)$$

$$J_Z^\mu = \frac{e}{2s_W c_W} \bar{f} \gamma^\mu (\tau^3 - 2s_W^2 Q_f) f, \quad (25)$$

$$\begin{aligned} J_X^\mu = & ig_X (\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) + 2ig_X (\phi_2^\dagger \partial^\mu \phi_2 - \phi_2 \partial^\mu \phi_2^\dagger) \\ = & ig_X (\phi_1^\dagger \partial^\mu \phi_1 - \phi_1 \partial^\mu \phi_1^\dagger) - 2g_X (s \partial^\mu a - a \partial^\mu s). \end{aligned} \quad (26)$$

Here, we note that the gauge kinetic mixing has been included for the linear interactions of the extra gauge boson in addition to those in Eq. (22).

Moreover, the SM Higgs h , (from $H^T = \frac{1}{\sqrt{2}}(0, v + h)$), mixes with the dark Higgs h_χ by

$$h_1 = \cos \theta h_X - \sin \theta h, \quad h_2 = \sin \theta h_X + \cos \theta h, \quad (27)$$

with

$$\tan 2\theta = \frac{\lambda_{\chi H} v_\chi v}{\lambda_H v^2 - \lambda_\chi v_\chi^2}. \quad (28)$$

Then, we also obtain the effective couplings for the dark Higgs-like scalar h_1 and the SM Higgs-like scalar h_2 to multi-component dark matter and the SM fermions, as follows,

$$\begin{aligned} \mathcal{L}_{h_1,h_2} = & -y_{h_1 \phi_1^\dagger \phi_1} h_1 |\phi_1|^2 - y_{h_2 \phi_1^\dagger \phi_1} h_2 |\phi_1|^2 \\ & - \frac{1}{2} y_{h_1 ss} h_1 s^2 - \frac{1}{2} y_{h_1 aa} h_1 a^2 \end{aligned}$$

$$-\frac{1}{2}y_{h_2ss}h_2s^2 - \frac{1}{2}y_{h_2aa}h_2a^2 - (\lambda_{f1}h_1 + \lambda_{f2}h_2)\bar{f}f, \quad (29)$$

with

$$y_{h_1\phi_1^\dagger\phi_1} = \lambda_{\chi 1}v_\chi \cos\theta - \lambda_{H1}v \sin\theta, \quad (30)$$

$$y_{h_2\phi_1^\dagger\phi_1} = \lambda_{\chi 1}v_\chi \sin\theta + \lambda_{H1}v \cos\theta, \quad (31)$$

$$y_{h_1ss} = (\lambda_{\chi 2}v_\chi + \sqrt{2}\kappa_1) \cos\theta - \lambda_{H2}v \sin\theta, \quad (32)$$

$$y_{h_2ss} = (\lambda_{\chi 2}v_\chi + \sqrt{2}\kappa_1) \sin\theta + \lambda_{H2}v \cos\theta, \quad (33)$$

$$y_{h_1aa} = (\lambda_{\chi 2}v_\chi - \sqrt{2}\kappa_1) \cos\theta - \lambda_{H2}v \sin\theta, \quad (34)$$

$$y_{h_2aa} = (\lambda_{\chi 2}v_\chi - \sqrt{2}\kappa_1) \sin\theta + \lambda_{H2}v \cos\theta, \quad (35)$$

$$\lambda_{f1} = -\frac{m_f}{v} \sin\theta, \quad (36)$$

$$\lambda_{f2} = \frac{m_f}{v} \cos\theta. \quad (37)$$

For $m_s = m_a = m_2$, namely, for $\kappa_1 = 0$, we can identify the effective couplings for the dark matter ϕ_2 by $y_{h_i\phi_2^\dagger\phi_2} = y_{h_iss}$, with $i = 1, 2$.

In the later discussion, the u -channel resonances appear for multi-component dark matter with $m_s \simeq 2m_1$ and/or $m_a \simeq 2m_1$, but we focus on the benchmark models with $m_s = m_a \simeq 2m_1$ for the DM self-scattering, the relic density and the direct and indirect detections, for which the scalar self-interactions in Eqs. (19) and (21) and the gauge interactions in Eq. (22) as well as gauge-portal and Higgs-portal interactions in Eqs. (23) and (29) are relevant.

3 DM self-scattering with u -channel resonances

We discuss the dark matter co-scattering processes with u -channel dominance at tree level and present the Bethe–Salpeter formalism for the non-perturbative results for them. In the following discussion, we assume the case $m_s = m_a$, without loss of generality.

3.1 Dark matter co-scattering at tree level

We consider the elastic scattering, $\phi_1s \rightarrow \phi_1s$ and its complex conjugate, which can have a non-perturbative enhancement near $m_s \sim 2m_1$ [6, 7]. The dark Higgs also contributes to the elastic scattering in the t -channel but we don't consider it in the following discussion by assuming that the dark Higgs is not so light as compared to the DM mass.

For the elastic scattering process, $\phi_1(q)s(p) \rightarrow \phi_1(q')s(p')$, the tree-level scattering amplitude $\tilde{\Gamma}_{s,u}(p, q; p, q')$ is given by

$$\begin{aligned} \tilde{\Gamma}_{s,u}(p, q; p', q') &= -\frac{4g_s^2m_1^2}{(p-q')^2 - m_1^2} \\ &= \frac{4g_s^2m_1^2}{|\vec{p} - \vec{q}'|^2 + m_1^2 - \omega^2} \end{aligned} \quad (38)$$

with $g_s \equiv \frac{1}{\sqrt{2}}(g_1 + g_2)$ and $\omega = p_0 - q'_0$ being the energy exchange. Then, in the non-relativistic limit for initial particles, we get

$$\begin{aligned} m_1^2 - \omega^2 &= m_1^2 - \left(\sqrt{m_s^2 + \vec{p}^2} - \sqrt{m_1^2 + \vec{q}'^2} \right)^2 \\ &\approx m_s(2m_1 - m_s) + \left(-1 + \frac{m_1}{m_s} \right) \vec{p}^2 \\ &\quad + \left(-1 + \frac{m_s}{m_1} \right) \vec{q}'^2. \end{aligned} \quad (39)$$

We note that the energy exchange ω does not vanish in the non-relativistic limit, unlike the case for the elastic scattering between the same particles, so the process is not instantaneous.

As a result, the tree-level scattering amplitude $\tilde{\Gamma}_{s,u}(p, q; p', q')$ becomes

$$\begin{aligned} \tilde{\Gamma}_{s,u}(p, q; p', q') &\approx \frac{4g_s^2m_1^2}{\left(\sqrt{\frac{m_1}{m_s}}\vec{p} - \sqrt{\frac{m_s}{m_1}}\vec{q}' \right)^2 + m_s(2m_1 - m_s)}. \end{aligned} \quad (40)$$

Then, we find that the effective squared mass of the u -channel mediator is semi-positive definite for $m_s \leq 2m_1$. For $\vec{p} = \vec{q}' = 0$, the above tree-level scattering amplitude diverges at $m_s = 2m_1$.

Similarly, for the elastic scattering process, $\phi_1(q)a(p) \rightarrow \phi_1(q')a(p')$, the tree-level scattering amplitude $\tilde{\Gamma}_{a,u}(p, q; p, q')$ is given by

$$\begin{aligned} \tilde{\Gamma}_{a,u}(p, q; p', q') &= -\frac{4g_a^2m_1^2}{(p-q')^2 - m_1^2} \\ &= \frac{4g_a^2m_1^2}{|\vec{p} - \vec{q}'|^2 + m_1^2 - \omega^2} \end{aligned} \quad (41)$$

with $g_a \equiv \frac{1}{\sqrt{2}}(g_1 - g_2)$. Following a similar step for the non-relativistic limit for the initial particles, we can approximate the above result to

$$\begin{aligned} \tilde{\Gamma}_{a,u}(p, q; p', q') &\approx \frac{4g_a^2m_1^2}{\left(\sqrt{\frac{m_1}{m_a}}\vec{p} - \sqrt{\frac{m_a}{m_1}}\vec{q}' \right)^2 + m_a(2m_1 - m_a)}. \end{aligned} \quad (42)$$

So, the above tree-level scattering amplitude diverges at $m_a = 2m_1$ for $\vec{p} = \vec{q}' = 0$.

For $m_s = m_a = m_2$, for the DM co-scattering processes of our focus in the later discussion, $\phi_1(q)\phi_2(p) \rightarrow \phi_1(q')\phi_2(p')$ and $\phi_1(q)\phi_2^\dagger(p) \rightarrow \phi_1(q')\phi_2^\dagger(p')$ and their

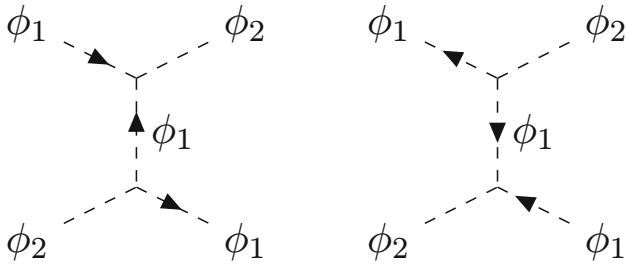


Fig. 1 Tree-level Feynman diagrams for the DM co-scattering processes with u -channel, $\phi_1\phi_2 \rightarrow \phi_1\phi_2$ and $\phi_1^\dagger\phi_2 \rightarrow \phi_1^\dagger\phi_2$; $\phi_1\phi_2^\dagger \rightarrow \phi_1\phi_2^\dagger$ and $\phi_1^\dagger\phi_2^\dagger \rightarrow \phi_1^\dagger\phi_2^\dagger$

complex conjugates, the corresponding u -channel amplitudes at tree level shown in Fig. 1 are given by

$$\tilde{\Gamma}_{\phi_1\phi_2 \rightarrow \phi_1\phi_2}(p, q; p', q') = -\frac{4g_1^2 m_1^2}{(p - q')^2 - m_1^2}, \quad (43)$$

$$\tilde{\Gamma}_{\phi_1\phi_2^\dagger \rightarrow \phi_1\phi_2^\dagger}(p, q; p', q') = -\frac{4g_1 g_2 m_1^2}{(p - q')^2 - m_1^2}. \quad (44)$$

Similarly, for $\phi_1(q)\phi_2^\dagger(p) \rightarrow \phi_1(q')\phi_2^\dagger(p')$ and $\phi_1(q)\phi_2(p) \rightarrow \phi_1(q')\phi_2(p')$ and their complex conjugates, as shown in Fig. 1, we also obtain

$$\tilde{\Gamma}_{\phi_1\phi_2^\dagger \rightarrow \phi_1\phi_2^\dagger}(p, q; p', q') = -\frac{4g_2^2 m_1^2}{(p - q')^2 - m_1^2}, \quad (45)$$

$$\tilde{\Gamma}_{\phi_1\phi_2 \rightarrow \phi_1\phi_2^\dagger}(p, q; p', q') = -\frac{4g_1 g_2 m_1^2}{(p - q')^2 - m_1^2}. \quad (46)$$

Then, the tree-level u -channel scattering amplitudes with a complex scalar ϕ_2 in Eqs. (43)–(46) diverge at $m_2 = 2m_1$ for $\vec{p} = \vec{q}' = 0$. Other DM co-scattering processes with u -channels are complex conjugates of the above processes.

In the non-relativistic limit for two-component dark matter, the DM co-scattering cross section for $\phi_1 C \rightarrow \phi_1 C$, with $C = s, a$ or ϕ_2 , is, at tree level, given by

$$\sigma_{\phi_1 C \rightarrow \phi_1 C} = \frac{1}{16\pi(m_1 + m_C)^2} |\tilde{\Gamma}_{u, \phi_1 C \rightarrow \phi_1 C}|^2. \quad (47)$$

However, as will be discussed shortly, we need to take into account the non-perturbative effects from the u -channels for the DM annihilation.

For $m_s = m_a = m_2$, the non-perturbative co-scattering cross section can be written with s -wave dominance [6, 7], as

$$\sigma_{\psi_1 \rightarrow \phi_1\phi_2}^{\text{total}} = \frac{4\pi}{k^2} \sin^2 \delta_0, \quad (48)$$

and its complex conjugate, with ψ_1 being a mixture of two DM pairs, $(\phi_1\phi_2)$ and $(\phi_1\phi_2^\dagger)$, $k = \mu v$, μ being the reduced mass for $\phi_1 - \phi_2^{(\dagger)}$, and δ_0 being the phase shift for the elastic co-scattering, which is determined by solving the Bethe–Salpeter equation for ψ_1 or its complex conjugate, as will be shown in the next subsection.

3.2 Bethe–Salpeter equations for DM co-scattering

In this section, we use the Bethe–Salpeter(BS) formalism for the enhanced self-scattering cross section as well as the DM annihilation cross section, based on the u -channel dominance.

For simplicity, we consider the case that the real and imaginary parts of the complex scalar ϕ_2 have degenerate masses, namely, for $m_s = m_a = m_2$. However, the results of the following discussion are applicable to the general case with split masses for ϕ_2 , as far as the resonant mass condition is satisfied between ϕ_1 and the lighter component of ϕ_2 . In this case, we can work in the basis with two complex scalars, (ϕ_1, ϕ_2) .

Then, we only have to consider four pairs of combinations of fields for self-scattering with u -channel dominance, namely, $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2\phi_1$, $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2^\dagger\phi_1$, $(\phi_2^\dagger, \phi_2)\phi_1^\dagger \rightarrow \phi_2\phi_1^\dagger$ and $(\phi_2^\dagger, \phi_2)\phi_1^\dagger \rightarrow \phi_2^\dagger\phi_1^\dagger$.

First, we consider a pair of self-scattering processes, $\phi_2(p)\phi_1(q) \rightarrow \phi_2(p')\phi_1(q')$ and $\phi_2^\dagger(p)\phi_1(q) \rightarrow \phi_2^\dagger(p')\phi_1(q')$. Then, the corresponding non-perturbative scattering amplitudes satisfy the following relations,

$$\begin{aligned} i\chi_A(p, q) &\simeq -G_2(p)G_1(q) \int \frac{d^4k}{(2\pi)^4} \\ &\times \left[\tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2\phi_1}(p, q; p + q - k, k) \chi_A(p + q - k, k) \right. \\ &\left. + \tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2^\dagger\phi_1}(p, q; p + q - k, k) \chi_B(p + q - k, k) \right], \end{aligned} \quad (49)$$

$$\begin{aligned} i\chi_B(p, q) &\simeq -G_2(p)G_1(q) \int \frac{d^4k}{(2\pi)^4} \\ &\times \left[\tilde{\Gamma}_{\phi_2^\dagger\phi_1 \rightarrow \phi_2\phi_1}(p, q; p + q - k, k) \chi_A(p + q - k, k) \right. \\ &\left. + \tilde{\Gamma}_{\phi_2^\dagger\phi_1 \rightarrow \phi_2^\dagger\phi_1}(p, q; p + q - k, k) \chi_B(p + q - k, k) \right] \end{aligned} \quad (50)$$

where $G_{1,2}(p) = \frac{i}{p^2 - m_{1,2}^2}$ are the propagators for dark scalars, $\tilde{\Gamma}$'s are the tree-level scattering amplitudes, and

$$\chi_A(p, q) \equiv G_2(p)G_1(q)\Gamma_{\phi_2\phi_1 \rightarrow \phi_2\phi_1}(p, q; p', q'), \quad (51)$$

$$\chi_B(p, q) \equiv G_2(p)G_1(q)\Gamma_{\phi_2^\dagger\phi_1 \rightarrow \phi_2\phi_1}(p, q; p', q'). \quad (52)$$

By using the relations between the tree-level scattering amplitudes,

$$\tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2^\dagger\phi_1} = \tilde{\Gamma}_{\phi_2^\dagger\phi_1 \rightarrow \phi_2\phi_1} = \frac{g_2}{g_1} \tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2\phi_1}, \quad (53)$$

$$\tilde{\Gamma}_{\phi_2^\dagger\phi_1 \rightarrow \phi_2^\dagger\phi_1} = \left(\frac{g_2}{g_1}\right)^2 \tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2\phi_1}, \quad (54)$$

we can rewrite the BS equations in Eqs. (55) and (56) as

$$i\chi_A(p, q) \simeq -G_2(p)G_1(q) \int \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2\phi_1} \times (p, q; p+q-k, k) \times \left[\chi_A(p+q-k, k) + \frac{g_2}{g_1} \chi_B(p+q-k, k) \right], \quad (55)$$

$$i\chi_B(p, q) \simeq -\frac{g_2}{g_1} G_2(p)G_1(q) \int \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2\phi_1} \times (p, q; p+q-k, k) \times \left[\chi_A(p+q-k, k) + \frac{g_2}{g_1} \chi_B(p+q-k, k) \right] \quad (56)$$

Thus, from Eqs. (55) and (56), we find that

$$\chi_B(p, q) \simeq \frac{g_2}{g_1} \chi_A(p, q). \quad (57)$$

Therefore, plugging Eq. (57) into Eqs. (55) or (56), both $\chi_B(p, q)$ and $\chi_A(p, q)$ satisfy the same BS equations,

$$i\chi_{A,B}(p, q) \simeq -\left(1 + \frac{g_2^2}{g_1^2}\right) G_2(p)G_1(q) \times \int \frac{d^4k}{(2\pi)^4} \tilde{\Gamma}_{\phi_2\phi_1 \rightarrow \phi_2\phi_1}(p, q; p+q-k, k) \times \chi_{A,B}(p+q-k, k). \quad (58)$$

Then, redefining $\chi_{A,B}(p, q) = \tilde{\chi}_{A,B}(P, Q)$ with

$$P = \frac{1}{2}(p+q), \quad Q = \mu \left(\frac{p}{m_2} - \frac{q}{m_1} \right), \quad (59)$$

with $\mu = m_1 m_2 / (m_1 + m_2)$ and using the BS wave functions in momentum space,

$$\tilde{\psi}_{A,B}(\vec{Q}) = \int \frac{dQ_0}{2\pi} \tilde{\chi}_{A,B}(P, Q), \quad (60)$$

we obtain the following coupled BS equations,

$$\left(\frac{\vec{Q}^2}{2\mu} - E \right) \tilde{\psi}_{A,B}(\vec{Q}) = -\left(1 + \frac{g_2^2}{g_1^2}\right) \int \frac{d^3k'}{(2\pi)^3} \tilde{V} \left(\sqrt{\frac{m_1}{m_2}} \vec{Q} + \sqrt{\frac{m_2}{m_1}} \vec{k}' \right) \tilde{\psi}_{A,B}(\vec{k}'), \quad (61)$$

with

$$\tilde{V} \left(\sqrt{\frac{m_1}{m_2}} \vec{Q} + \sqrt{\frac{m_2}{m_1}} \vec{k}' \right) \equiv -\frac{1}{4m_1 m_2} \cdot \frac{4g_1^2 m_1^2}{\left(\sqrt{\frac{m_1}{m_2}} \vec{Q} + \sqrt{\frac{m_2}{m_1}} \vec{k}' \right)^2 + m_2(2m_1 - m_2)}. \quad (62)$$

As a result, in terms of the BS wave functions in position space,

$$\psi_{A,B}(\vec{x}) = \int \frac{d^3\vec{Q}}{(2\pi)^3} e^{i\vec{Q}\cdot\vec{x}} \tilde{\psi}_{A,B}(\vec{Q}), \quad (63)$$

the coupled BS equations in Eq. (61) can be recast into the following,

$$-\frac{1}{2\mu} \nabla^2 \psi_{A,B}(\vec{x}) + V_{\text{eff}}(\vec{x}) \psi_{A,B} \left(-\frac{m_2}{m_1} \vec{x} \right) = E \psi_A(\vec{x}), \quad (64)$$

where

$$V_{\text{eff}}(\vec{x}) = -\left(1 + \frac{g_2^2}{g_1^2}\right) \frac{\alpha}{r} e^{-Mr}, \quad (65)$$

with $\alpha \equiv \frac{g_1^2}{4\pi}$ and $M \equiv m_2 \sqrt{2 - \frac{m_2}{m_1}}$. Here, from Eq. (57), the BS wavefunctions in position space are related by

$$\psi_B(\vec{x}) \simeq \frac{g_2}{g_1} \psi_A(\vec{x}). \quad (66)$$

It is illuminating to consider the BS equations before inserting Eq. (57) into Eqs. (55) or (56), in the form of the following coupled BS equations,

$$\left(\frac{\vec{Q}^2}{2\mu} - E_A \right) \tilde{\psi}_A(\vec{Q}) = -\int \frac{d^3k'}{(2\pi)^3} \tilde{V} \left(\sqrt{\frac{m_1}{m_2}} \vec{Q} + \sqrt{\frac{m_2}{m_1}} \vec{k}' \right) \times \left(\tilde{\psi}_A(\vec{k}') + \frac{g_2}{g_1} \tilde{\psi}_B(\vec{k}') \right), \quad (67)$$

$$\left(\frac{\vec{Q}^2}{2\mu} - E_B \right) \tilde{\psi}_B(\vec{Q}) = -\frac{g_2}{g_1} \int \frac{d^3k'}{(2\pi)^3} \tilde{V} \left(\sqrt{\frac{m_1}{m_2}} \vec{Q} + \sqrt{\frac{m_2}{m_1}} \vec{k}' \right) \times \left(\tilde{\psi}_A(\vec{k}') + \frac{g_2}{g_1} \tilde{\psi}_B(\vec{k}') \right), \quad (68)$$

which can be recast into the following,

$$-\frac{1}{2\mu} \nabla^2 \begin{pmatrix} \psi_A(\vec{x}) \\ \psi_B(\vec{x}) \end{pmatrix} + V(\vec{x}) \begin{pmatrix} 1 & \frac{g_2}{g_1} \\ \frac{g_2}{g_1} & \left(\frac{g_2}{g_1} \right)^2 \end{pmatrix} \begin{pmatrix} \psi_A \left(-\frac{m_2}{m_1} \vec{x} \right) \\ \psi_B \left(-\frac{m_2}{m_1} \vec{x} \right) \end{pmatrix} = \begin{pmatrix} E_A & 0 \\ 0 & E_B \end{pmatrix} \begin{pmatrix} \psi_A(\vec{x}) \\ \psi_B(\vec{x}) \end{pmatrix}. \quad (69)$$

Then, for the BS wave functions with the same energy eigenvalues, $E_A = E_B = E$, we find that only one of the linear combinations of the BS wave functions, $\psi_1 = \frac{1}{\sqrt{N}} (\psi_A + \frac{g_2}{g_1} \psi_B)$, with $N = 1 + \frac{g_2^2}{g_1^2}$, has a nonzero effective Yukawa-type potential given by Eq. (65), while the orthogonal combination, $\psi_2 \equiv \frac{1}{\sqrt{N}} (\psi_B - \frac{g_2}{g_1} \psi_A)$, is free from the Yukawa potential. Therefore, for the analysis of the enhanced cross sections with the u -channel resonances, we can set $\psi_B = \frac{g_2}{g_1} \psi_A$, which is the same as Eq. (66).

We note that in our model with the remaining Z_4 symmetry, the result for a toy model for self-resonant dark matter in Refs. [6, 7] is modified by the overall factor, $(1 + \frac{g_2^2}{g_1^2})$, in the effective Yukawa potential in the BS equation as in Eq. (65).

In addition to $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2\phi_1$, which was considered above, there are three other pairs of coupled self-scattering processes with u -channel enhancement which preserve the Z_4 symmetry: $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2^\dagger\phi_1$, $(\phi_2^\dagger, \phi_2)\phi_1^\dagger \rightarrow \phi_2\phi_1^\dagger$

and $(\phi_2^\dagger, \phi_2)\phi_1^\dagger \rightarrow \phi_2^\dagger\phi_1^\dagger$. Then, we find that there is an universal u -channel enhancement in all the DM co-scattering processes, because the linear combinations of the DM pairs for the other channels, $\psi_1 = \frac{1}{\sqrt{N}}(\psi_A + \frac{g_2}{g_1}\psi_B)$, satisfy the same BS equations as for the case with $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2\phi_1$. Actually, $(\phi_2^\dagger, \phi_2)\phi_1^\dagger \rightarrow \phi_2^\dagger\phi_1^\dagger$ is the CP conjugate of $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2\phi_1$, whereas $(\phi_2, \phi_2^\dagger)\phi_1 \rightarrow \phi_2^\dagger\phi_1$ is the CP conjugate of $(\phi_2^\dagger, \phi_2)\phi_1^\dagger \rightarrow \phi_2^\dagger\phi_1^\dagger$.

For the DM co-scattering, we can use the numerical solution for the DM co-scattering cross section, $\sigma_{\psi_1 \rightarrow \phi_1\phi_2}^{\text{total}}$, and perform the velocity average for dark matter in the halo. Then, we can define the effective u -channel scattering rate for dark matter as

$$\Gamma_{u,\text{scatt}} = \frac{n_1 n_2}{n_{\text{DM}}^2} \langle n_{\text{DM}} \sigma_{\psi_1 \rightarrow \phi_1\phi_2}^{\text{total}} v \rangle \equiv \rho_{\text{DM}} \frac{\langle \sigma v \rangle_{u,\text{eff}}}{m_1} \quad (70)$$

where $\langle \sigma v \rangle_{u,\text{eff}} = \langle \sigma_{\psi_1 \rightarrow \phi_1\phi_2}^{\text{total}} v \rangle r_1 (1 - r_1) / (\frac{m_2}{m_1} r_1 + 1 - r_1)$, with r_1 being the fraction of ϕ_1 in the DM relic abundance, namely, $r_1 = \Omega_1 / \Omega_{\text{DM}}$. Here, we note that $n_{\text{DM}} = n_1 + n_2$ and $\rho_{\text{DM}} = m_1 n_1 + m_2 n_2$.

In Fig. 2, we depict the effective u -channel self-scattering cross section for $\phi_1\phi_2 \rightarrow \phi_1\phi_2$ as a function of the averaged velocity $\langle v \rangle$, for $m_1 = 3, 7 \text{ GeV}$ on left and right plots, respectively. We chose $g_{\text{eff}} \equiv \sqrt{g_1^2 + g_2^2} = 0.3, 0.45$ on the left and right plots, respectively, and $r_1 = \frac{1}{2}$ for both plots. For $\Delta \equiv 1 - m_2/(2m_1)$, we also took $\Delta = 10^{-6}, 3 \times 10^{-6}$ on left, and $\Delta = 10^{-6}, 2 \times 10^{-6}$ on right, from top to bottom. In Fig. 2, we showed the fitted data used from Ref. [9] for comparison, but we don't pursue fitting the u -channel self-scattering cross section. Instead we just illustrate how the self-scattering cross section can be made velocity-dependent due to the u -channel resonance such that the self-scattering gets suppressed at large DM velocities for galaxy clusters for perturbative DM couplings [6, 7].

3.3 Some limiting cases of the Bethe–Salpeter equation

By taking the BS wave function for the DM self-scattering with the u -channel resonances [6, 7] as

$$\psi_{\text{BS}}(\vec{x}) = \frac{u_l(x)}{x} Y_l^m(\theta, \phi), \quad (71)$$

with $x \equiv \frac{1}{2}\mu\alpha'r$ and $\alpha' \equiv (1 + g_2^2/g_1^2)\alpha$, we can rewrite the BS equation in Eq. (64) with $E = \frac{1}{2}\mu v^2$ as

$$\left[\frac{d^2}{dx^2} - \frac{l(l+1)}{x^2} \right] u_l(x) + \frac{4}{bx} e^{-cx} (-1)^l u_l(bx) + a^2 u_l(x) = 0 \quad (72)$$

¹ A and B stand for the ordered pair of the initial states in the DM co-scattering.

where

$$a = \frac{2v}{\alpha'}, \quad b = \frac{m_2}{m_1}, \quad c = \frac{2M}{\mu\alpha'}. \quad (73)$$

Then, for $cx \lesssim 1$, we can approximate $e^{-cx} \simeq 1 - cx$, so the s -wave solution, $u_0(x)$, satisfies the following approximate equation,

$$\frac{d^2 u_0(x)}{dx^2} + \frac{4}{bx} u_0(bx) + \left(a^2 u_0(x) - \frac{4c}{b} u_0(bx) \right) = 0. \quad (74)$$

As a result, for $a^2 \gtrsim 4c/b$, namely,

$$v \gtrsim \sqrt{\frac{2\alpha' M m_1}{m_2 \mu}} \simeq \sqrt{\frac{3\alpha' M}{2m_1}} \quad (75)$$

where $m_2 \simeq 2m_1$ is taken on the right-hand side, the BS equation for s -wave in Eq. (74) becomes Coulomb-like or weakly-coupled (Coulomb limit), as far as the non-linear wave function is comparable to the linear counterpart, $u_0(bx) \sim u_0(x)$, for $cx \lesssim 1$. In this case, we can apply the standard results for the DM self-scattering or the Sommerfeld factor for the case with the Coulomb potential [19, 20]. We note that the lower bound on the DM velocity in the Coulomb limit can be rewritten as the upper bound on $\Delta \equiv 1 - m_2/(2m_1)$, as follows,

$$\Delta \lesssim \frac{v^4}{18\alpha'^2}. \quad (76)$$

Thus, for a given averaged DM velocity in the dwarf galaxies, the amount of detuning from $m_2 = 2m_1$ is constrained.

On the other hand, in the opposite limit with $a^2 \lesssim 4c/b$ or $v \lesssim \sqrt{\frac{3\alpha' M}{2m_1}}$, the BS equation in Eq. (74) becomes the radial equation for the hydrogen atom for $u_0(bx) \sim u_0(x)$, so the DM bound states between ϕ_1 and ϕ_2 form only for $4c/b \sim \frac{1}{n^2}$ with n being integer or

$$\frac{8Mm_1}{\alpha'\mu m_2} \sim \frac{1}{n^2}. \quad (77)$$

Then, for $m_2 \simeq 2m_1$, the above resonant condition for the DM bound states with the u -channel becomes

$$m_1 \sim \frac{12M}{\alpha'} n^2 \quad (78)$$

or

$$\Delta \sim \frac{\alpha'^2}{1152n^4}. \quad (79)$$

We note that the combination of the above resonant condition with the non-perturbative limit, namely, $v \lesssim \sqrt{\frac{3\alpha' M}{2m_1}}$, gives rise to the upper bound on the DM velocity by $v \lesssim \frac{1}{2\sqrt{2}n}\alpha'$.

Even if the DM bound states do not form, the self-scattering cross section [19–21] and the DM annihilation cross section [19, 20, 22] can be enhanced significantly near the resonant conditions. In particular, the resonant condition

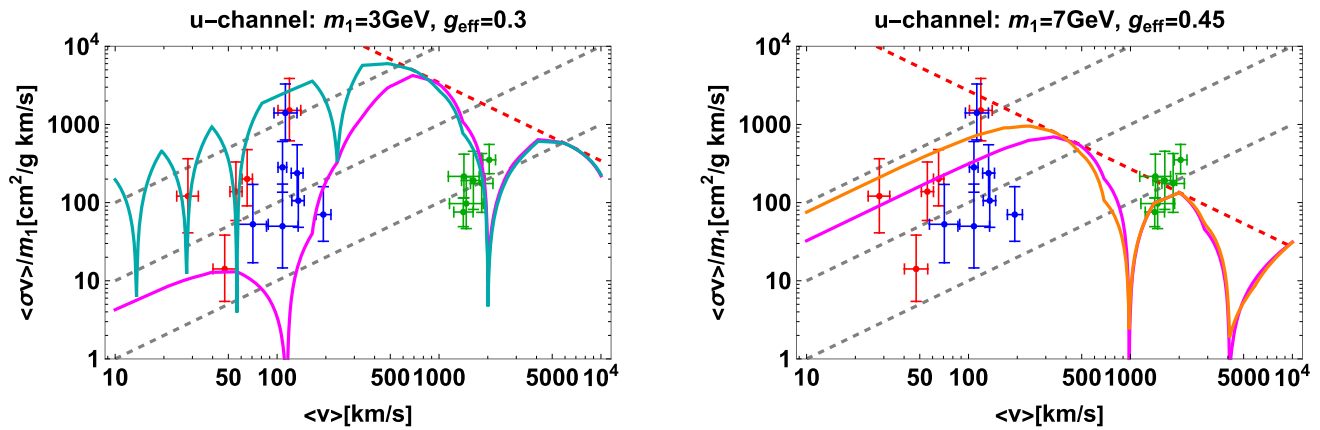


Fig. 2 The effective u -channel self-scattering cross section for $\phi_1\phi_2 \rightarrow \phi_1\phi_2$ as a function of the averaged velocity $\langle v \rangle$. We chose the DM self-couplings as $g_{\text{eff}} \equiv \sqrt{g_1^2 + g_2^2} = 0.3, 0.45$ on the left and

right plots, respectively, and $r_1 = \frac{1}{2}$ for both plots. The black dashed lines correspond to $\sigma_{\text{self}}/m_1 = 10, 1, 0.1 \text{ cm}^2/\text{g}$ from top to bottom, and the regions above the red dashed lines exceed the unitarity bound

in Eq. (78) leads to the phase shift for the DM co-scattering with the u -channel being $\delta_0 \simeq \frac{\pi}{2}$ [19–21], so the corresponding DM co-scattering cross section in Eq. (48) and the co-scattering rate in Eq. (70) becomes maximized, because $\sigma_{\psi_1 \rightarrow \phi_1\phi_2}^{\text{total}} \simeq \frac{4\pi}{k^2}$.

3.4 Comparison with t -channel case and enhancement factors for u -channel case

We consider the analytic results for the Yukawa type potential, $V(r) = -\frac{\alpha'}{r} e^{-Mr}$, in the case of the t -channel resonances for the DM self-scattering and interpret them for the DM co-scattering cross section and the Sommerfeld factor for the DM annihilation in the case of the u -channel resonances in our work.

There exist analytic solutions to the BS equation in the case for the DM self-scattering with the t -channel resonances, when the Yukawa potential is replaced by the Hulthén potential,

$$V_H(r) = -\frac{\alpha' \delta e^{-\delta r}}{1 - e^{-\delta r}}, \quad (80)$$

with $\delta = \frac{\pi^2}{6} M$. Then, the phase shift δ_0 for the s -wave self-scattering for dark matter [19–21] is given by

$$\delta_0 = \arg\left(\frac{i\Gamma(\lambda_+ + \lambda_- - 2)}{\Gamma(\lambda_+)\Gamma(\lambda_-)}\right) \quad (81)$$

where

$$\lambda_{\pm} = 1 + iw \pm w\sqrt{\frac{y}{w} - 1}, \quad (82)$$

with $w = \frac{k}{\delta}$ and $y = \frac{\alpha'}{v}$. Moreover, the Sommerfeld factor for the s -wave annihilation for dark matter is also given [19, 20, 22] by

$$S_0 = \frac{\left| \frac{\Gamma(\lambda_+)\Gamma(\lambda_-)}{\Gamma(\lambda_+ + \lambda_- - 1)} \right|}{\pi y \sinh(2\pi w)} = \frac{1}{\cosh(2\pi w) - \cos\left(2\pi w\sqrt{\frac{y}{w} - 1}\right)}. \quad (83)$$

We note that the Sommerfeld factor can be recast into another form, which is more convenient when the Coulomb limit with $y/w \lesssim 1$ is taken, in the following,

$$S_0 = \frac{\pi y \sinh(2\pi w)}{2 \sinh\left[\pi w\left(1 - \sqrt{1 - \frac{y}{w}}\right)\right] \sinh\left[\pi w\left(1 + \sqrt{1 - \frac{y}{w}}\right)\right]}. \quad (84)$$

Using the above results from the t -channel resonances, we interpret the conditions for the large self-scattering and the Sommerfeld factor for dark matter in our case. We first note that $w \gtrsim 1$ is the classical regime and $w \lesssim 1$ is the quantum regime; $y/w \lesssim 1$ is the Coulomb limit (or weakly-coupled regime) and $y/w \gtrsim 1$ is the strongly-coupled regime [23].

First, in the Coulomb limit with $y/w \lesssim 1$, which corresponds to $\alpha'\delta/(kv) \lesssim 1$, the Sommerfeld factor becomes

$$S_0 \simeq \frac{\pi\alpha'}{v} \frac{1}{1 - e^{-\frac{\pi\alpha'}{v}}}, \quad (85)$$

and the phase shift becomes

$$\begin{aligned} \delta_0 &= \arg\left(\frac{\Gamma(\lambda_+ + \lambda_- - 1)}{\Gamma(\lambda_+)\Gamma(\lambda_-)}\right) \\ &\simeq \arg\left(\Gamma\left(1 - i\frac{y}{2}\right)\right) + \frac{1}{2}y \ln(2w) \end{aligned} \quad (86)$$

where we used $\Gamma(\lambda_+ + \lambda_- - 1) = (\lambda_+ + \lambda_- - 2)\Gamma(\lambda_+ + \lambda_- - 2) = 2iw\Gamma(\lambda_+ + \lambda_- - 2)$ in the first line and we took $w = k/\delta \gtrsim 1$ (classical) in the second line. Then, the first term in Eq. (86) is the l -dependent phase shift and the second

term corresponds to the universal phase shift [24], which diverges in the limit of $\delta \rightarrow 0$. As a result, for $y = \alpha'/v \gtrsim 1$ (non-relativistic), we obtain

$$\delta_0 \simeq \frac{\alpha'}{2v} \ln \left(\frac{kv}{\delta\alpha'} \right) + \frac{\alpha'}{2v} - \frac{\pi}{4}. \quad (87)$$

Therefore, for $\alpha'/v \gtrsim 1$, the Sommerfeld factor becomes larger than unity and the phase shift varies between 0 and $\frac{\pi}{2}$.

Second, from Eq. (81), the resonance condition for the DM self-scattering, $\delta_0 = \frac{\pi}{2}$, is achieved for $\lambda_- \simeq -n$ with n being zero or negative integer, namely,

$$w \sqrt{\frac{y}{w} - 1} \simeq n + 1, \quad (88)$$

for $w \lesssim 1$ (quantum). Thus, for $y/w \gtrsim 1$ (strongly-coupled), the resonance condition becomes

$$M = \frac{4\alpha'm_1}{\pi^2} \frac{1}{(n+1)^2}, \quad n = 0, -1, -2, -3, \dots, \quad (89)$$

which corresponds to Eq. (78) in the previous subsection. Here, we took $\mu \simeq \frac{2}{3}m_1$ for $m_2 \simeq 2m_1$. We note that under the condition given by Eq. (88), the Sommerfeld factor in Eq. (83) becomes

$$\begin{aligned} S_0 &\simeq \frac{\pi y \sinh(2\pi w)}{\cosh(2\pi w) - 1} \\ &\simeq \frac{y}{w} \simeq \frac{\alpha'^2}{\pi^2 v^2} \end{aligned} \quad (90)$$

where we took $w \lesssim 1$ and Eq. (89) in the second line. Therefore, for $w \lesssim 1$, the phase shift can be made close to $\frac{\pi}{2}$ by the resonance condition and the Sommerfeld factor becomes larger than unity for $\alpha'/(v\pi) \gtrsim 1$.

We also remark that away from the poles of the gamma function, a low momentum expansion of the phase shift can be taken in the effective range theory, as follows,

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2}r_0 k^2, \quad (91)$$

with

$$\begin{aligned} a &= \frac{1}{\delta} \left(\psi^{(0)}(1+\eta) + \psi^{(0)}(1-\eta) + 2\gamma \right), \quad (92) \\ r_0 &= \frac{2}{3}a - \frac{1}{3\delta\eta} \left[\psi^{(0)}(1+\eta) + \psi^{(0)}(1-\eta) + 2\gamma \right]^{-2} \\ &\quad \times \left[3 \left(\psi^{(1)}(1+\eta) - \psi^{(1)}(1-\eta) \right) \right. \\ &\quad \left. + \eta \left(\psi^{(2)}(1+\eta) + \psi^{(2)}(1-\eta) + 16\zeta(3) \right) \right]. \end{aligned} \quad (93)$$

Here, $\eta \equiv yw = \frac{\alpha'\mu}{\delta}$ is a velocity-dependent quantity, $\gamma \simeq 0.5772$ is the Euler-Mascheroni constant, $\psi^{(n)}(z)$ are the polygamma functions of order n , and $\zeta(3)$ is the Riemann zeta function. As a result, applying to the DM co-scattering cross section in Eq. (48), we get

$$\sigma_{\psi_1 \rightarrow \phi_1 \phi_2}^{\text{total}} \simeq \frac{4\pi a^2}{1 + k^2(a^2 - ar_0) + \frac{1}{4}a^2 r_0^2 k^4}. \quad (94)$$

We note that the effective range term can be ignored for $k \ll 1/a$ in the Born approximation, so $\sigma_{\psi_1 \rightarrow \phi_1 \phi_2}^{\text{total}} \simeq 4\pi a^2$, which is velocity-independent.

4 DM self-scattering with s -channel resonances

We discuss the implications of the resonant condition taken in the u -channel co-scattering processes for the s -channel processes for self-annihilation processes. We perform the velocity averaged self-scattering cross section for the lighter component of dark matter and compare it with the u -channel co-scattering case.

4.1 Dark matter self-scattering at tree level

We also consider the elastic scattering, $\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}$ and its complex conjugate, for which the dark scalars, s, a , contributes to the s -channel. In this case, the contribution from the dark scalars s, a can be important because they are absolutely stable for $m_s, m_a < 2m_1$. On the other hand, in the case of $\phi_1^* \phi_1 \rightarrow \phi_1^* \phi_1$, $ss \rightarrow ss, aa \rightarrow aa$, and $sa \rightarrow sa$ (or $\phi_2 \phi_2 \rightarrow \phi_2^{(\dagger)} \phi_2^{(\dagger)}$ and $\phi_2^\dagger \phi_2 \rightarrow \phi_2^\dagger \phi_2$ for degenerate masses for s and a), there are also s -channel contributions from the dark Higgs h_X or the extra gauge boson X , but they are limited by nonzero decay widths. Thus, we focus on $\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}$ with the DM s -channel resonances.

For the DM scattering process, $\phi_1(q)\phi_1(p) \rightarrow \phi_1(q')\phi_1(p')$, with s, a in the s -channels, the corresponding tree-level scattering amplitude $\tilde{\Gamma}_{s,1}(p, q; p', q')$ is given by

$$\begin{aligned} \tilde{\Gamma}_{s,1}(p, q; p', q') &= -\frac{4g_s^2 m_1^2}{(p+q)^2 - m_s^2} - \frac{4g_a^2 m_1^2}{(p+q)^2 - m_a^2} \\ &= \frac{4g_s^2 m_1^2}{|\vec{p} + \vec{q}|^2 + m_s^2 - E_{\text{cm}}^2} + \frac{4g_a^2 m_1^2}{|\vec{p} + \vec{q}|^2 + m_a^2 - E_{\text{cm}}^2}, \end{aligned} \quad (95)$$

with $E_{\text{cm}} = p_0 + q_0$ being the COM energy. For the non-relativistic limit for dark matter, $E_{\text{cm}} \approx 2m_1 + \frac{1}{2m_1}(\vec{p}^2 + \vec{q}^2)$, so we can approximate Eq. (95) in the COM frame to

$$\begin{aligned} \tilde{\Gamma}_{s,1}(p, q; p', q') &\approx \frac{4g_s^2 m_1^2}{m_s^2 - 4m_1^2 - 4\vec{p}^2} \\ &\quad + \frac{4g_a^2 m_1^2}{m_a^2 - 4m_1^2 - 4\vec{p}^2}. \end{aligned} \quad (96)$$

Thus, for $\vec{p} = 0$, the above tree-level scattering amplitude diverges at $m_s = 2m_1$ or $m_a = 2m_1$, similarly to the case with the u -channel resonances. In general, we can take $m_s, m_a < 2m_1$ such that the lighter one between s and a is stable, so there is no decay width appearing in the propagator for either s or a .

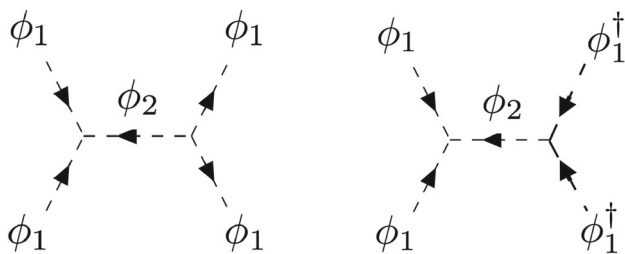


Fig. 3 Tree-level Feynman diagrams for the DM self-scattering processes with s -channel, $\phi_1 \phi_1 \rightarrow \phi_1 \phi_1$ and $\phi_1 \phi_1 \rightarrow \phi_1^\dagger \phi_1^\dagger$

Similarly, for the DM scattering process, $\phi_1(q)\phi_1(p) \rightarrow \phi_1^\dagger(q')\phi_1^\dagger(p')$, with s, a in the s -channels, the corresponding tree-level scattering amplitude $\tilde{\Gamma}_{s,2}(p, q; p', q')$ is given by

$$\begin{aligned} \tilde{\Gamma}_{s,2}(p, q; p', q') &= -\frac{4g_s^2 m_1^2}{(p+q)^2 - m_s^2} + \frac{4g_a^2 m_1^2}{(p+q)^2 - m_a^2} \\ &= \frac{4g_s^2 m_1^2}{|\vec{p} + \vec{q}|^2 + m_s^2 - E_{\text{cm}}^2} - \frac{4g_a^2 m_1^2}{|\vec{p} + \vec{q}|^2 + m_a^2 - E_{\text{cm}}^2}. \end{aligned} \quad (97)$$

Then, in the non-relativistic limit for dark matter, we can approximate Eq. (97) in the COM frame to

$$\tilde{\Gamma}_{s,2}(p, q; p', q') \approx \frac{4g_s^2 m_1^2}{m_s^2 - 4m_1^2 - 4\vec{p}^2} - \frac{4g_a^2 m_1^2}{m_a^2 - 4m_1^2 - 4\vec{p}^2}. \quad (98)$$

Thus, for $\vec{p} = 0$, the above tree-level scattering amplitude diverges at $m_s = 2m_1$ or $m_a = 2m_1$, similarly to the case with the u -channel resonances. As compared to $\tilde{\Gamma}_{s,1}$, there is a deconstructive interference between s and a contributions, so $\tilde{\Gamma}_{s,2}$ vanishes in the limit of an unbroken $U(1)'$, namely, for $g_s = g_a$ and $m_s = m_a$.

In the following discussion, as in the u -channel case, we focus the case that the real and imaginary parts of the complex scalar ϕ_2 have degenerate masses, namely, for $m_s = m_a = m_2$. In this case, from the tree-level Feynman diagrams shown in Fig. 3, the s -channel self-scattering amplitudes become

$$\tilde{\Gamma}_{s,1}(p, q; p', q') = -\frac{4(g_1^2 + g_2^2)m_1^2}{(p+q)^2 - m_2^2}, \quad (99)$$

$$\tilde{\Gamma}_{s,2}(p, q; p', q') = -\frac{8g_1 g_2 m_1^2}{(p+q)^2 - m_2^2}. \quad (100)$$

Then, we can take $m_2 < 2m_1$ such that ϕ_2 is absolutely stable, so there is no decay width appearing in the propagator for ϕ_2 . For the general case with split masses for ϕ_2 , as far as the resonant mass condition is satisfied between ϕ_1 and the lighter component of ϕ_2 , the current discussion on the s -channel case is still valid.

The DM self-scattering cross section with s -channel dominance, $\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}$, is, at tree level, given by

$$\sigma_{\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}} = \frac{1}{16\pi s} \left| \tilde{\Gamma}_{s, \phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}} \right|^2. \quad (101)$$

Then, again considering the case with $m_s = m_a = m_2$ and taking the non-relativistic limit for ϕ_1 , the above DM self-scattering cross section becomes

$$\sigma_{\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}} \simeq \frac{\tilde{g}^4}{8\pi m_1^2} \frac{1}{(v_c^2 + v^2)^2}, \quad (102)$$

with $v_c^2 = 4\Delta(2 - \Delta)$, $\Delta \equiv 1 - m_2/(2m_1) \geq 0$, and

$$\tilde{g} \equiv \begin{cases} \sqrt{g_1^2 + g_2^2}, & \phi_1 \phi_1 \rightarrow \phi_1 \phi_1, \\ \sqrt{2g_1 g_2}, & \phi_1 \phi_1 \rightarrow \phi_1^\dagger \phi_1^\dagger. \end{cases} \quad (103)$$

We comment on the s -channel self-scattering in comparison to the u -channel co-scattering discussed in the previous section. Comparing the u -channel co-scattering cross section in Eq. (48) and the s -channel self-scattering cross section for $\phi_1 \phi_1 \rightarrow \phi_1 \phi_1$ in Eq. (102), the former is dominant if

$$\frac{4\pi}{k^2} \sin^2 \delta_0 \gtrsim \frac{2\pi \alpha'^2}{m_1^2 (v_c^2 + v^2)^2}. \quad (104)$$

For $v \gtrsim v_c \simeq \sqrt{8\Delta}$, we need $v^2 \gtrsim \frac{2}{9} \alpha'^2 / \sin^2 \delta_0$, for which the enhancement factor for the u -channel is mild for $\delta_0 \simeq \frac{\pi}{2}$, as discussed in the previous section. On the other hand, for $v \lesssim v_c \simeq \sqrt{8\Delta}$ and $\delta_0 \simeq \frac{\pi}{2}$, we need $v^2 \lesssim 288(\Delta/\alpha')^2$. Therefore, taking $v^2 \sim 10^{-8}$ at dwarf galaxies, we find that the u -channel co-scattering is dominant over the s -channel self-scattering, as long as $\alpha' \lesssim 0.17(\Delta/10^{-6}) |\sin \delta_0|$. However, for a small δ_0 , namely, off the resonance, we also need to take into account the s -channel self-scattering. A similar conclusion can be drawn for the cross section for another s -channel self-scattering process, $\phi_1 \phi_1 \rightarrow \phi_1^\dagger \phi_1^\dagger$, with $g_1 = g_2$.

4.2 Velocity-averaged self-scattering

There are several definitions for the averaged self-scattering cross section, such as the velocity-averaged cross section $\langle \sigma v \rangle$, the momentum-transfer cross section $\langle \sigma v^2 \rangle$, the energy-transfer cross section, $\langle \sigma v^3 \rangle$ [23], etc. In this work, we consider the velocity-averaged self-scattering cross section, as follows,

$$\langle \sigma v \rangle = \int_0^{v_{\text{esp}}} f(v, v_0) (\sigma v) dv, \quad (105)$$

where the Maxwell–Boltzmann distribution for the dark matter velocity is given by

$$f(v, v_0) = \frac{4v^2}{v_0^3 \sqrt{\pi}} e^{-v^2/v_0^2}, \quad (106)$$

with v_0 being related to the averaged velocity by $\langle v \rangle \simeq 2v_0/\sqrt{\pi}$, and v_{esp} is the escape velocity in the dark matter halo. Here, σ can be $\sigma_{\phi_1 A \rightarrow \phi_1 A}^{\text{total}}$ in Eq. (48) for the DM co-scattering or $\sigma_{\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}}$ in Eq. (102) for the DM self-scattering.

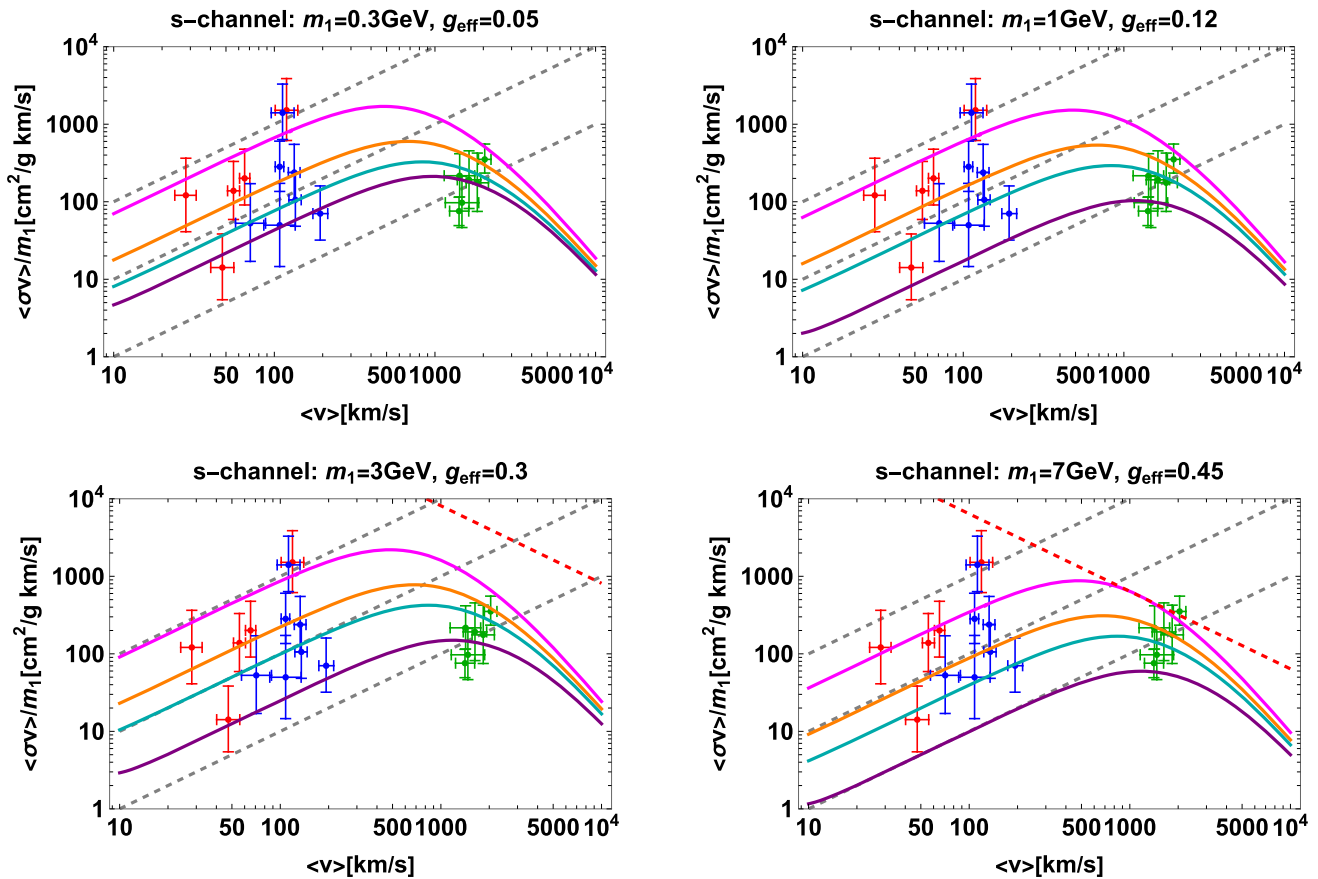


Fig. 4 The effective s -channel self-scattering cross section for $\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}$ as a function of the averaged velocity $\langle v \rangle$. The effective coupling is given by $g_{\text{eff}} = (g_1^4 + g_2^4 + 6g_1^2 g_2^2)^{\frac{1}{4}}$. We

chose $r_1 = \frac{1}{2}$ for all the plots. The black dashed lines correspond to $\sigma_{\text{self}}/m_1 = 10, 1, 0.1 \text{ cm}^2/\text{g}$ from top to bottom, and the regions above the red dashed lines exceed the unitarity bound

For the DM self-scattering, we obtain the velocity-averaged cross section as

$$\langle \sigma v \rangle_{\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}} = \frac{\tilde{g}^4}{2\pi^{3/2} m_1^2 v_0^3} \cdot F\left(\frac{v_c^2}{v_0^2}\right), \quad (107)$$

with

$$F(a^2) = \int_0^{\frac{v_{\text{esp}}}{v_0}} \frac{t^3 e^{-t^2}}{(a^2 + t^2)^2} dt \simeq \frac{1}{2} \left((1+a) e^a \Gamma(0, a) - 1 \right). \quad (108)$$

Here, $\Gamma(0, a)$ is the incomplete gamma function given by

$$\Gamma(0, a) = \int_a^\infty \frac{e^{-t}}{t} dt. \quad (109)$$

Therefore, from the effective s -channel scattering rate for dark matter,

$$\Gamma_{s, \text{scatt}} = \frac{n_1^2}{n_{\text{DM}}^2} \langle n_1 \sigma_s^{\text{total}} v \rangle \equiv \rho_{\text{DM}} \frac{\langle \sigma v \rangle_{s, \text{eff}}}{m_1}, \quad (110)$$

with $\sigma_s^{\text{total}} = \sigma_{\phi_1 \phi_1 \rightarrow \phi_1 \phi_1} + \sigma_{\phi_1 \phi_1 \rightarrow \phi_1^{(\dagger)} \phi_1^{(\dagger)}}$, we define the effective s -channel self-scattering cross section as $\langle \sigma v \rangle_{s, \text{eff}} = \langle \sigma_s^{\text{total}} v \rangle r_1^3 / (\frac{m_2}{m_1} r_1 + 1 - r_1)^2$.

We comment on the limits of the velocity-averaged cross section for the s -channel self-scattering. Using $\Gamma(0, a) \simeq -\gamma - \ln a$ for $a \ll 1$, we obtain the velocity integral for $v_c \ll v_0$ as

$$F\left(\frac{v_c^2}{v_0^2}\right) \simeq \frac{1}{2} \ln \frac{v_0^2}{v_c^2} \simeq \frac{1}{2} \ln \frac{v_0^2}{8\Delta} \quad (111)$$

where we approximated $v_c^2 \simeq 8\Delta$ for $\Delta \ll 1$. Therefore, there is a logarithmic enhancement for the DM self-scattering cross section due to the s -channel resonance, but it is bounded by unitarity as $\langle \sigma v \rangle \leq 4\pi \langle \frac{v}{k^2} \rangle \equiv \langle \sigma v \rangle_{\text{max}}$. For $k = \frac{1}{2} m_1 v$, we obtain $\langle \sigma v \rangle_{\text{max}} = 32\sqrt{\pi}/(m_1^2 v_0)$. On the other hand, for $v_c \gg v_0$, the velocity integral becomes

$$F\left(\frac{v_c^2}{v_0^2}\right) \simeq \frac{v_0^2}{2v_c^2} = \frac{v_0^2}{16\Delta(2-\Delta)}. \quad (112)$$

In this case, there is no enhancement for the DM self-scattering.

In Fig. 4, we depict the effective s -channel self-scattering cross section for $\phi_1\phi_1 \rightarrow \phi_1^{(\dagger)}\phi_1^{(\dagger)}$ as a function of the averaged velocity $\langle v \rangle$, for $r_1 = \frac{1}{2}$ and various choices of m_1 and $g_{\text{eff}} = (g_1^4 + g_2^4 + 6g_1^2g_2^2)^{\frac{1}{4}}$. We also chose $\Delta = 10^{-6}, 2 \times 10^{-6}, 3 \times 10^{-6}, 6 \times 10^{-6}$, in the order of magenta, orange, cyan and purple lines (namely, from top to bottom). In the plots in the lower panel of Fig. 4, the regions above the red dashed lines exceed the unitarity bound. In Fig. 4, we also showed the simulation data used from Ref. [9] for comparison, but we don't pursue fitting the s -channel self-scattering cross section.

As a result, we find that the self-scattering cross section becomes velocity-dependent due to the s -channel resonance such that it becomes suppressed at large DM velocities for galaxy clusters. However, for perturbative DM couplings as in the upper panel in Fig. 4, for a sub-GeV scale DM mass, we need $\Delta \sim 10^{-6}$ to get the velocity-dependent self-scattering from the s -channel consistent with the observed data. For instance, for $m_1 = 3$ GeV and $g_{\text{eff}} = 0.3$, we can take a wide range of Δ such as $\Delta = 1 - 6 \times 10^{-6}$ for the s -channel case, while a smaller Δ such as $\Delta = (1 - 3) \times 10^{-6}$ is favored for the u -channel case. Therefore, the s -channel enhancement for the DM self-scattering is less efficient than in the case with the u -channels.

5 Relic density for dark matter

We present the Boltzmann equations governing the relic densities for two-component dark matter in our model and show some benchmark models for DM and mediator masses of weak-scale or sub-GeV-to-GeV scale, satisfying the correct relic density.

5.1 Dark matter annihilations

Multi-component dark matter in our model can annihilate through the self-interactions in the dark sector in Eq. (19) such as a $U(1)'$ invariant term, $\phi_2^\dagger\phi_1^2$, a Z_4 invariant term, $\phi_2\phi_1^2$, and the interactions involving the dark Higgs in Eq. (21) and the extra gauge boson in Eq. (22).

We consider the benchmark models with $m_s \simeq 2m_1$ or $m_a \simeq 2m_1$ for the u -channel resonances in our model. Thus, the self-interactions for dark matter give rise to the self-annihilations, $ss(aa) \rightarrow \phi_1\phi_1^\dagger$; the dark Higgs and the extra gauge boson lead to the extra self-annihilations, $ss(aa), \phi_1\phi_1^\dagger \rightarrow f\bar{f}, VV, h_1h_1, h_2h_2, h_1h_2, XX$, with f being the SM fermions and $V = W, Z$, and the co-annihilations, $sa \rightarrow f\bar{f}, \phi_1^{(\dagger)}s(a) \rightarrow \phi_1^{(\dagger)}h_1(h_2), \phi_1^{(\dagger)}s(a) \rightarrow \phi_1^{(\dagger)}X, \phi_1s, \phi_1a \rightarrow \phi_1^\dagger h_1(h_2), \phi_1^{(\dagger)}s(a) \rightarrow \phi_1^{(\dagger)}a(s)$.

In particular, for $m_s = m_a = m_2$, the self-interactions for dark matter give rise to the self-annihilations, $\phi_2\phi_2^\dagger \rightarrow \phi_1\phi_1^\dagger, \phi_2\phi_2 \rightarrow \phi_1\phi_1^\dagger, \phi_2^\dagger\phi_2^\dagger \rightarrow \phi_1\phi_1^\dagger$; the dark Higgs and the extra gauge boson lead to the extra self-annihilations, $\phi_2\phi_2^\dagger, \phi_1\phi_1^\dagger \rightarrow f\bar{f}, VV, h_1h_1, h_2h_2, h_1h_2, XX$, and the co-annihilations, $\phi_2\phi_1^\dagger \rightarrow \phi_1X(h_1, h_2), \phi_2^\dagger\phi_1 \rightarrow \phi_1X(h_1, h_2)$, and their complex conjugates.

5.2 Boltzmann equations

For simplicity, we consider the case with $m_s = m_a = m_2$. We also assume that two-component dark matter keeps in kinetic equilibrium during the freeze-out process.² For instance, relatively small couplings from dark photon or Higgs portals in our model are sufficient to maintain the two-component dark matter in kinetic equilibrium [27, 28]. Then, the evolution of the number densities are governed by the Boltzmann equations,

$$\begin{aligned} \dot{n}_{\phi_1} + 3Hn_{\phi_1} = & -\langle\sigma v\rangle_{\phi_1\phi_1^\dagger \rightarrow f\bar{f}, VV, h_1h_1, h_2h_2, h_1h_2, XX} \\ & \times \left(n_{\phi_1}n_{\phi_1^\dagger} - n_{\phi_1}^{\text{eq}}n_{\phi_1^\dagger}^{\text{eq}} \right) \\ & + \langle\sigma v\rangle_{\phi_2\phi_2^\dagger \rightarrow \phi_1\phi_1^\dagger} \left(n_{\phi_2}n_{\phi_2^\dagger} - n_{\phi_1}n_{\phi_1^\dagger} \right) \\ & + \langle\sigma v\rangle_{\phi_2\phi_2 \rightarrow \phi_1\phi_1^\dagger} \left(n_{\phi_2}^2 - n_{\phi_1}n_{\phi_1^\dagger} \right) \\ & + \langle\sigma v\rangle_{\phi_2^\dagger\phi_2^\dagger \rightarrow \phi_1\phi_1^\dagger} \left(n_{\phi_2^\dagger}^2 - n_{\phi_1}n_{\phi_1^\dagger} \right) \\ & - 2\langle\sigma v\rangle_{\phi_1\phi_1 \rightarrow \phi_1^\dagger\phi_1^\dagger} \left(n_{\phi_1}^2 - (n_{\phi_1^\dagger})^2 \right) \\ & + \sum_{i=X, h_1, h_2} \left[\langle\sigma v\rangle_{\phi_2\phi_1^\dagger \rightarrow \phi_1 i} \left(n_{\phi_2}n_{\phi_1^\dagger} - n_{\phi_1}n_i \right) \right. \\ & \left. + \langle\sigma v\rangle_{\phi_2^\dagger\phi_1 \rightarrow \phi_1 i} \left(n_{\phi_2^\dagger}n_{\phi_1} - n_{\phi_1}n_i \right) \right] \\ & - \sum_{i=X, h_1, h_2} \left[\langle\sigma v\rangle_{\phi_2^\dagger\phi_1 \rightarrow \phi_1^\dagger i} \left(n_{\phi_2^\dagger}n_{\phi_1} - n_{\phi_1^\dagger}n_i \right) \right. \\ & \left. + \langle\sigma v\rangle_{\phi_2\phi_1 \rightarrow \phi_1^\dagger i} \left(n_{\phi_2}n_{\phi_1} - n_{\phi_1^\dagger}n_i \right) \right], \quad (113) \end{aligned}$$

$$\begin{aligned} \dot{n}_{\phi_2} + 3Hn_{\phi_2} = & -\langle\sigma v\rangle_{\phi_2\phi_2^\dagger \rightarrow f\bar{f}, VV, h_1h_1, h_2h_2, h_1h_2, XX} \\ & \times \left(n_{\phi_2}n_{\phi_2^\dagger} - n_{\phi_2}^{\text{eq}}n_{\phi_2^\dagger}^{\text{eq}} \right) \\ & - \langle\sigma v\rangle_{\phi_2\phi_2^\dagger \rightarrow \phi_1\phi_1^\dagger} \left(n_{\phi_2}n_{\phi_2^\dagger} - n_{\phi_1}n_{\phi_1^\dagger} \right) \\ & - \langle\sigma v\rangle_{\phi_1\phi_2 \rightarrow \phi_1\phi_2^\dagger} n_{\phi_1} \left(n_{\phi_2} - n_{\phi_2^\dagger} \right) \\ & - 2\langle\sigma v\rangle_{\phi_2\phi_2 \rightarrow \phi_1\phi_1^\dagger} \left(n_{\phi_2}^2 - n_{\phi_1}n_{\phi_1^\dagger} \right) \\ & - 2\langle\sigma v\rangle_{\phi_2\phi_2 \rightarrow \phi_2^\dagger\phi_2^\dagger} \left(n_{\phi_2}^2 - (n_{\phi_2^\dagger})^2 \right) \end{aligned}$$

² The co-annihilations, $\phi_2\phi_1^\dagger \rightarrow \phi_1X(h_1, h_2), \phi_2^\dagger\phi_1 \rightarrow \phi_1X(h_1, h_2)$, could make the dark matter ϕ_1 heated due to the mass due to the mass difference between ϕ_2 and the mediator particles. But, the Sommerfeld effects for the co-annihilation processes during the freeze-out are small and the elastic scattering between ϕ_1 and the SM particles can maintain the kinetic equilibrium for dark matter, so the heating effect is small.

$$\begin{aligned}
& -\langle\sigma v\rangle_{\phi_2\phi_2\rightarrow\phi_2^\dagger\phi_2}n_{\phi_2}(n_{\phi_2}-n_{\phi_2^\dagger}) \\
& -\sum_{i=X,h_1,h_2}\left[\langle\sigma v\rangle_{\phi_2\phi_1^\dagger\rightarrow\phi_1i}(n_{\phi_2}n_{\phi_1^\dagger}-n_{\phi_1}n_i)\right. \\
& \left.+\langle\sigma v\rangle_{\phi_2\phi_1\rightarrow\phi_1^\dagger i}(n_{\phi_2}n_{\phi_1}-n_{\phi_1^\dagger}n_i)\right], \quad (114)
\end{aligned}$$

and the corresponding Boltzmann equations for $n_{\phi_1^\dagger}$ and $n_{\phi_2^\dagger}$. Here, we assumed that h_X , X are in thermal equilibrium with the SM plasma.

Assuming no CP violation in the dark sector, we can take $n_{\phi_1} = n_{\phi_1^\dagger} = \frac{1}{2}n_1$ and $n_{\phi_2} = n_{\phi_2^\dagger} = \frac{1}{2}n_2$ and $n_i = n_i^{\text{eq}}$ for the co-annihilation channels, so the Boltzmann equations in Eqs. (113) and (114) become

$$\begin{aligned}
\dot{n}_1 + 3Hn_1 = & -\frac{1}{2}\langle\sigma v\rangle_{\phi_1\phi_1^\dagger\rightarrow f\bar{f},VV,h_1h_1,h_2h_2,h_1h_2,XX} \\
& \times (n_1^2 - (n_1^{\text{eq}})^2) \\
& + \frac{1}{2}\langle\sigma v\rangle_{\phi_2\phi_2^\dagger,\phi_2\phi_2,\phi_2^\dagger\phi_2^\dagger\rightarrow\phi_1\phi_1^\dagger}(n_2^2 - n_1^2), \quad (115)
\end{aligned}$$

$$\begin{aligned}
\dot{n}_2 + 3Hn_2 = & -\frac{1}{2}\langle\sigma v\rangle_{\phi_2\phi_2^\dagger\rightarrow f\bar{f},VV,h_1h_1,h_2h_2,h_1h_2,XX}(n_2^2 - (n_2^{\text{eq}})^2) \\
& -\frac{1}{2}(\langle\sigma v\rangle_{\phi_2\phi_2^\dagger\rightarrow\phi_1\phi_1^\dagger} + 2\langle\sigma v\rangle_{\phi_2\phi_2\rightarrow\phi_1\phi_1^\dagger})(n_2^2 - n_1^2) \\
& -\frac{1}{2}\sum_{i=X,h_1,h_2}(\langle\sigma v\rangle_{\phi_2\phi_1^\dagger\rightarrow\phi_1i} + \langle\sigma v\rangle_{\phi_2\phi_1\rightarrow\phi_1^\dagger i})n_1 \\
& \times (n_2 - n_2^{\text{eq}}). \quad (116)
\end{aligned}$$

As a result, the abundances for both ϕ_1 and ϕ_2 components depend on the annihilations of the heavier pair into the lighter pair and the self-annihilation channels into the SM and dark sector particles. But, only the abundance for ϕ_2 is affected directly by the semi-annihilation channels between two components of dark matter. When the gauge kinetic mixing and the dark Higgs-portal couplings are small, the self-annihilation channels of dark matter into the SM and dark sectors are suppressed, so the self-interactions between two components of dark matter and the associated dark Higgs couplings are dominant in determining the relic density. In this case, the fraction of ϕ_2 in the total relic abundance gets highly suppressed, due to the strong annihilations of the heavier component ϕ_2 .

In Figs. 5 and 6, we depict the evolutions of the relic abundances for two-component dark matter by varying the DM and mediator masses as well as the DM couplings. In the left of Fig. 5 (BM1), we took a relatively small dark gauge coupling but turned on nonzero couplings between dark matter and dark Higgs (or SM Higgs). So, in this case, the relic abundance for ϕ_2 is relatively large. On the other hand, in the right of Fig. 5 (BM2), we took a sizable dark gauge coupling and turned off the couplings between dark matter and dark Higgs (or SM Higgs), so the dark matter can annihilate dominantly into a pair of dark photons. Thus, in this case,

the dark matter abundance is dominantly determined by ϕ_1 . Similarly, in Fig. 6 (BM3 and BM4), we considered the case with smaller masses for DM and mediators, finding that relatively small couplings for dark matter are necessary for the correct relic density.

In Table 2, including the benchmark models in Figs. 5 and 6, we make a summary of the predictions for the semi-annihilation cross sections for multi-component dark matter at tree level, the DM fraction of ϕ_1 , the u -channel Sommerfeld factor, as well as the effective self-scattering cross sections for u -channel or s -channel. We note that the correct DM relic density and other direct and indirect DM constraints are satisfied for the benchmark models.

The effective self-scattering cross sections for u -channel and s -channel and the Sommerfeld factor with the u -channel resonance depend on the detuning parameter from the resonance mass condition, $\Delta = 1 - m_2/(2m_1)$, which is positive and small for the u -channel resonance. In Table 2, we chose $\Delta = 10^{-6}$ for all the benchmark models and the choice of the other parameters is explained in Table 2. Then, we computed numerically the effective self-scattering cross sections for u -channel or s -channel at the scales of dwarf galaxies for the DM relative velocity chosen to $v = 20$ km/s. As a result, we find that the effective self-scattering cross section for u -channel is much larger than the one for the s -channel in most of the benchmark models except BM6. For a larger value of Δ , both u -channel and s -channel enhancements become less significant, because the effective mass of the u -channel mediator (ϕ_1) gets heavier and the s -channel gets off the resonance.

On the other hand, we also obtained the Sommerfeld factor in each benchmark model in Table 2 by setting the DM velocity to $v = 10^{-3}$ at galaxies. Then, we can set the CMB bound on the decay branching ratios of the dark photon or dark Higgs into e^+e^- , as will be discussed in Sect. 6.2. We note that at the scales of dwarf galaxies with the DM velocity $v = 10^{-4}$, the Sommerfeld factor reads in the order of benchmark models in Table 2: $S_0 = 1.20 \times 10^5, 224, 267, 88.8, 1.34, 1.08$. Thus, the Sommerfeld factor increases slightly in dwarf galaxies as compared to the case at galaxies, except BM1.

We note that the Sommerfeld factor is calculated numerically for the u -channel case, but we can refer to the t -channel case in Sects. 3.3 and 3.4 about the qualitative and quantitative discussion on a large Sommerfeld factor. As a result, we found that a sizable Sommerfeld factor can be obtained on or off the resonances for the benchmark models (BM1–BM4) in Table 2. On the other hand, the Sommerfeld factor is $S_0 \simeq 1$ for the other benchmark models, BM5–BM6, in Table 2, because of small self-interactions for dark matter, i.e. $\alpha' = (g_1^2 + g_2^2)/(4\pi)$, which is much smaller than the DM velocity, $v \sim 10^{-4} - 10^{-3}$, at dwarf galaxies and galaxies.

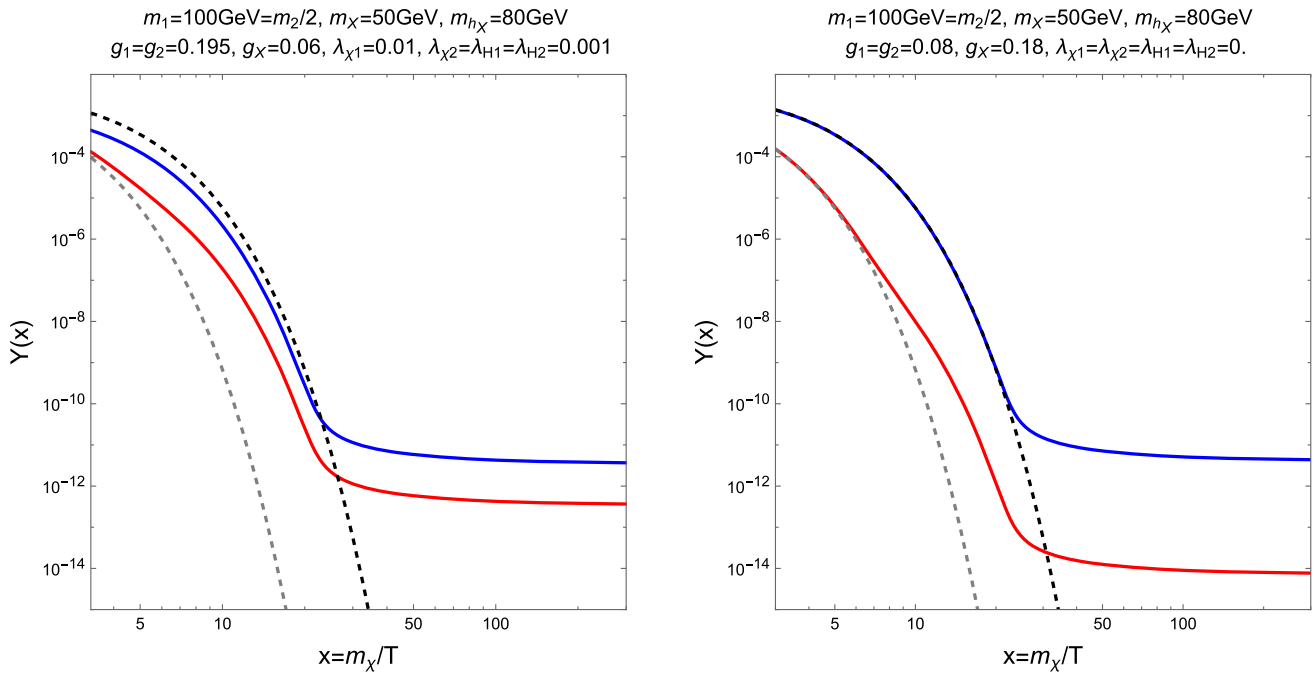


Fig. 5 The relic abundances for two-component dark matter, ϕ_1 and ϕ_2 , in blue and red lines, respectively. We took $m_1 = m_2/2 = 100 \text{ GeV} = 2m_X$, $m_{h_1} = 80 \text{ GeV}$, and $g_1 = g_2 = 0.195$, $g_X = 0.06$, and the nonzero quartic couplings are $\lambda_{X1} = 0.01$, $\lambda_{X2} = 0.001$, and $\lambda_{H1} = \lambda_{H2} = 0.001$, on left (BM1). We took $m_1 = m_2/2 =$

$100 \text{ GeV} = 2m_X = m_{h_1}$, and $g_1 = g_2 = 0.08$, $g_X = 0.18$, and the other dark sector couplings in the potential are set to zero, on right (BM2). The dashed lines correspond to the cases where ϕ_1, ϕ_2 are in thermal equilibrium

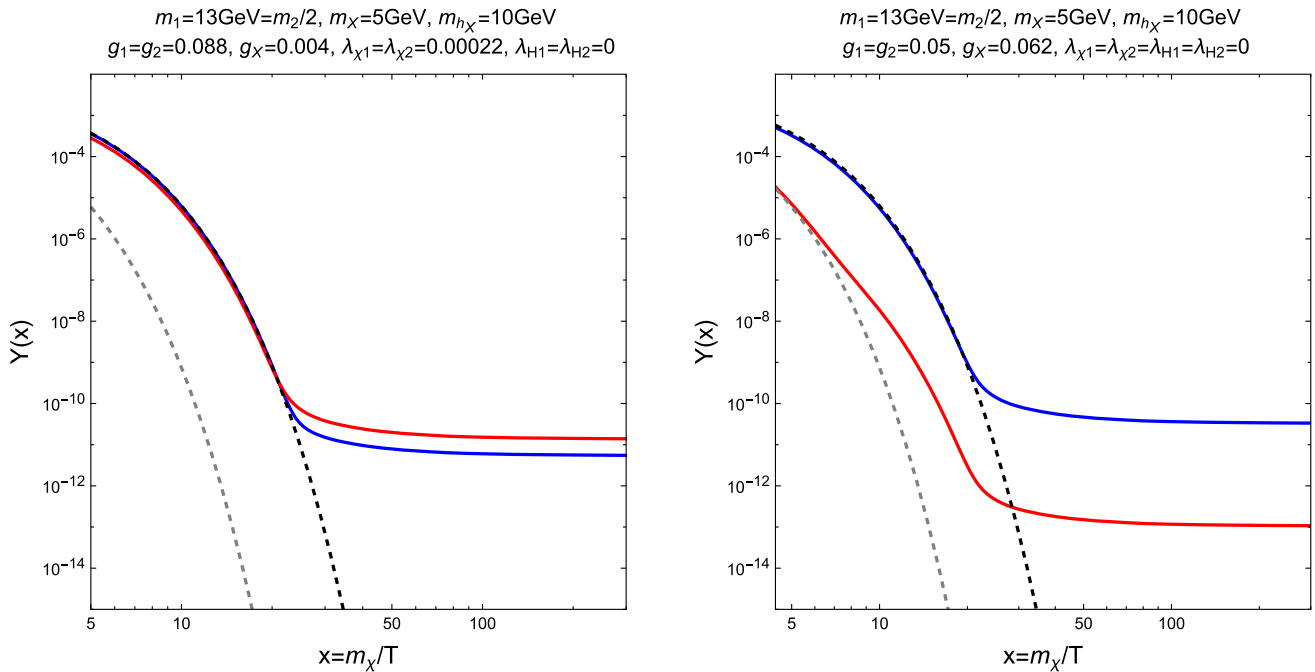


Fig. 6 The relic abundances for two-component dark matter, ϕ_1 and ϕ_2 , in blue and red lines, respectively. We took $m_1 = m_2/2 = 13 \text{ GeV}$, $m_X = 5 \text{ GeV}$, $m_{h_1} = 10 \text{ GeV}$, for both plots. We chose $g_1 = g_2 = 0.088$, $g_X = 0.004$, and the nonzero quartic couplings

are $\lambda_{X1} = \lambda_{X2} = 0.00022$, on left (BM3). We took $g_1 = g_2 = 0.05$, $g_X = 0.062$, and the other dark sector couplings in the potential are set to zero, on right (BM4). The dashed lines correspond to the cases where ϕ_1, ϕ_2 are in thermal equilibrium

Table 2 The Sommerfeld factor and the effective self-scattering cross sections in benchmark models. We took $m_2 \simeq 2m_1$ with $\Delta = 10^{-6}$ and $v = 20$ km/s for the effective self-scattering cross sections, in all the benchmark models. The other parameters for BM1-BM4 are chosen as in Figs. 5 and 6. For BM5 and BM6, we took $\varepsilon = 10^{-3}$,

	m_2 [GeV]	m_X [GeV]	m_{h_1} [GeV]	$g_{1,2}$	g_X	$\langle\sigma v\rangle_X^0$ [cm ³ /s]	$\langle\sigma v\rangle_{h_1}^0$ [cm ³ /s]	r_1 $= \frac{\Omega_1}{\Omega_{DM}}$	S_0	$\sigma_{u,\text{eff}}/m_1$ [cm ² /g]	$\sigma_{s,\text{eff}}/m_1$ [cm ² /g]
BM1	200	50	80	0.195	0.06	1.7×10^{-25}	1.6×10^{-25}	0.834	1.42×10^5	0.347	0.00108
BM2	200	50	80	0.08	0.18	2.5×10^{-25}	2.5×10^{-25}	0.996	181	0.00844	4.41×10^{-5}
BM3	26	5	10	0.088	0.004	1.6×10^{-26}	1.3×10^{-26}	0.165	243	1.52	3.92×10^{-4}
BM4	26	5	10	0.05	0.062	1.2×10^{-24}	1.1×10^{-24}	0.994	28.8	6.00	0.00305
BM5	1	1.5	0.8	0.014	1.8	–	5.1×10^{-22}	0.9998	1.019	37.8	0.333
BM6	0.2	0.6	0.04	0.0039	0.5	–	3.4×10^{-23}	0.9996	1.016	0.0231	0.252

6 Probes of self-resonant dark matter

We propose ways of testing the self-resonant dark matter models, such as direct detection for halo dark matter, CMB recombination, direct detection for boosted dark matter, etc., and show how the proposed benchmark models can be constrained.

6.1 Direct detection for halo dark matter

The DM-nucleus elastic scattering, $\phi_1^{(\dagger)} N \rightarrow \phi_1^{(\dagger)} N$, is possible through the exchanges of the dark Higgs or the extra gauge boson [29]. On the other hand, the dark Higgs is responsible for $sN \rightarrow sN$ or $aN \rightarrow aN$, but $sN \rightarrow aN$ or $aN \rightarrow sN$ with the extra gauge boson exchange is kinematically forbidden in the standard halo model if $\delta \equiv |m_s - m_a| \gtrsim 100$ keV [17,30]. But, for $\delta \equiv |m_s - m_a| \lesssim 100$ keV, the inelastic processes, $sN \rightarrow aN$ or $aN \rightarrow sN$, are kinematically allowed, so the gauge kinetic mixing can be constrained by direct detection experiments for the inelastic processes.

For $m_s = m_a = m_2$, both $\phi_1^{(\dagger)} N \rightarrow \phi_1^{(\dagger)} N$ and $\phi_2^{(\dagger)} N \rightarrow \phi_2^{(\dagger)} N$ are relevant for direct detection through the dark Higgs and the extra gauge boson exchanges [29].

We also note that the DM-electron elastic scatterings, $\phi_1^{(\dagger)} e \rightarrow \phi_1^{(\dagger)} e$, $s(a)e \rightarrow s(a)e$ or $s(a)e \rightarrow a(s)e$, are also relevant for light dark matter with sub-GeV mass [31,32] or boosted dark matter coming from the DM co-annihilations [29]. For $m_s = m_a = m_2$, both $\phi_1^{(\dagger)} e \rightarrow \phi_1^{(\dagger)} e$ and $\phi_2^{(\dagger)} e \rightarrow \phi_2^{(\dagger)} e$ are present due to the dark Higgs and the extra gauge boson exchanges [29].

For $m_s = m_a = m_2$, from the results in the appendix A, the averaged cross section for the ϕ_i -nucleon coherent scattering is given by

$$\sigma_{\phi_i, \phi_i^\dagger - A}^{\text{coh}} = \frac{\mu_{A,i}^2}{4\pi m_i^2} r_i \times \left[\left(Z c_p^{(i)} f_p + (A - Z) c_n^{(i)} f_n \right)^2 + Z^2 (g_p^{(i)})^2 \right] \quad (117)$$

and other nonzero parameters to $\lambda_{X1} = \lambda_{X2} = 10^{-4}$ in BM5, and $\lambda_{X1} = \lambda_{X2} = 3 \times 10^{-7}$ in BM6. We note that $\langle\sigma v\rangle_X^0 \equiv \langle\sigma v\rangle_{\phi_1^\dagger \phi_2^\dagger \rightarrow \phi_1 X}^0$, $\langle\sigma v\rangle_{h_1}^0 \equiv \langle\sigma v\rangle_{\phi_1^\dagger \phi_2^\dagger \rightarrow \phi_1 h_1}^0$, and $\sigma_{u,\text{eff}}$ and $\sigma_{s,\text{eff}}$ are the effective self-scattering cross sections for u -channel and s -channel, respectively

where $r_i = \Omega_i / \Omega_{DM}$, $\mu_{A,i} = m_i m_A / (m_A + m_i)$ are the reduced masses for the system with ϕ_i and a target nucleus A , and $c_{p,n}^{(i)}$, $g_p^{(i)}$ are the effective couplings to nucleons, and $f_{p,n}$ are the nucleon form factors. For instance, for ^{131}Xe nucleus, $A = 131$ and $N = 54$.

Similarly, the elastic scattering cross section between dark matter ϕ_i and electron is given by

$$\sigma_{\phi_i, \phi_i^\dagger - e} = \frac{\mu_{e,i}^2}{4\pi m_i^2} r_i \times \left[\left(\frac{\lambda_{e2} y_{h_2} \phi_i^\dagger \phi_i}{m_{h_2}^2} + \frac{\lambda_{e1} y_{h_1} \phi_i^\dagger \phi_i}{m_{h_1}^2} \right)^2 + \frac{4e^2 q_i^2 g_X^2 \varepsilon^2 m_i^2}{m_X^4} \right] \quad (118)$$

where $\mu_{e,i} = m_i m_e / (m_e + m_i)$ is the reduced mass of the ϕ_i -electron system.

6.2 CMB recombination

The dark matter annihilation, $\phi_1^\dagger \phi_2^{(\dagger)} \rightarrow \phi_1 Y$ with $Y = X, h_1$, can be constrained by the CMB bound [25]. We define the effective efficiency parameter for CMB recombination [26] as

$$f_{\text{eff}}(m_2) = \frac{\int_0^{m_2/2} dE_e E_e 2 f_{\text{eff}}^{e^+e^-} \frac{dN_e}{dE_e}}{m_2} \quad (119)$$

where $f_{\text{eff}}^{e^+e^-}$ is the efficiency factor³ for $Y \rightarrow e^+e^-$, and we took the range of the lepton energy to $m_2/2$ when two leptons coming from the Y decay are back-to-back. As compared to the DM self-annihilation, $DM + DM \rightarrow e^+e^-$, where the rate for the energy deposition is given by $\langle\sigma v\rangle m_{DM} n_{DM}^2$, the

³ Efficiency factors are computed for e^+e^- or photons produced from the DM annihilations in Ref. [26], but we used the results only for e^+e^- because they are produced from the direct decays of the mediator particles in our work.

total deposit energy, $2m_{\text{DM}}$, is replaced by m_2 and the interaction rate, $\langle\sigma v\rangle n_{\text{DM}}^2/2 = \langle\sigma v\rangle \rho_{\text{DM}}^2/(2m_{\text{DM}}^2)$, is replaced by $\langle\sigma v\rangle n_1 n_2 = \langle\sigma v\rangle r_1(1-r_1)\rho_{\text{DM}}^2/(4m_1 m_2)$. Thus, at the CMB recombination, the relative velocity for dark matter gets small [6, 7], so the Sommerfeld factors for the enhanced cross sections for the semi-annihilation channels in our model can be constrained by the Planck data [6, 7, 25] as in the following,⁴

$$\langle\sigma v\rangle_{\phi_1^\dagger\phi_2^{(\dagger)}\rightarrow\phi_1 Y} \cdot \text{BR}(Y \rightarrow e^+e^-) < 4.1 \times 10^{-25} \text{ cm}^3/\text{s} \\ \times \left(\frac{f_{\text{eff}}}{0.1}\right)^{-1} \cdot \frac{4}{r_1(1-r_1)} \cdot \left(\frac{m_1}{100 \text{ GeV}}\right), \quad (120)$$

with $r_1 = \Omega_1/\Omega_{\text{DM}}$.

In Table 3, taking the same benchmark models as in Table 2, we show the CMB bounds on the branching ratios of the mediators, $X/h_1 \rightarrow e^+e^-$. In most of the benchmark models, the DM abundances are highly asymmetric abundances for two-component dark matter, the tree-level semi-annihilation rates are already small due to the lack of one of the components of dark matter. Thus, the CMB limits on the decay branching ratios of the mediator particles into electron and positron become weak in most of the benchmark models, except BM1, for which a very large Sommerfeld factor is achieved.

6.3 Direct detection for boosted dark matter

The co-annihilation channels, $\phi_1^{(\dagger)}s(a) \rightarrow \phi_1^{(\dagger)}h_1$, $\phi_1^{(\dagger)}s(a) \rightarrow \phi_1^{(\dagger)}X$, $\phi_1s, \phi_1a \rightarrow \phi_1^\dagger h_1$, can make the resultant dark matter $\phi_1^{(\dagger)}$ boosted for $m_{h_1}, m_X \lesssim m_{s(a)}$. So, it leads to an interesting consequence for direct detection [33–35]. Moreover, the corresponding annihilation cross sections can be enhanced by the u -channel resonances at present, so the DM flux as well as the indirect signals from the decay products of h_1 or X can be increased accordingly [6, 7, 36, 37].

Focusing on the case with $m_s = m_a = m_2$, we consider the DM co-annihilation channels, $\phi_2\phi_1^\dagger \rightarrow \phi_1X(h_1)$, $\phi_2^\dagger\phi_1^\dagger \rightarrow \phi_1X(h_1)$ and their complex conjugates, so the resultant dark matter $\phi_1^{(\dagger)}$ gets boosted for $m_{h_1}, m_X \lesssim m_2$. In this case, as discussed in Sect. 2.2, only if the initial states for the co-annihilation channels is a mixture of two DM pairs, $\psi_1 \sim \psi_A + \frac{g_2}{g_1}\psi_B$ or its conjugate, with $A = (\phi_1\phi_2)$ and $B = (\phi_1\phi_2^\dagger)$, there is a Sommerfeld enhancement due to the DM co-scattering processes, $(\phi_2^\dagger, \phi_2) \rightarrow \phi_2\phi_1^\dagger$ and $(\phi_2^\dagger, \phi_2) \rightarrow \phi_2^\dagger\phi_1^\dagger$. As a result, the DM co-annihilation cross sections get enhanced by an universal Sommerfeld factor S_0 ,

as follows,

$$\langle\sigma v\rangle_{\phi_2\phi_1^\dagger\rightarrow\phi_1X(h_1)} = S_0 \langle\sigma v\rangle_{\phi_2\phi_1^\dagger\rightarrow\phi_1X(h_1)}^0, \quad (121)$$

$$\langle\sigma v\rangle_{\phi_2^\dagger\phi_1^\dagger\rightarrow\phi_1X(h_1)} = S_0 \langle\sigma v\rangle_{\phi_2^\dagger\phi_1^\dagger\rightarrow\phi_1X(h_1)}^0, \quad (122)$$

with $\langle\sigma v\rangle_{\phi_2^{(\dagger)}\phi_1^\dagger\rightarrow\phi_1X(h_1)}^0$ being the tree-level DM co-annihilation cross sections.

Moreover, we consider the differential flux for boosted dark matter (BDM) $\phi_1^{(\dagger)}$ at the location of the Solar System, coming from the semi-annihilation processes for dark matter in our galaxy, $\phi_2\phi_1^\dagger \rightarrow \phi_1Y$ with $Y = h_1, X$, or their CP conjugate processes, is given by

$$\frac{d\Phi_{\phi_1}}{dE_1} = \frac{1}{8\pi m_1 m_2} \langle\sigma v\rangle_{\psi_1^\dagger\rightarrow\phi_1 Y} \cdot \frac{dN_{\phi_1}}{dE_1} \frac{1}{2} r_1(1-r_1) \\ \int d\Omega \int_{\text{l.o.s.}} ds \rho_{\text{DM}}^2(r) = \frac{d\Phi_{\phi_1^\dagger}}{dE_1} \quad (123)$$

where $r(s, \theta) = \sqrt{r_\odot^2 + s^2 - 2r_\odot s \cos \theta}$ with $r_\odot = 8.33 \text{ kpc}$ being the distance of the Sun to the galactic center and $r_1 = \Omega_1/\Omega_{\text{DM}}$. The differential number of the boosted dark matter ϕ_1 is given [6, 7] by

$$\frac{dN_{\phi_1}}{dE_1} = \delta(E_1 - E'_1), \quad (124)$$

where

$$E'_1 = \frac{E_{\text{cm}}^2 + m_1^2 - m_Y^2}{2E_{\text{cm}}}, \quad E_{\text{cm}} \equiv m_1 + m_2 + \frac{1}{2}\mu v_{\text{rel}}^2, \quad (125)$$

with $\mu = m_1 m_2 / (m_1 + m_2)$ and v_{rel} being the relative velocity between $\phi_1^\dagger$ and ϕ_2 , so the kinetic energy of the boosted dark matter, $T'_1 = E'_1 - m_1$, is also given by

$$T'_1 = \frac{(E_{\text{cm}} - m_1)^2 - m_Y^2}{2E_{\text{cm}}}. \quad (126)$$

Here, since there is no directional information in the current direct detection experiments, we need to integrate the solid angle over the whole sky in the computation of the flux.

We note that for $K \ll m_1, m_2$ and $K \ll (m_2^2 - m_Y^2)/(2m_2)$, with $K \equiv \frac{1}{2}\mu v_{\text{rel}}^2$, we get the energy of the boosted dark matter as

$$E'_1 \simeq \frac{2m_1^2 + 2m_1 m_2 + m_2^2 - m_Y^2}{2(m_1 + m_2)} \quad (127)$$

or the kinetic energy as

$$T'_1 \simeq \frac{m_2^2 - m_Y^2}{2(m_1 + m_2)}. \quad (128)$$

This is a good approximation, as far as $(m_2 - m_Y)/m_2 \gtrsim v_{\text{rel}}^2 \sim 10^{-6}$ for $v_{\text{rel}} \sim 10^{-3}$ in our galaxy.

⁴ We have corrected a typo in the formula in Refs. [6, 7], from m_2 to $4m_1$.

Table 3 CMB bounds on the branching ratios of the mediator decays, $X/h_1 \rightarrow e^+e^-$

	m_2 [GeV]	m_X [GeV]	m_{h_1} [GeV]	S_0	$\text{BR}(X \rightarrow e^+e^-)$	$\text{BR}(h_1 \rightarrow e^+e^-)$
BM1	200	50	80	1.42×10^5	2.3×10^{-4}	2.5×10^{-4}
BM2	200	50	80	181	0.18	0.18
BM3	26	5	10	243	0.14	0.16
BM4	26	5	10	28.8	0.35	0.38
BM5	1	1.5	0.8	1.019	–	0.017
BM6	0.2	0.6	0.04	1.016	–	0.014

As a result, we can rewrite the differential flux in the following form,

$$\frac{d\Phi_{\phi_1}}{dT_1} = \Phi_{\text{BSM}} \delta(T_1 - T'_1) \quad (129)$$

with

$$\Phi_{\text{BSM}} = \frac{m_1}{2m_2} r_1(1 - r_1) \cdot (3.2 \times 10^{-3} \text{ cm}^{-2} \text{ s}^{-1}) \times \left(\frac{m_1}{100 \text{ MeV}} \right)^{-2} \left(\frac{\langle \sigma v \rangle_{\phi_2 \phi_1^\dagger \rightarrow \phi_1 Y}}{10^{-26} \text{ cm}^3 \text{ s}^{-1}} \right). \quad (130)$$

The differential rate for the boosted DM, ϕ_1 or $\phi_1^\dagger$, per target mass for direct detection experiments is given by

$$\frac{dR_A}{dT_A} = \frac{1}{m_A} \int_{T_{1,\min}}^{\infty} dT_1 \times \frac{d\sigma_{\text{SI}}}{dT_A} \frac{d\Phi_{\phi_1}}{dT_1}, \quad (131)$$

with

$$\frac{d\sigma_{\text{SI}}}{dT_A} \equiv \frac{1}{2} \left(\frac{d\sigma_{\phi_1-A}}{dT_A} + \frac{d\sigma_{\phi_1^\dagger-A}}{dT_A} \right). \quad (132)$$

First, we note that $T_{1,\min}$ is the minimal kinetic energy for the boosted dark matter for the kinetic energy of the nucleus T_A , given by

$$T_{1,\min} = \left(\frac{T_A}{2} - m_1 \right) \left[1 \pm \sqrt{1 + \frac{2T_A}{m_A} \frac{(m_1 + m_A)^2}{(T_A - 2m_1)^2}} \right], \quad (133)$$

in which either (+) or (−) signs are taken for $T_A > 2m_1$ or $T_A < 2m_1$. Moreover, the total differential cross section is composed of coherent and incoherent parts [35], as follows,

$$\frac{d\sigma_{\text{SI}}}{dT_A} = \left(\frac{d\sigma_{\text{SI}}}{dT_A} \right)_{\text{coh}} + \left(\frac{d\sigma_{\text{SI}}}{dT_A} \right)_{\text{inc}} \quad (134)$$

with

$$\left(\frac{d\sigma_{\text{SI}}}{dT_A} \right)_{\text{coh}} = \frac{\sigma_{\text{SI}}^{\text{coh}}}{T_{A,\max}} |F_{\text{SI}}(q)|^2, \quad (135)$$

$$\left(\frac{d\sigma_{\text{SI}}}{dT_A} \right)_{\text{inc}} = \frac{\sigma_{\text{SI}}^{\text{inc}}}{T_{A,\max}} (1 - |F_{\text{SI}}(q)|^2). \quad (136)$$

Here, the maximal kinetic energy of the target nucleus after scattering is given by

$$T_{A,\max} = \frac{T_1(T_1 + 2m_1)}{(m_1 + m_A)^2/(2m_A) + T_1}, \quad (137)$$

and the form factor for the target nucleus is, in the dipole nucleon approximation [38],

$$F_{\text{SI}}(q) = \left(1 + \frac{q^2}{\Lambda_A^2} \right)^{-2}, \quad (138)$$

with $\Lambda_A \simeq 0.843 \text{ GeV} (0.8791 \text{ fm}/R_A)$ and R_A being the charge radius of the target nucleus. Moreover, the spin-independent cross sections for coherent and incoherent scatterings are given by

$$\begin{aligned} \sigma_{\text{SI}}^{\text{coh}} &\equiv \frac{2}{r_1} \sigma_{\phi_1, \phi_1^\dagger-A}^{\text{coh}} \\ &= \frac{\mu_{A,1}^2}{2\pi m_1^2} \left[\left(Z c_p^{(1)} f_p + (A - Z) c_n^{(1)} f_n \right)^2 + Z^2 (g_p^{(1)})^2 \right], \end{aligned} \quad (139)$$

$$\begin{aligned} \sigma_{\text{SI}}^{\text{inc}} &\equiv 2 \sigma_{\phi_1, \phi_1^\dagger-A}^{\text{inc}} \\ &= \frac{\mu_{A,1}^2}{2\pi m_1^2} \left[Z \left(c_p^{(1)} f_p \right)^2 + Z (g_p^{(1)})^2 + (A - Z) \left(c_n^{(1)} f_n \right)^2 \right], \end{aligned} \quad (140)$$

respectively. Then, the total event rate in units of the number events per day and kg is given by

$$R = \sum_A \int dT_A \frac{dR_A}{dT_A}. \quad (141)$$

In Fig. 7, we show the differential event rate for the spin-independent elastic scattering of boosted dark matter per day per kg in units of keV^{-1} in the case of the Xenon target nucleus. We fixed the semi-annihilation cross section for $\phi_1^\dagger \phi_2^{(\dagger)} \rightarrow \phi_1 Y$ with $Y = X, h_1$ for the benchmark models, BM5, BM6, respectively, and we also took the gauge kinetic mixing to $\varepsilon e = 2 \times 10^{-4}$ and the u -channel Sommerfeld factor to $S_0 = 1$. If we increase the Sommerfeld factor, the differential event rate in Fig. 7 increases by a factor of S_0 .

We note that there is a maximal kinetic energy for the target nucleus given in Eq. (137). As the nuclear recoil becomes undetectable below the detector threshold T_{th} , there is a lower

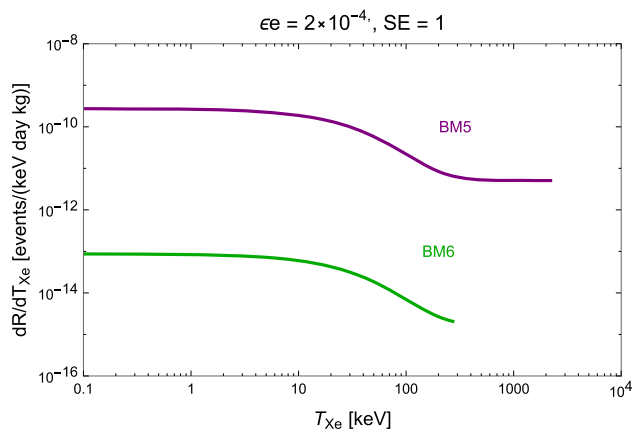


Fig. 7 The differential event rate for the spin-independent elastic scattering of boosted dark matter as a function of the kinetic energy of the Xenon target nucleus. We took benchmark models for light dark matter, BM5 and BM6, in Table 2

bound on the DM kinetic energy T_1 , from $T_{A,\max} \geq T_{\text{th}}$. For instance, taking the kinetic energy of the boosted dark matter from Eq. (128) with $m_2 \simeq 2m_1$ and $m_Y \ll m_1$, we obtain the lower bound on the DM mass for direct detection [35], as follows,

$$m_{1,\min} \simeq \frac{15m_A}{\frac{32m_A}{T_{\text{th}}} - 9} \left(1 + \frac{4}{5} \sqrt{1 + \frac{2m_A}{T_{\text{th}}}} \right). \quad (142)$$

Therefore, we need to take the DM mass to be greater than $m_{1,\min}$ for a direct detection of boosted dark matter at XENONnT. For instance, for the case of the XENONnT experiment, the detector threshold is given by $T_{\text{th}} = 3.3 \text{ keV}$ [2], so we obtain $m_{1,\min} \simeq 10.7 \text{ MeV}$. For $m_Y \lesssim m_2$, we would need a larger DM mass to pass the detector thresh-

old. However, we increase m_1 , the DM flux coming from our galaxy gets suppressed. Thus, we can constrain our model by the direct detection of boosted dark matter with relatively small masses.

We also remark that the semi-annihilation processes, $\phi_2\phi_1^\dagger \rightarrow \phi_1 X(h_1)$, $\phi_2^\dagger\phi_1^\dagger \rightarrow \phi_1 X(h_1)$, and their conjugates, can be enhanced by the Sommerfeld factor with the u -channel resonance in our model as discussed in Sect. 3, so we could take a larger cross section for the semi-annihilation processes at present than the thermal cross section. In this case, we could take a larger DM mass well to go above the detector threshold, as the DM flux can be sizable even for a larger DM mass due to the enhanced annihilation cross sections.

In the left of Fig. 8, we show the parameter scan for the relic density for self-resonant light dark matter in the parameter space for DM self-couplings ($g_1 = g_2$) vs m_1 . We took $m_2 \simeq 2m_1$ for the self-resonant condition and $m_{h_1} = 1.6m_1 < m_2$ and $m_X > m_2$, for which the semi-annihilation channels with dark Higgs, $\phi_2\phi_1^\dagger \rightarrow \phi_1 h_1$ and $\phi_2^\dagger\phi_1^\dagger \rightarrow \phi_1 h_1$, are kinematically open. In this case, there is more freedom to make the decay branching fraction of the dark Higgs into e^+e^- small enough in the presence of new light particles in the dark sector, being consistent with the CMB bound. We also chose the other parameters to $g_X = 1.8$ and $\lambda_{\chi_1} = \lambda_{\chi_2} = 10^{-4}$ and set the Higgs-portal couplings and Higgs mixing angle to zero. The parameter scan results contain the benchmark model BM6 on the red line along which the observed relic density is obtained. The color gradation in the left of Fig. 8 indicates the total abundance of self-resonant dark matter in the parameter space.

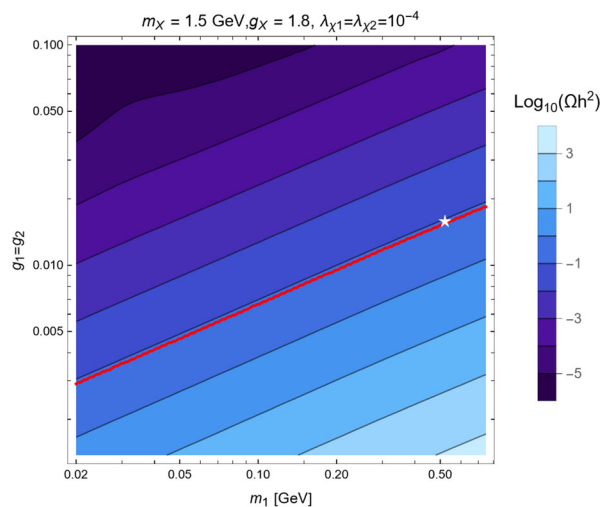
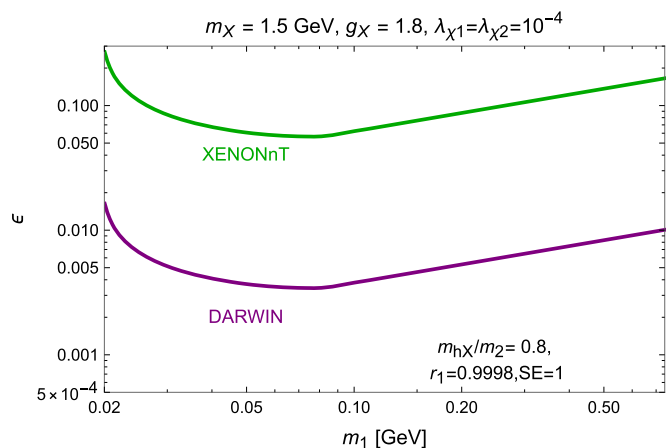


Fig. 8 (Left) Parameter scan for the relic density for self-resonant light dark matter in the DM self-couplings ($g_1 = g_2$) vs $m_1 \simeq m_2/2$. The benchmark model, BM5, (marked in white star), lies on the red line which indicates the contour line satisfying the observed relic density.



(Right) XENONnT and DARWIN (projected) bounds for boosted dark matter in the gauge kinetic mixing (ϵ) vs m_1 . We consider the X gauge portal for BDM-nucleus elastic scattering

On the other hand, in the right of Fig. 8, we also show the limits on the gauge kinetic mixing (ε) vs the DM mass m_1 from the direct detection for boosted dark matter at XENONnT [2] and DARWIN (in prospect) [39]. The flux for boosted dark matter relies on the semi-annihilations into the dark Higgs while the direct detection for BDM-nucleus elastic scattering is dominated by the gauge kinetic mixing of the X gauge boson. But, we ignored the Higgs portal couplings for the dark Higgs. Here, we took $m_2 \simeq 2m_1$, $m_X = 1.5$ GeV and $\lambda_{X1} = \lambda_{X2} = 10^{-4}$, as in the left plot, and set $g_1 = g_2 = 0.014$, $m_{h_X}/m_2 = 0.8$, $r_1 = 0.9998$, and the u -channel Sommerfeld factor to $S_0 = 1$. Thus, the direct detection limit can constrain the parameter space for boosted dark matter extended from the benchmark model BM7.

We remark that for a large Sommerfeld factor, the bounds on ε from XENONnT and DARWIN get more stringent by the factor of $\sqrt{S_0}$. For instance, if we choose larger values for g_1, g_2 for sub-GeV scale dark matter, the relic density would be smaller due to larger DM annihilation cross sections. However, if there is a way to reproduce dark matter from the decays of a heavy particle that dominates at late times, larger DM annihilation cross sections would be helpful for the correct relic density [40].

As a result, we find that the gauge kinetic mixing ε is bounded to be below $10^{-4} - 10^{-3}$ in the range, $m_1 = 20 - 750$ MeV. For $\varepsilon \gtrsim 0.1$, a large BDM-nucleus scattering cross section would attenuate the kinetic energy of boosted dark matter during the propagation throughout the Earth below the detection threshold [35]. We also note that for sub-GeV scale dark photon, the parameter space up to $\varepsilon = 10^{-5} - 10^{-3}$ is already excluded by the direct detection for halo dark matter (non-boosted dark matter) and collider data such as Belle-II [41–44], BaBar [45, 46], NA64, [47–49], etc, depending on the decay branching ratios of the dark photon.

We also remark that the couplings of the mediator particles with sub-GeV scale masses can be constrained by Big Bang Nucleosynthesis (BBN) or CMB (N_{eff}), depending on their lifetimes and the fraction of the energy density [50–53]. For instance, if the dark photon of masses below GeV scale decay dominantly into the SM particles, the BBN bound on the lifetime of the dark photon would be $\tau_X \lesssim 100$ s [51, 52]. Thus, the dark photon must decay into the dark sector such that the energy injection into the SM during BBN gets suppressed. This is consistent with the limit from CMB discussed in Sect. 6.2 where a small decay branching fraction of the dark photon into e^+e^- is required for a sizable u -channel Sommerfeld factor.

7 Conclusions

We presented a new model for two-component scalar dark matter, consisting of ϕ_1 and ϕ_2 , where both the DM compo-

nents are stable due to the remaining Z_4 gauge symmetry, as far as the DM masses satisfy $m_2 < 2m_1$. The Z_4 symmetry in the model ensures the stability of two-component dark matter. This is in contrast to the case for the Z_2 model [6, 7] where ϕ_1 is a complex singlet scalar field charged under the $U(1)'$ and ϕ_2 is a neutral singlet scalar under the $U(1)'$, being subject to loop-induced decays due to Z' interactions.

For DM masses with m_2 being slightly smaller than $2m_1$, we obtained the resonance condition for the elastic co-scattering processes between ϕ_1 and $\phi_2^{(\dagger)}$ (u -channel processes) for which the corresponding cross sections at tree-level become divergent at a zero momentum transfer for dark matter. Thus, applying the Bethe–Salpeter formalism for the resummation of the ladder diagrams for the u -channel with two dark matter scalars, we derived a nonzero effective Yukawa potential with an effective mass given by the amount of detuning m_2 from $2m_1$, only for one of the linear combinations of the initial states for dark matter.

We showed that the effective co-scattering processes for dark matter become enhanced and velocity-dependent to solve the small-scale problems at galaxies and they are more important than the DM self-scattering processes for ϕ_1 , which are similarly enhanced by the s -channel mechanism. The same DM couplings responsible for the co-scattering processes give rise to the semi-annihilation processes for ϕ_1 and $\phi_2^{(\dagger)}$ annihilating into $\phi_1^\dagger$ and dark photon/Higgs, which are enhanced by the u -channel Sommerfeld factor. Thus, there is a nontrivial upper bound on the Sommerfeld factor from CMB recombination, depending the decay branching fractions of dark photon/Higgs. Focusing on the benchmark models for two-component dark matter satisfying the observed relic density, we considered the possibility of producing boosted dark matter with sub-GeV scale mass from the semi-annihilation processes at the galactic center and showed the parameter space for the gauge kinetic mixing for dark photon versus the DM mass, which is consistent with the XENONnT and DARWIN experiments.

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Appendix A: Dark matter annihilation cross sections

We list the annihilation cross sections for multi-component dark matter composed of ϕ_1, ϕ_2 , which is the case with $\kappa_1 = 0$ in our model.

The annihilation cross section of $\phi_i^\dagger \phi_i$ ($i = 1, 2$) into a pair of SM fermions is given by

$$\langle \sigma v \rangle_{\phi_i \phi_i^\dagger \rightarrow f \bar{f}} = \frac{\varepsilon^2 e^2 q_i^2 g_X^2 N_c}{\pi x_i} \frac{m_i^2 + \frac{1}{2} m_f^2}{(4m_i^2 - m_X^2)^2 + m_X^2 \Gamma_X^2} \times \sqrt{1 - \frac{m_f^2}{m_i^2}} + \frac{N_c}{4\pi} \left(1 - \frac{m_f^2}{m_i^2}\right)^{3/2} \left| \frac{\lambda_{f1} y_{h1} \phi_i^\dagger \phi_i}{4m_i^2 - m_{h1}^2} + \frac{\lambda_{f2} y_{h2} \phi_i^\dagger \phi_i}{4m_i^2 - m_{h2}^2} \right|^2, \quad (\text{A.1})$$

with $x_i \equiv m_i/T$ and $q_i = 1, 2$ being the $U(1)'$ charges for ϕ_i with $i = 1, 2$.

The cross section for $\phi_i^\dagger \phi_i$ ($i = 1, 2$) annihilating into VV with $V = Z, W$ are given by

$$\langle \sigma v \rangle_{\phi_i \phi_i^\dagger \rightarrow VV} = \frac{\delta_V e^4}{256\pi \sin^4 \theta_W \cos^4 \theta_W} \frac{v^2 m_i^2}{m_V^4} \times \left(4 - 4 \frac{m_V^2}{m_i^2} + 3 \frac{m_V^4}{m_i^4}\right) \times \sqrt{1 - \frac{m_V^2}{m_i^2}} \left(\frac{v \lambda_{f1}}{m_f} \frac{y_{h1} \phi_i^\dagger \phi_i}{4m_i^2 - m_{h1}^2} + \frac{v \lambda_{f2}}{m_f} \frac{y_{h2} \phi_i^\dagger \phi_i}{4m_i^2 - m_{h2}^2} \right)^2 \quad (\text{A.2})$$

with $\delta_V = 1$ ($2 \cos^4 \theta_W$) for $V = Z$ (W). The cross-section for dark matter annihilating into $\gamma\gamma$ is loop-induced, given by

$$\langle \sigma v \rangle_{\phi_i \phi_i^\dagger \rightarrow \gamma\gamma} = \frac{e^4 m_i^2}{64\pi^5 v^2} \left| \sum_f N_c Q_f^2 A_{1/2}(x_f) + A_1(x_w) \right|^2 \times \left(\frac{v \lambda_{f1}}{m_f} \frac{y_{h1} \phi_i^\dagger \phi_i}{4m_i^2 - m_{h1}^2} + \frac{v \lambda_{f2}}{m_f} \frac{y_{h2} \phi_i^\dagger \phi_i}{4m_i^2 - m_{h2}^2} \right)^2. \quad (\text{A.3})$$

We note that for a vanishing Higgs mixing angle, the DM couplings to the Higgs-like scalars become $y_{h1} \phi_1^\dagger \phi_1 = \lambda_{\chi 1} v_\chi$,

$y_{h2} \phi_2^\dagger \phi_2 = \lambda_{\chi 2} v_\chi$, and $y_{h2} \phi_2^\dagger \phi_2 = \lambda_{H2} v$; $\lambda_{f1} = 0$ and $\lambda_{f2} = \frac{m_f}{v}$.

We also list the cross sections for the additional DM annihilations, in the limit of vanishing Higgs mixing (i.e. $\lambda_{\chi H} = 0$) and gauge kinetic mixing (i.e. $\xi = 0$), as follows,

$$\langle \sigma v \rangle_{ij \rightarrow kl} = \frac{|\mathcal{M}_{ij \rightarrow kl}|^2}{8\pi(m_i + m_j)^2} \sqrt{1 - \frac{(m_k - m_l)^2}{(m_i + m_j)^2}} \times \sqrt{1 - \frac{(m_k + m_l)^2}{(m_i + m_j)^2}} \left(1 - \frac{(m_i - m_j)^2}{(m_i + m_j)^2}\right)^{-1}, \quad (\text{A.4})$$

with

$$|\mathcal{M}_{\phi_1 \phi_1^\dagger \rightarrow h_1 h_1}|^2 = \lambda_{\chi 1}^2 \left(\frac{(2\lambda_{\chi 1} v_\chi^2 + m_{h_1}^2 - 2m_1^2)^2}{2(m_{h_1}^2 - 2m_1^2)^2} - \frac{3\sqrt{2}m_{h_2} v_\chi (2\lambda_{\chi 1} v_\chi^2 + m_{h_1}^2 - 2m_1^2)}{m_{h_1}^4 - 6m_{h_1}^2 m_1^2 + 8m_1^4} + \frac{9m_{h_2}^2 v_\chi^2}{(m_{h_1}^2 - 4m_1^2)^2} \right), \quad (\text{A.5})$$

$$|\mathcal{M}_{\phi_2 \phi_2^\dagger \rightarrow h_1 h_1}|^2 = \lambda_{\chi 2}^2 \left(\frac{(2\lambda_{\chi 2} v_\chi^2 + m_{h_1}^2 - 2m_2^2)^2}{2(m_{h_1}^2 - 2m_2^2)^2} - \frac{3\sqrt{2}m_{h_2} v_\chi (2\lambda_{\chi 2} v_\chi^2 + m_{h_1}^2 - 2m_2^2)}{m_{h_1}^4 - 6m_{h_1}^2 m_2^2 + 8m_2^4} + \frac{9m_{h_2}^2 v_\chi^2}{(m_{h_1}^2 - 4m_2^2)^2} \right), \quad (\text{A.6})$$

$$|\mathcal{M}_{\phi_1 \phi_1^\dagger \rightarrow h_1 h_2}|^2 = \frac{16\lambda_{H1}^2 \lambda_{\chi 1}^2 v^2 v_\chi^2}{(m_{h_1}^2 + m_{h_2}^2 - 4m_1^2)^2}, \quad (\text{A.7})$$

$$|\mathcal{M}_{\phi_2 \phi_2^\dagger \rightarrow h_1 h_2}|^2 = \frac{16\lambda_{H2}^2 \lambda_{\chi 2}^2 v^2 v_\chi^2}{(m_{h_1}^2 + m_{h_2}^2 - 4m_2^2)^2}, \quad (\text{A.8})$$

$$|\mathcal{M}_{\phi_1 \phi_1^\dagger \rightarrow h_2 h_2}|^2 = \lambda_{H1}^2 \left(2m_{h_2}^2 (v(\lambda_{H1} v + m_{h_1}) - 3m_1^2) - 4m_1^2 v(2\lambda_{H1} v + m_{h_1}) + m_{h_2}^4 + 8m_1^4 \right)^2 = \frac{(2m_{h_2}^2 - 2m_1^2)^2 (m_{h_2}^2 - 4m_1^2)^2}{2(m_{h_2}^2 - 2m_1^2)^2 (m_{h_2}^2 - 4m_1^2)^2}, \quad (\text{A.9})$$

$$|\mathcal{M}_{\phi_2 \phi_2^\dagger \rightarrow h_2 h_2}|^2 = \lambda_{H2}^2 \left(2m_{h_2}^2 (v(\lambda_{H2} v + m_{h_1}) - 3m_2^2) - 4m_2^2 v(2\lambda_{H2} v + m_{h_1}) + m_{h_2}^4 + 8m_2^4 \right)^2 = \frac{(2m_{h_2}^2 - 2m_2^2)^2 (m_{h_2}^2 - 4m_2^2)^2}{2(m_{h_2}^2 - 2m_2^2)^2 (m_{h_2}^2 - 4m_2^2)^2}, \quad (\text{A.10})$$

(A.17)

$$\begin{aligned}
|\mathcal{M}_{\phi_2^\dagger \phi_1^\dagger \rightarrow h_1 \phi_1}|^2 &= \left[\sqrt{2} \kappa_2 m_2 (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_{h_1}^2) \right. \\
&\times (m_2^2 (2m_1 + m_2) - m_{h_1}^2 m_1) \\
&- 2g_2 m_1 v_\chi (\lambda_{\chi 2} m_2 (m_1 + m_2) (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_{h_1}^2) \\
&- \lambda_{\chi 1} (m_{h_1}^2 + m_2 (2m_1 + m_2)) (m_{h_1}^2 m_1 - m_2^2 (2m_1 + m_2))) \left. \right]^2 \\
&\times \left(m_2^2 (2m_1 + m_2)^2 \right. \\
&\left. (m_{h_1}^2 - m_1 (2m_1 + m_2))^2 (m_{h_1}^2 m_1 - m_2^2 (2m_1 + m_2))^2 \right)^{-1},
\end{aligned} \quad (\text{A.11})$$

$$\begin{aligned}
|\mathcal{M}_{\phi_2 \phi_1^\dagger \rightarrow h_1 \phi_1}|^2 &= 4g_1^2 m_1^2 v_\chi^2 \\
&\times \left(\lambda_{\chi 2} m_2 (m_1 + m_2) (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_{h_1}^2) \right. \\
&- \lambda_{\chi 1} (m_{h_1}^2 + m_2 (2m_1 + m_2)) (m_{h_1}^2 m_1 - m_2^2 (2m_1 + m_2)) \left. \right)^2 \\
&\times \left(m_2^2 (2m_1 + m_2)^2 \right. \\
&\left. (m_{h_1}^2 - m_1 (2m_1 + m_2))^2 (m_{h_1}^2 m_1 - m_2^2 (2m_1 + m_2))^2 \right)^{-1},
\end{aligned} \quad (\text{A.12})$$

$$\begin{aligned}
|\mathcal{M}_{\phi_2^\dagger \phi_1^\dagger \rightarrow h_2 \phi_1}|^2 &= 4g_2^2 m_1^2 v^2 \\
&\times \left(\lambda_{H2} m_2 (m_1 + m_2) (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_{h_2}^2) \right. \\
&- \lambda_{H1} (m_{h_2}^2 + m_2 (2m_1 + m_2)) (m_{h_2}^2 m_1 - m_2^2 (2m_1 + m_2)) \left. \right)^2 \\
&\times \left(m_2^2 (2m_1 + m_2)^2 (m_{h_2}^2 - m_1 (2m_1 + m_2))^2 \right. \\
&\left. (m_{h_2}^2 m_1 - m_2^2 (2m_1 + m_2))^2 \right)^{-1},
\end{aligned} \quad (\text{A.13})$$

$$\begin{aligned}
|\mathcal{M}_{\phi_2 \phi_1^\dagger \rightarrow h_2 \phi_1}|^2 &= 4g_1^2 m_1^2 v^2 \\
&\times \left(\lambda_{H2} m_2 (m_1 + m_2) (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_{h_2}^2) \right. \\
&- \lambda_{H1} (m_{h_2}^2 + m_2 (2m_1 + m_2)) (m_{h_2}^2 m_1 - m_2^2 (2m_1 + m_2)) \left. \right)^2 \\
&\times \left(m_2^2 (2m_1 + m_2)^2 (m_{h_2}^2 - m_1 (2m_1 + m_2))^2 \right. \\
&\left. (m_{h_2}^2 m_1 - m_2^2 (2m_1 + m_2))^2 \right)^{-1},
\end{aligned} \quad (\text{A.14})$$

$$|\mathcal{M}_{\phi_1 \phi_1^\dagger \rightarrow X X}|^2 = \frac{2g_X^4 q_1^4 (8m_1^4 - 8m_1^2 m_X^2 + 3m_X^4)}{(m_X^2 - 2m_1^2)^2}, \quad (\text{A.15})$$

$$|\mathcal{M}_{\phi_2 \phi_2^\dagger \rightarrow X X}|^2 = \frac{2g_X^4 q_2^4 (8m_2^4 - 8m_2^2 m_X^2 + 3m_X^4)}{(m_X^2 - 2m_2^2)^2}, \quad (\text{A.16})$$

$$\begin{aligned}
|\mathcal{M}_{\phi_2^\dagger \phi_1^\dagger \rightarrow X \phi_1}|^2 &= 4g_2^2 g_X^2 m_1^2 (m_2^2 - m_X^2) \left((2m_1 + m_2)^2 - m_X^2 \right) \\
&\times \left(q_2 m_2^2 (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_X^2) \right. \\
&\left. + q_1 (2m_1 (2m_1 + m_2) - m_X^2) (2m_1 m_2^2 - m_1 m_X^2 + m_2^3) \right)^2
\end{aligned}$$

$$\begin{aligned}
&\times \left(m_2^2 m_X^2 (2m_1 + m_2)^2 (m_X^2 - m_1 (2m_1 + m_2))^2 \right. \\
&\left. (2m_1 m_2^2 - m_1 m_X^2 + m_2^3)^2 \right)^{-1},
\end{aligned}$$

$$\begin{aligned}
|\mathcal{M}_{\phi_2 \phi_1^\dagger \rightarrow X \phi_1}|^2 &= 4g_1^2 g_X^2 m_1^2 (m_2^2 - m_X^2) \left((2m_1 + m_2)^2 - m_X^2 \right) \\
&\times \left(q_2 m_2^2 (2m_1 + m_2) (m_1 (2m_1 + m_2) - m_X^2) \right. \\
&- q_1 (2m_1 (2m_1 + m_2) - m_X^2) (2m_1 m_2^2 - m_1 m_X^2 + m_2^3) \left. \right)^2 \\
&\times \left(m_2^2 m_X^2 (2m_1 + m_2)^2 (m_X^2 - m_1 (2m_1 + m_2))^2 \right. \\
&\left. (2m_1 m_2^2 - m_1 m_X^2 + m_2^3)^2 \right)^{-1},
\end{aligned} \quad (\text{A.18})$$

$$\begin{aligned}
|\mathcal{M}_{\phi_2 \phi_2^\dagger \rightarrow \phi_1 \phi_1^\dagger}|^2 &= \left(\lambda_{12} - \frac{4m_1^2 (g_1^2 + g_2^2)}{m_2^2} - \frac{\lambda_{H1} \lambda_{H2} v^2}{m_{h_2}^2 - 4m_2^2} - \frac{\lambda_{\chi 1} \lambda_{\chi 2} v_\chi^2}{m_{h_1}^2 - 4m_2^2} \right)^2,
\end{aligned} \quad (\text{A.19})$$

$$|\mathcal{M}_{\phi_2 \phi_2 \rightarrow \phi_1 \phi_1^\dagger}|^2 = \frac{64g_1^2 g_2^2 m_1^4}{m_2^4}. \quad (\text{A.20})$$

Appendix B: Direct detection cross sections

We list the cross section formulas for direct detection [29].

For $m_s = m_s = m_2$, we consider the direct direction of multi-component dark matter with ϕ_1 and ϕ_2 in our model. The effective Lagrangian for dark matter-quark elastic scattering is given by

$$\begin{aligned}
\mathcal{L}_{q,\text{eff}} &= \sum_{i=1,2} \left(\frac{\lambda_{q2} y_{h_2 \phi_i^\dagger \phi_i}}{m_{h_2}^2} + \frac{\lambda_{q1} y_{h_1 \phi_i^\dagger \phi_i}}{m_{h_1}^2} \right) |\phi_i|^2 \bar{q} q \\
&+ \sum_{i=1,2} \frac{g_X q_i e \varepsilon Q_q}{m_{Z'}^2} i (\phi_i \partial_\mu \phi_i^\dagger - \phi_i^\dagger \partial_\mu \phi_i) \bar{q} \gamma^\mu q,
\end{aligned} \quad (\text{B.1})$$

with $q_1 = 1$, $q_2 = 2$. Then, the relevant matching conditions between quark and nucleon operators are given by

$$\langle N | \bar{q} q | N \rangle = \frac{m_N}{m_q} f_{Tq}^{(N)}, \quad (\text{B.2})$$

for light quarks ($q = u, d, s$), and

$$\langle N | \bar{q} q | N \rangle = \frac{2}{27} \frac{m_N}{m_q} f_{Tq}^{(N)}, \quad f_{TG}^{(N)} = 1 - \sum_{q=u,d,s} f_{Tq}^{(N)}, \quad (\text{B.3})$$

for heavy quarks ($q = c, b, t$), and those for vector operators are given by

$$\begin{aligned}\langle N|\bar{u}\gamma^\mu u|N\rangle &= 2\bar{N}\gamma^\mu N, & N = p, \\ \langle N|\bar{d}\gamma^\mu d|N\rangle &= \bar{N}\gamma^\mu N, & N = p, \\ \langle N|\bar{u}\gamma^\mu u|N\rangle &= \bar{N}\gamma^\mu N, & N = n, \\ \langle N|\bar{d}\gamma^\mu d|N\rangle &= 2\bar{N}\gamma^\mu N, & N = n.\end{aligned}\quad (\text{B.4})$$

As a result, we get the effective Lagrangian for dark matter-nucleon elastic scattering as follows,

$$\begin{aligned}\mathcal{L}_{N,\text{eff}} &= \sum_{i=1,2} \frac{m_N}{m_q} \left(\frac{\lambda_{q2} y_{h_2 \phi_i^\dagger \phi_i}}{m_{h_2}^2} + \frac{\lambda_{q1} y_{h_1 \phi_i^\dagger \phi_i}}{m_{h_1}^2} \right) \\ &\times \left(\sum_{q=u,d,s} f_{Tq}^{(N)} + \frac{2}{27} f_{TG}^{(N)} \times 3 \right) |\phi_i|^2 \bar{N} N \\ &+ \sum_{i=1,2} \frac{g_X q_i e \varepsilon}{m_X^2} i(\phi_i \partial_\mu \phi_i^\dagger - \phi_i^\dagger \partial_\mu \phi_i) \\ &\times \left((2Q_u + Q_d) \bar{p} \gamma^\mu p + (Q_u + 2Q_d) \bar{n} \gamma^\mu n \right).\end{aligned}\quad (\text{B.5})$$

Thus, from $2Q_u + Q_d = +1$ and $Q_u + 2Q_d = 0$, there is a nonzero interaction only for dark matter-proton scattering with X -portal. Consequently, we obtain the total spin-independent cross sections for the ϕ_i -nucleus coherent scattering [29], as follows,

$$\begin{aligned}\sigma_{\phi_i-A}^{\text{coh}} &= \frac{\mu_{A,i}^2}{4\pi m_i^2} \\ &\times \left[Z \left(c_p^{(i)} f_p + g_p^{(i)} \right) + (A-Z) \left(c_n^{(i)} f_n + g_n^{(i)} \right) \right]^2,\end{aligned}\quad (\text{B.6})$$

where $\mu_{A,i} = m_i m_A / (m_A + m_i)$ is the reduced mass of the ϕ_i -nucleus system with m_A being the target nucleus mass, Z, A are the number of protons and the atomic number, respectively, and the effective couplings and form factors are given by

$$c_N^{(i)} \equiv \frac{m_N}{m_q} \left(\frac{\lambda_{q2} y_{h_2 \phi_i^\dagger \phi_i}}{m_{h_2}^2} + \frac{\lambda_{q1} y_{h_1 \phi_i^\dagger \phi_i}}{m_{h_1}^2} \right), \quad (\text{B.7})$$

$$f_p \equiv \sum_{q=u,d,s} f_{Tq}^{(p)} + \frac{2}{9} f_{TG}^{(p)} \simeq 0.28, \quad (\text{B.8})$$

$$f_n \equiv \sum_{q=u,d,s} f_{Tq}^{(n)} + \frac{2}{9} f_{TG}^{(n)} \simeq 0.28, \quad (\text{B.9})$$

$$g_p^{(i)} = -\frac{2eq_i g_X \varepsilon m_i}{m_X^2}, \quad (\text{B.10})$$

$$g_n^{(i)} \approx 0. \quad (\text{B.11})$$

Similarly, the $\phi_i^\dagger$ -nucleus scattering cross sections have the X contributions flipped in sign, given by

$$\begin{aligned}\sigma_{\phi_i^\dagger-A}^{\text{coh}} &= \frac{\mu_{A,i}^2}{4\pi m_i^2} \\ &\times \left[Z \left(c_p^{(i)} f_p - g_p^{(i)} \right) + (A-Z) \left(c_n^{(i)} f_n - g_n^{(i)} \right) \right]^2.\end{aligned}\quad (\text{B.12})$$

Then, the averaged spin-independent cross section for dark matter-nucleus coherent scattering is given

$$\begin{aligned}\sigma_{\phi_i, \phi_i^\dagger-A}^{\text{coh}} &= \frac{r_i}{2} \left(\sigma_{\phi_i-A}^{\text{coh}} + \sigma_{\phi_i^\dagger-A}^{\text{coh}} \right) \\ &= \frac{\mu_{A,i}^2}{4\pi m_i^2} r_i \left[\left(Z c_p^{(i)} f_p + (A-Z) c_n^{(i)} f_n \right)^2 + Z^2 (g_p^{(i)})^2 \right],\end{aligned}\quad (\text{B.13})$$

with $r_i = \Omega_i / \Omega_{\text{DM}}$. As a result, the above dark matter-nucleus scattering cross section is related to the normalized-to-proton scattering cross section by

$$\sigma_{\phi_i, \phi_i^\dagger-p}^{\text{coh}} = \left(\frac{\mu_{N,i}}{\mu_{A,i}} \right)^2 \frac{\sigma_{\phi_i, \phi_i^\dagger-A}}{A^2}, \quad (\text{B.14})$$

with $\mu_{N,i} = m_i m_N / (m_N + m_i)$.

On the other hand, if the dark matter-nucleus scattering is not coherent, such as for the boosted dark matter ϕ_1 , the total spin-independent cross sections in Eqs. (B.6) and (B.12) should be replaced by

$$\begin{aligned}\sigma_{\phi_1-A}^{\text{inc}} &= \frac{\mu_{A,i}^2}{4\pi m_1^2} \\ &\times \left[Z \left(c_p^{(1)} f_p + g_p^{(1)} \right)^2 + (A-Z) \left(c_n^{(1)} f_n + g_n^{(1)} \right)^2 \right],\end{aligned}\quad (\text{B.15})$$

$$\begin{aligned}\sigma_{\phi_1^\dagger-A}^{\text{inc}} &= \frac{\mu_{A,i}^2}{4\pi m_1^2} \\ &\times \left[Z \left(c_p^{(1)} f_p - g_p^{(1)} \right)^2 + (A-Z) \left(c_n^{(1)} f_n - g_n^{(1)} \right)^2 \right],\end{aligned}\quad (\text{B.16})$$

respectively. Then, the averaged spin-independent cross sections for dark matter-nucleus incoherent scattering are given by

$$\begin{aligned}\sigma_{\phi_1, \phi_1^\dagger-A}^{\text{inc}} &= \frac{1}{2} \left(\sigma_{\phi_1-A}^{\text{inc}} + \sigma_{\phi_1^\dagger-A}^{\text{inc}} \right) \\ &= \frac{\mu_{A,1}^2}{4\pi m_1^2} \left[Z \left(c_p^{(1)} f_p \right)^2 + Z (g_p^{(1)})^2 + (A-Z) \left(c_n^{(1)} f_n \right)^2 \right].\end{aligned}\quad (\text{B.17})$$

Then, the above dark matter-nucleus scattering cross section is related to the normalized-to-proton scattering cross section

by

$$\sigma_{\phi_1, \phi_1^\dagger \rightarrow p}^{\text{inc}} = \left(\frac{\mu_{N,1}}{\mu_{A,1}} \right)^2 \frac{\sigma_{\phi_1, \phi_1^\dagger \rightarrow A}^{\text{inc}}}{A}. \quad (\text{B.18})$$

The elastic scattering between dark matter and electron is mediated by CP-even scalars and the X gauge boson, so the corresponding cross sections are similarly given [29] by

$$\begin{aligned} \sigma_{\phi_i, \phi_i^\dagger \rightarrow e} &= \frac{\mu_{e,i}^2}{8\pi m_i^2} r_i \left[\left(\frac{\lambda_{e2} y_{h2} \phi_i^\dagger \phi_i}{m_{h2}^2} + \frac{\lambda_{e1} y_{h1} \phi_i^\dagger \phi_i}{m_{h1}^2} \right) + \frac{2eq_i g_X \varepsilon m_i}{m_X^2} \right]^2 \\ &+ \frac{\mu_{e,i}^2}{8\pi m_i^2} r_i \left[\left(\frac{\lambda_{e2} y_{h2} \phi_i^\dagger \phi_i}{m_{h2}^2} + \frac{\lambda_{e1} y_{h1} \phi_i^\dagger \phi_i}{m_{h1}^2} \right) - \frac{2eq_i g_X \varepsilon m_i}{m_X^2} \right]^2 \\ &= \frac{\mu_{e,i}^2}{4\pi m_i^2} r_i \left[\left(\frac{\lambda_{e2} y_{h2} \phi_i^\dagger \phi_i}{m_{h2}^2} + \frac{\lambda_{e1} y_{h1} \phi_i^\dagger \phi_i}{m_{h1}^2} \right)^2 + \frac{4e^2 q_i^2 g_X^2 \varepsilon^2 m_i^2}{m_X^4} \right] \end{aligned} \quad (\text{B.19})$$

where $\mu_{e,i} = m_i m_e / (m_e + m_i)$ is the reduced mass of the ϕ_i -electron system.

Appendix C: Kinematics for boosted dark matter

Consider the DM co-annihilation processes, $\phi_1^\dagger \phi_2^{(\dagger)} \rightarrow \phi_1 Y$ with $Y = h_1, X$, or its conjugate process.

In the galactic center of mass frame, from the momentum and the energy for Y ,

$$\begin{aligned} p_Y &= \frac{m_2(2m_1 + m_2)}{2(m_1 + m_2)} \left(1 - \frac{m_Y^2}{m_2^2} \right)^{1/2} \\ &\times \left(1 - \frac{m_Y^2}{(2m_1 + m_2)^2} \right)^{1/2}, \end{aligned} \quad (\text{C.1})$$

$$E_Y = \frac{m_2^2 + 2m_1 m_2 + m_Y^2}{2(m_1 + m_2)}, \quad (\text{C.2})$$

the velocity for the Y particle [6, 7] is given by

$$\begin{aligned} v_Y &= \frac{p_Y}{E_Y} = \frac{m_2(2m_1 + m_2)}{m_2^2 + 2m_1 m_2 + m_Y^2} \left(1 - \frac{m_Y^2}{m_2^2} \right)^{1/2} \\ &\times \left(1 - \frac{m_Y^2}{(2m_1 + m_2)^2} \right)^{1/2}. \end{aligned} \quad (\text{C.3})$$

Similarly, with the momentum, $p'_1 = p_Y$, and the energy for ϕ_1 appearing in the final states, given by

$$E'_1 = \frac{2m_1^2 + 2m_1 m_2 + m_2^2 - m_Y^2}{2(m_1 + m_2)}, \quad (\text{C.4})$$

the velocity for the boosted dark matter ϕ_1 from the DM co-annihilations is given by

$$\begin{aligned} v'_1 &= \frac{p_Y}{E'_1} = \frac{m_2(2m_1 + m_2)}{2m_1^2 + 2m_1 m_2 + m_2^2 - m_Y^2} \\ &\times \left(1 - \frac{m_Y^2}{m_2^2} \right)^{1/2} \left(1 - \frac{m_Y^2}{(2m_1 + m_2)^2} \right)^{1/2}. \end{aligned} \quad (\text{C.5})$$

Moreover, when the Y particle decays into a pair of leptons, ignoring the lepton masses, we get the energy of the leptons in the rest frame of the Y particle as $\bar{E}_f = \frac{m_Y}{2}$. However, the energy of the leptons in the galactic center of mass frame is boosted [6, 7] to

$$E_f = \frac{1}{\gamma_Y} \bar{E}_f (1 - v_Y \cos \theta)^{-1} \quad (\text{C.6})$$

where θ is the angle between the velocity of the Y particle and the lepton velocity, and

$$\gamma_Y = \frac{1}{\sqrt{1 - v_Y^2}} = \frac{E_Y}{m_Y}. \quad (\text{C.7})$$

Therefore, the leptons can carry the energies between E_f^- and E_f^+ with

$$E_f^\pm = \sqrt{\frac{1 \mp v_Y}{1 \pm v_Y}} \frac{m_Y}{2}, \quad (\text{C.8})$$

and the energy spectrum for the leptons is box-shaped with the width being given by $\Delta E_f = E_f^+ - E_f^- = v_Y \gamma_Y m_Y$.

We also comment on the cosmic ray signatures from boosted dark matter in our model.

Due to the same semi-annihilation processes discussed for boosted dark matter in the previous section, namely, $\phi_1^\dagger \phi_2^{(\dagger)} \rightarrow \phi_1 Y$ with $Y = X, h_1$ and their complex conjugate processes, the subsequent decays of the boosted particle $Y, Y \rightarrow f \bar{f}$, can produce leptons at galaxies, so they give rise to cosmic ray signals too.⁵ Then, we can consider the corresponding differential flux for the leptons as

$$\frac{d\Phi_f}{dE_f} = \frac{1}{4\pi m_1 m_2} \langle \sigma v \rangle_{\phi_1^\dagger \phi_2^{(\dagger)} \rightarrow \phi_1 Y, \text{c.c.}} \cdot \frac{dN_f}{dE_f} r_1 (1 - r_1) J \quad (\text{C.9})$$

where the differential number of leptons is given by

$$\frac{dN_f}{dE_f} = \frac{1}{E_f^+ - E_f^-} \Theta(E_f - E_f^-) \Theta(E_f^+ - E_f) \text{BR}(Y \rightarrow f \bar{f}), \quad (\text{C.10})$$

where

$$E_f^\pm = \sqrt{\frac{1 \mp v_Y}{1 \pm v_Y}} \frac{m_Y}{2}, \quad (\text{C.11})$$

with v_Y being the velocity of the Y particle in the galactic frame, given by

⁵ We also remark that the energetic leptons produced from the semi-annihilation processes can be also constrained by the CMB recombination [6, 7].

$$v_Y = \frac{m_2(2m_1 + m_2)}{m_2^2 + 2m_1m_2 + m_Y^2} \times \left(1 - \frac{m_Y^2}{m_2^2}\right)^{1/2} \left(1 - \frac{m_Y^2}{(2m_1 + m_2)^2}\right)^{1/2}. \quad (\text{C.12})$$

We note that we need to consider the J -factor for indirect detection, in the following way,

$$J = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds \rho_{\text{DM}}^2. \quad (\text{C.13})$$

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