

Letter

Small neutrino masses from a decoupled singlet scalar field

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ARTICLE INFO

Editor: G.F. Giudice

ABSTRACT

We propose a unified solution with Z_4 discrete symmetry for small neutrino masses and stability of dark matter. The Standard Model is extended with an inert doublet scalar, a dark singlet scalar, a spurion scalar and right-handed neutrinos, which all transform nontrivially under Z_4 . After the Z_4 symmetry is broken to Z_2 by the VEV of the spurion, much below the mass scale of the dark singlet scalar, a small lepton number violating coupling for the inert doublet is generated at tree level, so small neutrino masses are obtained at one-loops for relatively light new fields. We discuss the important roles of the Z_4 symmetry for neutrino masses, dark matter physics and thermal leptogenesis.

1. Introduction

Neutrino oscillations [1] indicate neutrinos masses of $m_\nu \sim 0.05$ eV. Moreover, the bounds from neutrinoless double beta decay experiments [2] and Cosmic Microwave Background anisotropies and Baryon Acoustic Oscillations [3] are $m_{\bar{\nu}_e} < 0.8$ eV and $\sum_i m_{\nu_i} < 0.12$ eV at 95% CL, respectively. In order to account for small neutrino masses, it is possible to extend the Standard Model (SM) by the dimension-5 operators for neutrino masses with a cutoff scale beyond the electroweak scale or light sterile neutrinos coupled to neutrinos much below the electroweak scale [4].

Small neutrino masses can be obtained by the mixing between active neutrinos and heavy Majorana neutrinos [5–8]. Alternatively, small neutrino masses can be generated by loop corrections, for instance, due to right-handed (RH) neutrinos and an extra doublet scalar [9], the latter of which serves as a dark matter candidate [10,11]. However, there is still a need of very small parameters in obtaining small neutrino masses in similar models [4].

In this article, we propose a novel model with a Z_4 discrete symmetry for the common origin of small neutrino masses and stability of dark matter. In our model, there are one inert doublet scalar, three right-handed neutrinos, a real spurion scalar and a singlet complex scalar field, all transforming nontrivially under Z_4 . The lepton number is respected in the renormalizable interactions due to Z_4 . Then, only after the Z_4 symmetry is broken to Z_2 due to the VEV of the spurion field, the lepton number is broken, so neutrino masses are generated at one-loops. Moreover, the lightest neutral dark scalar in the model becomes a

dark matter candidate. The spurion scalar can be regarded as a pseudo-Goldstone boson coming from the breaking of a global $U(1)$ symmetry into Z_4 at a higher scale, and Z_4 can be further down to Z_2 at a low scale.

Taking the decoupling limit where the singlet scalar field S becomes heavier than the Z_4 breaking scale, we show how a small mass splitting between the neutral components of the inert doublet arises, in turn giving rise to small neutrino masses at one-loops. In the present case, we argue that large masses of extra fields or small Yukawa couplings are not necessary, unlike the case with the Z_2 symmetry only. We discuss the roles of the new singlet scalars and constraints for neutrino masses, dark matter and thermal leptogenesis.

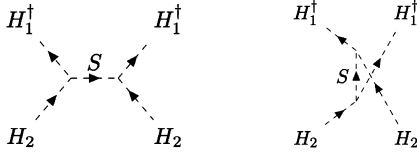
2. Discrete symmetry and dark matter scalars

We introduce a complex singlet scalar S and one extra electroweak doublet scalar, H_2 , other than the SM Higgs doublet H_1 , and three right-handed (RH) neutrinos, $N_{R,i}$, with $i = 1, 2, 3$, and a spurion real scalar field ϕ , beyond the SM. We impose a Z_4 symmetry for the extra fields such that $\phi_i \rightarrow e^{iQ_X \pi/2} \phi_i$ with $\phi_i = S, \phi, H_1, H_2, N_R$ and $Q_X(\phi_i) = (+1, +2, 0, +1, +1)$. But, the SM fermions are neutral under Z_4 so the model is different from the flavor-dependent Z_4 symmetry introduced in Ref. [12]. The Z_4 symmetry can be embedded into a global $U(1)$ Peccei-Quinn symmetry which is anomalous in the presence of the couplings to the SM quarks or extra quarks, but we don't pursue this possibility in detail in the current work. As the subgroup of the Z_4 symmetry, we can identify Z_2 parities as $Z_2(\phi_i) = (-, +, +, -, -)$. The

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Received 29 November 2024; Accepted 1 January 2025

Fig. 1. Feynman diagrams contributing to $\lambda_{5,\text{eff}}$.

spurion field φ transforms by $\varphi \rightarrow -\varphi$ under the Z_4 symmetry but it is invariant under the Z_2 subgroup.

Now we consider the relevant interaction Lagrangian for the model as

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -\sqrt{2}\kappa S^\dagger H_1^\dagger H_2 - \sqrt{2}\lambda'_{S\varphi} \varphi S H_1^\dagger H_2 \\ & - y_{N,ij} \bar{l}_i \tilde{H}_2 N_{R,j} - \frac{1}{2} \lambda_{N,ij} \varphi \overline{N_{R,i}^c} N_{R,j} + \text{h.c.} \end{aligned} \quad (1)$$

with $\tilde{H}_2 = i\sigma_2 H_2^*$. It is remarkable that $\frac{1}{2}\lambda_5(H_1^\dagger H_2)^2$ and its hermitian conjugate, vanish due to the Z_4 symmetry, and there is no bare mass terms for right-handed neutrinos due to the lepton number conservation. There are extra Z_4 invariant interaction terms in the potential such as $\mu\varphi S^2$, $\frac{1}{2}\lambda_{H_i\varphi}\varphi^2|H_i|^2$ ($i = 1, 2$), etc, as shown in the appendix, but it is sufficient to focus on eq. (1) for our discussion below.

After φ gets a VEV by $\langle\varphi\rangle = v_\varphi$, Z_4 is broken to Z_2 and we obtain the Z_2 invariant effective interactions and the mass terms for right-handed neutrinos,

$$\begin{aligned} \mathcal{L}_{\text{int,eff}} = & -\sqrt{2}\kappa S^\dagger H_1^\dagger H_2 - \sqrt{2}\kappa' S H_1^\dagger H_2 \\ & - y_{N,ij} \bar{l}_i \tilde{H}_2 N_{R,j} - \frac{1}{2} M_{N,ij} \overline{N_{R,i}^c} N_{R,j} + \text{h.c.} \end{aligned} \quad (2)$$

where $\kappa' \equiv \lambda'_{S\varphi} v_\varphi$ is the Z_4 breaking effective interaction and $M_{N,ij} \equiv \lambda_{N,ij} v_\varphi$ are the masses for RH neutrinos. Here, we assumed the basis where $\lambda_{N,ij}$ is diagonalized. After electroweak symmetry breaking with $\langle H_1 \rangle = (0, v_H)^T / \sqrt{2} \neq 0$, there appear mass mixings between dark matter scalars, namely, S and the neutral components of H_2 , so nonzero neutrino masses can be generated at one-loops, which will be discussed later. When the SM Higgs and the spurion field φ are expanded around the VEVs as $H_1 = (0, v_H + h)^T / \sqrt{2}$ and $\varphi = v_\varphi + \rho$, the dark Higgs ρ can mix with the SM Higgs h , so the mass eigenstates become $h_1 = c_\alpha h + s_\alpha \rho$ and $h_2 = -s_\alpha h + c_\alpha \rho$, with $c_\alpha \equiv \cos \alpha$, $s_\alpha \equiv \sin \alpha$.

We remark that S and H_2 do not get VEVs, leaving a Z_2 symmetry unbroken. S , H_2 and $N_{R,i}$ are odd under the remaining Z_2 , which ensures the stability of the lightest neutral Z_2 -odd particle among them as a dark matter candidate. For small neutrino masses without introducing small parameters, we consider the case where the singlet scalar S is decoupled, so either the lighter neutral component of H_2 or the lightest RH neutrino can be a dark matter candidate.

3. Decoupled scalar and inert doublet scalar masses

From the effective interactions in eq. (2), we can take the limit of integrating out the singlet scalar S whose squared mass becomes $m_S^2 \gg \kappa\kappa'$. As a result, we obtain a renormalizable but Z_4 breaking effective interaction for two Higgs doublets from κ, κ' terms, at tree level as shown in Fig. 1, in the following form,

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\lambda_{5,\text{eff}}(H_1^\dagger H_2)^2 + \text{h.c.} \quad (3)$$

with

$$\lambda_{5,\text{eff}} = -\frac{4\kappa\kappa'}{m_S^2}. \quad (4)$$

Here, κ is a Z_4 invariant dimensionful coupling, so it can be of order m_S . But, κ' is a Z_4 breaking coupling due to the VEV of the spurion field φ , so it is naturally suppressed as compared to m_S , that is, $\kappa' = \lambda'_{S\varphi} v_\varphi \ll m_S$.

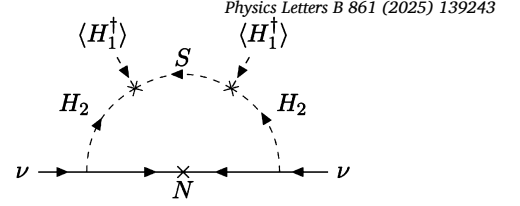


Fig. 2. Feynman diagrams contributing to neutrino masses at one-loop.

Therefore, the effective $\lambda_{5,\text{eff}}$ term in eq. (4) is also suppressed as a consequence of the decoupled scalar.

For a small λ_5 term in the effective theory below the threshold of the singlet scalar S , we obtain the masses for the components of the inert doublet, $H_2 = (H^+, \frac{1}{\sqrt{2}}(H_0 + iA_0))^T$, as follows,

$$m_{H^\pm}^2 = m_2^2 + \frac{1}{2}\lambda_3 v_H^2, \quad (5)$$

$$m_{H_0}^2 = m_2^2 + \frac{1}{2}(\lambda_L + \lambda_{5,\text{eff}})v_H^2, \quad (6)$$

$$m_{A_0}^2 = m_2^2 + \frac{1}{2}(\lambda_L - \lambda_{5,\text{eff}})v_H^2 \quad (7)$$

where $\lambda_L \equiv \lambda_3 + \lambda_4$, with λ_3, λ_4 being mixing quartic couplings in the scalar potential, $\lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1)$, and m_2^2 is the squared mass parameter for the inert doublet H_2 . Henceforth, we choose $\lambda_{5,\text{eff}} > 0$ without loss of generality such that A_0 is a dark matter candidate. The most general results for the mass eigenvalues and the mixing angles with the non-decoupled singlet scalar S are shown in eqs. (39), (50) and (51), in the appendix.

In the decoupling limit of the singlet scalar S , the small effective λ_5 term gives rise to a mass splitting between two real neutral scalars of H_2 is suppressed as $\Delta \equiv m_{H_0} - m_{A_0} \simeq \frac{\lambda_{5,\text{eff}} v_H^2}{2m_0}$, with $m_0^2 \equiv (m_{H_0}^2 + m_{A_0}^2)/2$. So, we can avoid the strong bound on DM-nucleon scattering with Z boson exchange, as far as $|\Delta| \gtrsim 100$ keV, which sets a lower bound on $\lambda_{5,\text{eff}}$ as $\lambda_{5,\text{eff}} \gtrsim 3.3 \times 10^{-7} (m_0/100 \text{ GeV})(|\Delta|/100 \text{ keV})$. Then, for $\kappa \sim m_S$, we need $\kappa'/m_S \gtrsim 8 \times 10^{-8}$ for $m_0 \sim 100$ GeV and $|\Delta| \gtrsim 100$ keV.

4. Decoupled scalar and small neutrino masses

In the decoupling limit of the singlet scalar S , from the combination of the Yukawa interactions for right-handed neutrinos and a small effective $\lambda_{5,\text{eff}}$ term in eq. (3), we obtain the one-loop corrections for neutrino masses from the Feynman diagram shown in Fig. 2 as

$$\begin{aligned} (\mathcal{M}_\nu)_{ij} \simeq & \frac{\lambda_{5,\text{eff}} v_H^2}{16\pi^2} \sum_k \frac{y_{N,ik} y_{N,jk} M_{N,k}}{m_0^2 - M_{N,k}^2} \\ & \times \left[1 - \frac{M_{N,k}^2}{m_0^2 - M_{N,k}^2} \ln \frac{m_0^2}{M_{N,k}^2} \right]. \end{aligned} \quad (8)$$

The above results match with those due to a nonzero small λ_5 in the case without a Z_4 symmetry in Ref. [9], and the same results can be obtained from the decoupling limit of the general results in eq. (53) in the appendix. Then, when the RH neutrinos are heavier than the inert doublet Higgs, namely, $M_{N,k} \gtrsim m_0$, the above expression with eq. (4) gets simplified to

$$(\mathcal{M}_\nu)_{ij} \simeq \frac{\lambda_{5,\text{eff}} v_H^2}{16\pi^2} \sum_k \frac{y_{N,ik} y_{N,jk}}{M_{N,k}} \left[\ln \frac{M_{N,k}^2}{m_0^2} - 1 \right]. \quad (9)$$

Therefore, for neutrino masses, $m_\nu \lesssim 0.1$ eV, choosing $y_{N,ij} = \mathcal{O}(1)$ and $M_{N,k} \sim 10^9$ GeV, we can set the upper bound on $\lambda_{5,\text{eff}}$ as $\lambda_{5,\text{eff}} \lesssim 8 \times 10^{-6}$, which is consistent with the lower bound on Δ from the direct detection. In other words, we need $\kappa'/m_S \lesssim 10^{-6}$ for $\kappa \sim m_S$ and $M_{N,k} \lesssim \kappa' \ll m_S$ for $\lambda_{N,kk} \lesssim \lambda'_{S\varphi}$. Then, we can keep the RH right-handed neutrinos in the effective theory while the singlet scalar S is

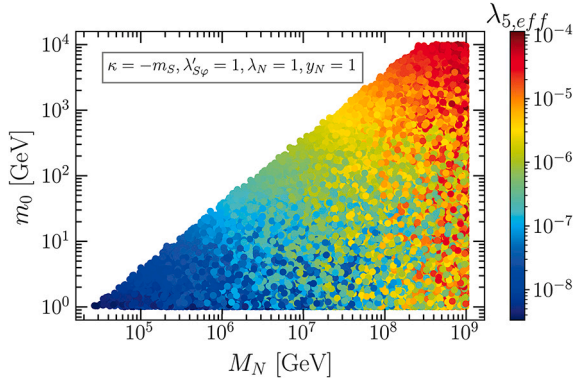


Fig. 3. Inert doublet scalar mass m_0 vs RH neutrino mass M_N for varying $\lambda_{5,\text{eff}}$.

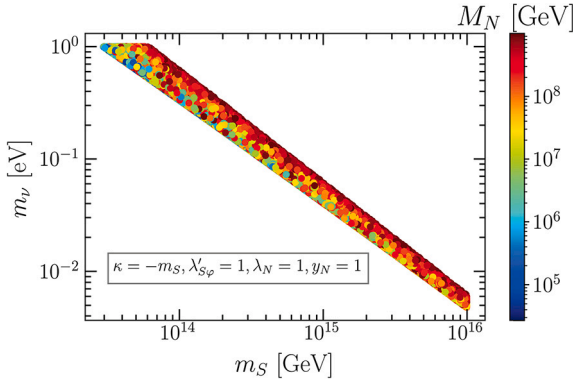


Fig. 4. Neutrino mass m_ν vs singlet scalar mass m_S for varying M_N .

decoupled. So, for a small $\lambda_{5,\text{eff}}$, it is sufficient to take $m_S \gtrsim M_{N,k} \sim 10^9$ GeV, unless we tune the Yukawa couplings $y_{N,ij}$ to small values.

In the case with $m_0 \gtrsim M_{N,k}$, the lightest RH neutrino is a dark matter candidate [13] whereas the scalars of the inert doublet decay into a pair of the SM lepton and the RH neutrino through the Yukawa interactions. In this limit, eq. (8) becomes approximated to $(M_\nu)_{ij} \simeq \frac{\lambda_{5,\text{eff}} v_H^2}{16\pi^2 m_0^2} \sum_k y_{N,ik} y_{N,jk} M_{N,k}$. For instance, for $m_\nu \lesssim 0.1$ eV, $y_{N,ij} = \mathcal{O}(1)$, $m_0 \sim 100$ GeV and $M_{N,k} \sim 10$ MeV, we can set the upper bound, $\lambda_{5,\text{eff}} \lesssim 8 \times 10^{-10}$ or $|\Delta| \lesssim 0.2$ keV. But, there is no more lower bound on $\lambda_{5,\text{eff}}$ because the scalars in the inert doublet decay. A small effective $\lambda_{5,\text{eff}}$ is achievable if $m_S \gtrsim 10^9$ GeV ($\lambda'_{S\phi}/\lambda_{N,kk}$) for $v_\phi = 10$ MeV/ $\lambda_{N,kk}$ for $\kappa \sim m_S$.

In Fig. 3, we show the parameter space for inert doublet scalar mass m_0 vs RH neutrino mass $M_N \equiv M_{N,1}$ satisfying $m_\nu = 0.1$ eV. The color code indicates a scan over $\lambda_{5,\text{eff}}$ between 10^{-8} and 10^{-4} , showing that $\lambda_{5,\text{eff}} \gtrsim 10^{-6}$ and $M_N \gtrsim 2 \times 10^7$ GeV are necessary for $m_0 \gtrsim 100$ GeV. In Fig. 4, we decode the correlation between neutrino mass m_ν and singlet scalar mass m_S for varying M_N between 10^4 GeV and 10^9 GeV. In both Figs. 3 and 4, we imposed $|\Delta| > 100$ keV and took into account the perturbativity, $|\lambda_{5,\text{eff}}| < 1$. Then, we need the singlet scalar mass m_S to be of order $10^{14} - 10^{16}$ GeV for $m_\nu = 0.01 - 0.1$ eV, which might hint at a connection of neutrino mass generation to the unification. If we choose smaller Yukawa couplings y_N , the diagonal boundary in Fig. 3 is shifted to the left, allowing for smaller M_N for a fixed m_0 , and the strip in Fig. 4 is shifted to the bottom, making m_S smaller for a fixed m_ν .

5. Dark matter direct detection

When the lighter neutral scalar from the inert doublet is dark matter, we can avoid the strong bound from direct detection if the mass splitting between A_0 and H_0 is large enough, i.e. $|\Delta| \gtrsim 100$ keV. Then, the cross section for spin-independent scattering between dark matter and

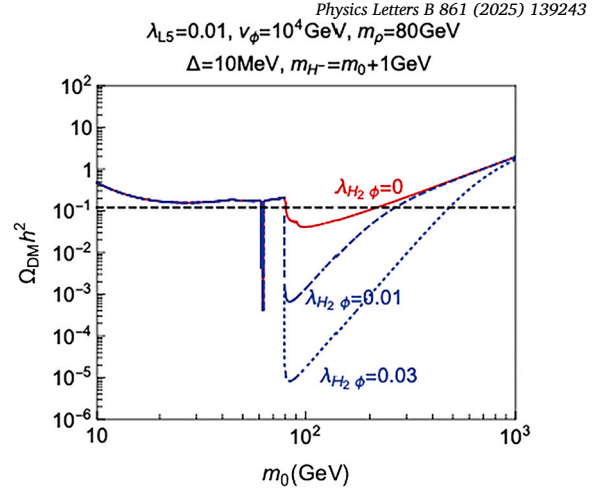


Fig. 5. DM relic density as a function of DM mass m_0 , depending on the inert doublet portal coupling, $\lambda_{H_2\phi}$.

nucleus is dominated by the tree-level exchanges of Higgs-like scalars [10,11], as follows,

$$\sigma_{A_0}^{\text{SI}} = \frac{\lambda_{L5}^2 \mu_N^2 (Z f_p + (A - Z) f_n)^2}{4\pi m_{A_0}^2 A^2} \left(\frac{c_\alpha^2}{m_{h_1}^2} - \frac{s_\alpha^2}{m_{h_2}^2} \right)^2 \quad (10)$$

where $\lambda_{L5} \equiv \lambda_L - \lambda_{5,\text{eff}}$, $m_{h_{1,2}}$ are the masses for SM Higgs-like and dark Higgs-like scalars, $Z, A - Z$ are the numbers of protons and neutrons in the detector nucleus, μ_N is the reduced mass for the DM-nucleus system, and $f_{p,n}$ are the nucleon form factors [14–16].

The DM relic density is determined by the DM annihilations into a pair of SM fermions, WW, ZZ and $h_i h_j$ with $i, j = 1, 2$ [10,11,17]. If $H_0, H^\pm$ have comparable masses to m_{A_0} , we also need to include the co-annihilation channels associated with them as eq. (57) in the appendix. In particular, there is room to adjust λ_L and dark Higgs-portal couplings, denoted by $\mathcal{L}_{\text{int}} \supset -\frac{1}{2} \sum_{i=1,2} \lambda_{H_i\phi} |H_i|^2 \phi^2$, to explain the DM relic density for relatively heavy DM due to $A_0 A_0 \rightarrow h_i h_j$ while the direct detection bound is satisfied [18]. DM masses above about 100 GeV are consistent with the bounds from direct detection, as far as $|\lambda_{L5}(c_\alpha^2 - s_\alpha^2 m_{h_1}^2/m_{h_2}^2)| \lesssim 10^{-2}$, apart from the resonance, and the bounds from the SM Higgs decays can be easily satisfied for $m_{A_0}, m_{h_2} \gtrsim m_{h_1}/2 = 62.5$ GeV. But, for a smaller value of $|\lambda_{L5}|$, as required near resonances, the one-loop corrections to the DM-nucleus cross section are relevant [19].

In Fig. 5, we depict the DM relic density as a function of DM mass for $\lambda_{H_2\phi} = 0, 0.01, 0.03$, in red solid, blue dashed and dotted lines, respectively. We chose a zero Higgs mixing, $\alpha = 0$, the singlet parameters are $\lambda_{H_1\phi} = 0$, $v_\phi = 10^4$ GeV and $m_{h_2} = m_\rho = 80$ GeV, and the parameters for the inert doublet are $\lambda_{L5} = 0.01$, $\Delta = 10$ MeV and $m_{H^\pm} = m_0 + 1$ GeV. Thus, as $\lambda_{H_2\phi}$ increases, the correct relic density can be explained for a heavier DM mass far beyond 100 GeV. The results are insensitive to Δ as far as $\Delta \lesssim T_f$ where T_f is the freeze-out temperature given by $T_f \simeq m_0/20$. Moreover, we remark that a nonzero $\lambda_{H_1\phi}$ also contributes to the dark matter annihilation into $\phi\phi$ with the SM Higgs exchange, but its contribution is suppressed for a small $|\lambda_{L5}|$.

6. Thermal leptogenesis

Baryogenesis can be realized in our model via thermal leptogenesis [20], due to the decays of RH neutrinos with hierarchical masses. The RH neutrinos can be once in thermal equilibrium with the SM plasma in the early universe due to their Yukawa couplings to the inert doublet or the spurion scalar, which mixes with the SM Higgs. As the asymmetries generated from the decays of the heavier RH neutrinos, $N_{2,3}$, are negligible due to strong washout effects by scattering, we focus on the

N_1 contribution to the lepton asymmetry. The baryon-to-photon ratio converted from the final $B - L$ asymmetry is given by

$$\eta_B = -C\varepsilon_1\kappa_1 \quad (11)$$

where $C \simeq 0.01$, ε_1 is the CP asymmetry parameter, and κ_1 is the efficiency factor approximated [21] to

$$\kappa_1(K_1) \simeq \frac{2}{z_B(K_1)K_1} \left(1 - e^{-\frac{1}{2}z_B(K_1)K_1}\right), \quad (12)$$

with $K_1 = \Gamma_1/H(z_1 = 1)$ being the decay parameter and z_B being the value of $z_1 = M_{N,1}/T$ at the completion of the baryon asymmetry generation, given [21] by

$$z_B(K) \simeq 1 + \frac{1}{2} \ln \left(1 + \frac{\pi K^2}{1024} \left[\ln \left(\frac{3125\pi K^2}{1024} \right) \right]^5 \right). \quad (13)$$

Here, Γ_1 is the decay rate for N_1 , given by $\Gamma_1 \simeq \frac{M_{N,1}}{8\pi} (y_N^\dagger y_N)_{11}$ for $m_0 \ll M_{N,1}$, and $H(z_1)$ is the Hubble parameter. In the Casas-Ibarra parametrization for the Yukawa matrix y_N [22], we can express $(y_N^\dagger y_N)_{ij} = \sqrt{\Lambda_i \Lambda_j} \tilde{m}_{ij}$ in terms of the effective neutrino mass matrix $\tilde{m} \equiv R D_{\mathcal{M}_\nu} R^\dagger$ with R being arbitrary complex rotation matrix, $D_{\mathcal{M}_\nu} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, and $\Lambda_i = \frac{4\pi^2 \xi_i M_{N,i}}{\lambda_{5,\text{eff}} v_H^2}$ [23]. Here, $\xi_i = \left(\frac{1}{8} \frac{M_{N,i}^2}{m_{H_0}^2 - m_{A_0}^2} [L(m_{H_0}^2) - L(m_{A_0}^2)] \right)^{-1}$ with $L(m^2) = \frac{m^2}{m^2 - M_{N,i}^2} \ln(m^2/M_{N,i}^2)$ are order one parameters in most of the parameter space. Thus, we get the decay parameter as $K_1 \simeq \frac{2\pi^2 \xi_1}{\lambda_{5,\text{eff}}} \sqrt{\frac{45}{64\pi^5 g_*}} \frac{M_P}{v_H^2} \tilde{m}_{11}$ for $m_0 \ll M_{N,1}$.

For the case with three RH neutrinos, the CP asymmetry parameter is bounded [23,24] by

$$\varepsilon_1 \lesssim \frac{3\pi}{4\lambda_{5,\text{eff}} v_H^2} \xi_3 (m_h - m_l) M_{N,1} \quad (14)$$

with m_h, m_l are the masses for the heaviest and lightest active neutrinos, respectively. After optimizing the efficiency factor κ_1 and the CP asymmetry parameter ε_1 , we obtain the relation, $\lambda_{5,\text{eff}} \simeq 4\pi \xi_1 (m_l/10^{-3} \text{ eV})$ [23], for which the baryon-to-photon ratio without washout effects from ΔL processes is given by

$$\eta_B \simeq 3.8 \times 10^{-10} \left(\frac{M_{N,1}}{10^4 \text{ GeV}} \right) \left(\frac{10^{-4}}{\lambda_{5,\text{eff}}} \right) \left(\frac{m_h}{0.1 \text{ eV}} \right). \quad (15)$$

The efficiency factor κ_1 is subject to the corrections coming from the $\Delta L = 2$ processes such as $lH_2 \leftrightarrow \bar{l}\bar{H}_2$ and $ll \leftrightarrow \bar{H}_2\bar{H}_2$. Then, if the $\Delta L = 2$ processes become important after the baryon asymmetry generation is finished, the total efficiency factor is given by $\kappa_{\text{tot}} = \kappa_1 \exp(-\int_{z_B}^\infty dz \Delta W)$ with $\Delta W = \Gamma_{\Delta L=2}/H z_1$ [21,23]. As $\Gamma_{\Delta L=2}$ is the scattering rate for the $\Delta L = 2$ processes, we obtain $\int_{z_B}^\infty dz \Delta W = 5.2 \times 10^{-13} \text{ GeV} \xi_3 M_{N,1} m_h M_P / (\sqrt{g_*} \lambda_{5,\text{eff}} v_H^4)$ [23], which is less than unity for $\lambda_{5,\text{eff}} \gtrsim 1.7 \times 10^{-6}$ for $M_{N,1} = 10^4 \text{ GeV}$ and $m_h = 0.05 \text{ eV}$. Then, in Fig. 6, using the analytic results, we show the parameter space for RH neutrino mass M_N vs $\lambda_{5,\text{eff}}$ where the maximum baryon-to-photon ratio for $m_h = 0.05 \text{ eV}$ is obtained as observed along the boundary of the purple region. The analysis in the region with $M_N \lesssim 10^5 \text{ GeV}$ can be improved by numeral solutions to the Boltzmann equations [23]. For comparison, we also showed the contours for $m_h = 0.05 \text{ eV}$ with $m_0 = 500 \text{ GeV}$, depending on the values of the Yukawa coupling, $y_N = 10^{-3} - 1$, in the same plot.

We remark that the asymmetry in the inert doublet or the DM asymmetry can reduce the lepton asymmetry, because $lH_2 \rightarrow \bar{l}\bar{H}_2$ is favored over $\bar{l}\bar{H}_2 \rightarrow lH_2$ [25]. However, for a sizable $\lambda_{5,\text{eff}}$, $H_2\bar{H}_1 \leftrightarrow \bar{H}_2H_1$ process is fast, so it can make the DM asymmetry Boltzmann sufficiently suppressed. It turns out that $\Delta L = 2$ processes are inefficient if $|\lambda_{5,\text{eff}}| \gtrsim 10^{-5}$ for $M_{N,1} \lesssim 10^9 \text{ GeV}$ [26].

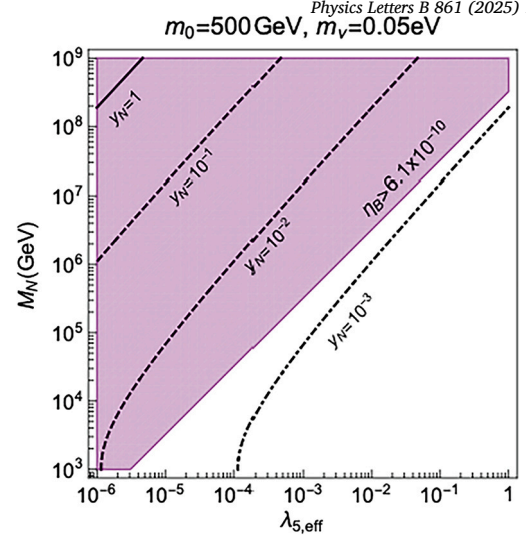


Fig. 6. Parameter space for RH neutrino mass M_N vs $\lambda_{5,\text{eff}}$, showing $\eta_B > 6.1 \times 10^{-10}$ in purple region. The contour lines show the parameter space for the heaviest neutrino mass equal to $m_\nu = 0.05 \text{ eV}$.

7. Conclusions

We presented a unified solution with Z_4 discrete symmetry for small neutrino masses and stability of dark matter in the radiative seesaw scenarios. Once the Z_4 symmetry is broken to Z_2 by the VEV of the spurion singlet scalar φ much below the mass of the singlet scalar S , a small effective lepton number violating interaction between two Higgs doublets is generated, being responsible for a small mass splitting of the neutral inert scalars at tree level and small neutrino masses at one-loop level.

Although the Z_2 symmetry is sufficient for stability of dark matter, its extension to the Z_4 symmetry in our model allows for explaining the origin of small neutrino masses, even if inert doublet scalar and RH neutrinos are relatively light and the Yukawa couplings for neutrinos are sizable. This option gives rise to phenomenologically a novel window for new particle searches. Moreover, we showed that the spurion scalar responsible for the Z_4 symmetry breaking opens up a new parameter space for obtaining the correct relic density for inert doublet-like dark matter and a successful leptogenesis is achieved.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Hyun Min Lee reports financial support was provided by National Research Foundation of Korea. Rojalin Padhan reports financial support was provided by National Research Foundation of Korea. Jongkuk Kim reports financial support was provided by National Research Foundation of Korea. Seong-Sik Kim reports financial support was provided by National Research Foundation of Korea. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The work is supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (NRF-2022R1A2C2003567 and RS-2024-00341419 (JKK)). This research was supported by the Chung-Ang University research grant in 2019.

Appendix A

Here we provide the details on the full Z_4 invariant Lagrangian and discuss the case with a general mixing between the singlet scalar S and the neutral components of H_2 . We also discuss the process of determining the dark matter relic density in the limit of decoupling the singlet scalar S while the excitation of a new singlet scalar φ is kept.

Nontrivial transformations of the fields under the Z_4 symmetry are given by $S \rightarrow iS$, $\varphi \rightarrow -\varphi$, $H_2 \rightarrow iH_2$ and $N_R \rightarrow iN_R$, whereas the SM fields are neutral under Z_4 .

We first note that the Z_4 invariant scalar potential V is composed of $V = V_1 + V_2$, with

$$V_1 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 (H_1^\dagger H_2) (H_2^\dagger H_1) \quad (16)$$

and

$$V_2 = -\sqrt{2}\kappa S^\dagger H_1^\dagger H_2 - \sqrt{2}\lambda'_{S\varphi} \varphi S H_1^\dagger H_2 + \mu \varphi S^2 + \text{h.c.} + m_S^2 |S|^2 + \lambda_S |S|^4 + \frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{4} \lambda_\varphi \varphi^4 + \frac{1}{2} \lambda_{S\varphi} |S|^2 \varphi^2 + \sum_{i=1,2} \left[\lambda_{H_i S} (H_i^\dagger H_i) |S|^2 + \frac{1}{2} \lambda_{H_i \varphi} (H_i^\dagger H_i) \varphi^2 \right]. \quad (17)$$

The relevant interaction Lagrangian for neutrino masses takes a more general form as compared to eq. (1), as follows,

$$\mathcal{L}_{\text{int}} = -\sqrt{2}\kappa S^\dagger H_1^\dagger H_2 - \sqrt{2}\lambda'_{S\varphi} \varphi S H_1^\dagger H_2 + \mu \varphi S^2 - y_{N,ij} \bar{l}_i \tilde{H}_2 N_{R,j} - \frac{1}{2} \lambda_{N,ij} \varphi \overline{N_{R,i}^c} N_{R,j} + \text{h.c.} \quad (18)$$

Then, after the Z_4 symmetry is spontaneously broken to Z_2 due to the VEV of φ , we obtain the effective interactions from eq. (1) as

$$\mathcal{L}_{\text{eff}} = \sqrt{2}\kappa S^\dagger H_1^\dagger H_2 + \sqrt{2}\kappa' S H_1^\dagger H_2 + \frac{1}{2} \hat{m}_S^2 S^2 - y_{N,ij} \bar{l}_i \tilde{H}_2 N_{R,j} - \frac{1}{2} \lambda_{N,ij} \varphi \overline{N_{R,i}^c} N_{R,j} + \text{h.c.} \quad (19)$$

where $\kappa' = \lambda'_{S\varphi} v_\varphi$ and $\hat{m}_S^2 = 2\mu v_\varphi$. Here, the mass for the singlet S is shifted by the contribution coming from $\lambda_{S\varphi} v_\varphi^2$, but we keep the notation for m_S^2 the same. Moreover, integrating out the singlet scalar S with $\hat{m}_S^2 = 2\mu v_\varphi \ll \mu_S^2$ and $\kappa' \ll m_S$, from the combination of $\sqrt{2}\kappa S^\dagger H_1^\dagger H_2$, $\sqrt{2}\kappa' S H_1^\dagger H_2$ and $\frac{1}{2} \hat{m}_S^2 S^2$ terms, we obtain the effective λ_5 term for two Higgs doublets, given by

$$\lambda_{5,\text{eff}} = \frac{2\hat{m}_S^2 (\kappa^2 + \kappa'^2)}{m_S^4} - \frac{4\kappa\kappa'}{m_S^2}, \quad (20)$$

which generalizes the result in eq. (4). Thus, there are extra contributions due to the μ term for the interactions between S and φ , but we can maintain a small value of the effective $\lambda_{5,\text{eff}}$ coupling as far as $\hat{m}_S^2 \ll m_S^2$, i.e. $v_\varphi \ll m_S$ for $\mu \sim m_S$.

Minimization of the potential

In unitarity gauge, two Higgs fields are expanded as

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v_H \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H_0 + iA_0) \end{pmatrix}. \quad (21)$$

On the other hand, the SM singlet scalar fields, S and φ , are expanded as

$$\varphi = v_\varphi + \rho, \quad S = \frac{1}{\sqrt{2}} (s + ia). \quad (22)$$

After the Z_4 symmetry is broken to Z_2 , domain walls can be formed in the early universe, so we need an explicit breaking of the Z_4 symmetry

with small soft mass terms or the Z_4 symmetry must be broken during inflation, in order to avoid domain wall problems.

We note that when the Z_4 symmetry is extended to Z_{2n} with $n = 3, 4, \dots$, containing Z_2 as a subgroup or an approximate $U(1)$ global symmetry. In both cases, a similar mechanism as for Z_4 is working for ensuring lepton number conservation and stability of dark matter and we need to promote φ to be a complex scalar field by $\varphi = v_\varphi + \rho + i\eta$ and change φ interactions in the previous discussion appropriately. Then, in the case of an approximate $U(1)$ global symmetry, there can be phenomenologically interesting consequences due to a pseudo-Goldstone boson η .

Then, to determine the minimum of the potential, we consider the potential only for the CP-even scalars, as follows,

$$V = \frac{1}{2} m_1^2 h^2 + \frac{1}{2} m_2^2 (H_0)^2 + \frac{1}{4} \lambda_1 h^4 + \frac{1}{4} \lambda_2 (H_0)^4 + \frac{1}{4} (\lambda_3 + \lambda_4) h^2 (H_0)^2 + \kappa s H_0 h + \frac{1}{2} \lambda'_{S\varphi} \rho h s H_0 + \mu \rho s^2 + \frac{1}{2} m_S^2 s^2 + \frac{1}{4} \lambda_S s^4 + \frac{1}{2} m_\varphi^2 \rho^2 + \frac{1}{4} \lambda_\varphi \rho^4 + \frac{1}{4} (\lambda_{H_1 S} h^2 + \lambda_{H_2 S} (H_0)^2) s^2 + \frac{1}{4} (\lambda_{H_1 \varphi} h^2 + \lambda_{H_2 \varphi} (H_0)^2) \rho^2 + \frac{1}{4} \lambda_{S\varphi} s^2 \rho^2. \quad (23)$$

Here, we can determine the VEVs by imposing the first derivatives of the CP-even scalar fields to vanish, i.e. $\frac{\partial V}{\partial \chi_i} = 0$, for $\chi_i = h, \rho, H_0, s$, with

$$\frac{\partial V}{\partial h} = m_1^2 h + \lambda_1 h^3 + \frac{1}{2} (\lambda_3 + \lambda_4) h (H_0)^2 + \kappa s H_0 + \frac{1}{2} \lambda'_{S\varphi} \rho s H_0 + \frac{1}{2} \lambda_{H_1 S} h s^2 + \frac{1}{2} \lambda_{H_1 \varphi} h \rho^2, \quad (24)$$

$$\frac{\partial V}{\partial \rho} = m_\varphi^2 \rho + \lambda_\varphi \rho^3 + \frac{1}{2} \lambda'_{S\varphi} h s H_0 + \mu s^2 + \frac{1}{2} \lambda_{H_1 \varphi} h^2 \rho + \frac{1}{2} \lambda_{H_2 \varphi} (H_0)^2 \rho + \frac{1}{2} \lambda_{S\varphi} s^2 \rho, \quad (25)$$

$$\frac{\partial V}{\partial H_0} = m_2^2 H_0 + \lambda_2 (H_0)^3 + \frac{1}{2} (\lambda_3 + \lambda_4) h^2 H_0 + \kappa s h + \frac{1}{2} \lambda'_{S\varphi} \rho h s + \frac{1}{2} \lambda_{H_2 S} H_0 s^2 + \frac{1}{2} \lambda_{H_2 \varphi} H_0 \rho^2, \quad (26)$$

$$\frac{\partial V}{\partial s} = m_S^2 s + \lambda_S s^3 + \kappa H_0 h + \frac{1}{2} \lambda'_{S\varphi} \rho h H_0 + 2\mu \rho s + \frac{1}{2} \lambda_{H_1 S} h^2 s + \frac{1}{2} \lambda_{H_2 S} (H_0)^2 s + \frac{1}{2} \lambda_{S\varphi} s \rho^2. \quad (27)$$

We now take the VEVs of h and ρ fields to v_H and v_φ , respectively, whereas setting the VEVs of the other fields to zero. Then, the minimization conditions for the potential become

$$m_1^2 = -\lambda_1 v_H^2 - \frac{1}{2} \lambda_{H_1 \varphi} v_\varphi^2, \quad (28)$$

$$m_\varphi^2 = -\lambda_\varphi v_\varphi^2 - \frac{1}{2} \lambda_{H_1 \varphi} v_H^2. \quad (29)$$

Therefore, we can determine the VEVs as

$$v_H = 2 \frac{\lambda_{H_1 \varphi} m_\varphi^2 - 2\lambda_\varphi m_1^2}{4\lambda_1 \lambda_\varphi - \lambda_{H_1 \varphi}^2}, \quad (30)$$

$$v_\varphi = 2 \frac{\lambda_{H_1 \varphi} m_1^2 - 2\lambda_1 m_\varphi^2}{4\lambda_1 \lambda_\varphi - \lambda_{H_1 \varphi}^2}. \quad (31)$$

By using the results in eqs. (30) and (31), the diagonal elements of the squared mass matrix for (h, ρ) , $\mathcal{M}_{h,\rho}^2$, become $2\lambda_1 v_H^2$ and $2\lambda_\varphi v_\varphi^2$, respectively, so $\det(\mathcal{M}_{h,\rho}^2) = (4\lambda_1 \lambda_\varphi - \lambda_{H_1 \varphi}^2) v_H^2 v_\varphi^2 > 0$. Thus, together with eqs. (30) and (31), we find the conditions for a local minimum as

$$4\lambda_1 \lambda_\varphi - \lambda_{H_1 \varphi}^2 > 0, \quad (32)$$

$$\lambda_{H_1 \varphi} m_\varphi^2 - 2\lambda_\varphi m_1^2 > 0, \quad (33)$$

$$\lambda_{H_1 \varphi} m_1^2 - 2\lambda_\varphi m_\varphi^2 > 0. \quad (34)$$

Higgs mixing

The SM Higgs mixes with the neutral component of singlet scalar φ from the mass matrix, given by

$$\mathcal{M}_{h,\rho}^2 = \begin{pmatrix} m_h^2 & \lambda_{H_1\varphi} v_H v_\varphi \\ \lambda_{H_1\varphi} v_H v_\varphi & m_\rho^2 \end{pmatrix} \quad (35)$$

where $m_h^2 = 2\lambda_1 v_H^2$ and $m_\rho^2 = 2\lambda_\varphi v_\varphi^2$. Then, we can diagonalize the mass matrix by introducing the mixing matrices, as follows,

$$\begin{pmatrix} h \\ \rho \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad (36)$$

with

$$\tan 2\alpha = \frac{2v_H v_\varphi \lambda_{H_1\varphi}}{m_h^2 - m_\rho^2}. \quad (37)$$

Then, the mass eigenvalues for neutral scalars are given by

$$m_{h_{1,2}}^2 = \frac{1}{2} \left[m_h^2 + m_\rho^2 \pm (m_h^2 - m_\rho^2) \sqrt{1 + \frac{4v_H^2 v_\varphi^2 \lambda_{H_1\varphi}^2}{(m_h^2 - m_\rho^2)^2}} \right]. \quad (38)$$

Dark matter mixing

Without mixing mass terms, we first list the scalar masses for the inert Higgs doublet and the real scalars, s, a , of the singlet S as

$$m_{H^\pm}^2 = m_2^2 + \frac{1}{2}\lambda_3 v_H^2 + \frac{1}{2}\lambda_{H_2\varphi} v_\varphi^2, \quad (39)$$

$$m_{H_0}^2 = m_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4) v_H^2 + \frac{1}{2}\lambda_{H_2\varphi} v_\varphi^2, \quad (40)$$

$$m_{A_0}^2 = m_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4) v_H^2 + \frac{1}{2}\lambda_{H_2\varphi} v_\varphi^2, \quad (41)$$

$$m_s^2 = m_S^2 + \frac{1}{2}\lambda_{H_1S} v_H^2 + \hat{m}_S^2 + \frac{1}{2}\lambda_{S\varphi} v_\varphi^2, \quad (42)$$

$$m_a^2 = m_s^2 - 2\hat{m}_S^2. \quad (43)$$

In the presence of the unbroken Z_4 symmetry, the masses of H_0 and A_0 fields are the same, that is, $m_{H_0}^2 = m_{A_0}^2$. We note that a negative λ_4 is taken for the extra neutral Higgs scalars to be lighter than the charged Higgs.

In our model, however, there are mass mixings between the neutral scalars of the inert doublet and the singlet scalars containing in S , leading to split masses between neutral scalar fields. First, the mass matrix for CP-even scalars, (H_0, s) , is given by

$$\mathcal{M}_{H_0,s}^2 = \begin{pmatrix} m_{H_0}^2 & (\kappa + \kappa') v_H \\ (\kappa + \kappa') v_H & m_s^2 \end{pmatrix}. \quad (44)$$

We also obtain the mass matrix for the CP-odd scalars, (A_0, a) , as

$$\mathcal{M}_{A_0,a}^2 = \begin{pmatrix} m_{H_0}^2 & (\kappa - \kappa') v_H \\ (\kappa - \kappa') v_H & m_a^2 \end{pmatrix}. \quad (45)$$

Then, we can diagonalize the mass matrices by introducing the mixing matrices, as follows,

$$\begin{pmatrix} H_0 \\ s \end{pmatrix} = \begin{pmatrix} \cos \theta_s & -\sin \theta_s \\ \sin \theta_s & \cos \theta_s \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}, \quad (46)$$

$$\begin{pmatrix} A_0 \\ a \end{pmatrix} = \begin{pmatrix} \cos \theta_a & -\sin \theta_a \\ \sin \theta_a & \cos \theta_a \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad (47)$$

with

$$\tan 2\theta_s = \frac{2(\kappa + \kappa') v_H}{m_{H_0}^2 - m_s^2}, \quad (48)$$

$$\tan 2\theta_a = \frac{2(\kappa - \kappa') v_H}{m_{H_0}^2 - m_a^2}. \quad (49)$$

Then, the mass eigenvalues for neutral scalars are given by

$$m_{H_{1,2}}^2 = \frac{1}{2} \left[m_{H_0}^2 + m_s^2 \pm (m_{H_0}^2 - m_s^2) \sqrt{1 + \frac{4(\kappa + \kappa')^2 v_H^2}{(m_{H_0}^2 - m_s^2)^2}} \right], \quad (50)$$

$$m_{A_{1,2}}^2 = \frac{1}{2} \left[m_{H_0}^2 + m_a^2 \pm (m_{H_0}^2 - m_a^2) \sqrt{1 + \frac{4(\kappa - \kappa')^2 v_H^2}{(m_{H_0}^2 - m_a^2)^2}} \right]. \quad (51)$$

Therefore, the lightest neutral scalar among $H_{1,2}$ and $A_{1,2}$ becomes a candidate for dark matter. In this case, even if $m_{H_0}^2 = m_{A_0}^2$ due to the Z_4 symmetry, there arises a mass splitting between the CP-even and CP-odd scalars due to $m_s^2 \neq m_a^2$ and $\kappa' \neq 0$, which stem from the Z_4 breaking mass terms for the singlet scalar. So, when the mass splitting is larger than about 100 keV, we can avoid the strong direct detection bound on the inert doublet-like DM in the case of the inelastic DM-nucleon scattering mediated by Z boson.

General formulas for neutrino masses

In the case with the non-decoupled singlet scalar S , we present the general results for one-loop neutrino masses in our model. From eq. (2), the relevant interactions for neutrino masses are

$$\mathcal{L}_{\nu,\text{eff}} = -\frac{1}{\sqrt{2}} y_{N,ij} \bar{\nu}_i \left(\cos \theta_s H_1 - \sin \theta_s H_2 - i \cos \theta_a A_1 + i \sin \theta_a A_2 \right) N_{R,j} - \frac{1}{2} M_{N,i} \overline{N_{R,i}^c} N_{R,i} + \text{h.c.} \quad (52)$$

As a result, we obtain the neutrino masses from one-loop diagrams as

$$(\mathcal{M}_\nu)_{ij} = \frac{1}{16\pi^2} \sum_a \sum_k y_{N,ik} y_{N,jk} M_{N,k} F_a \left[\frac{m_a^2}{m_a^2 - M_{N,k}^2} \ln \frac{m_a^2}{M_{N,k}^2} \right] \quad (53)$$

with $F_a \equiv (\cos^2 \theta_s, \sin^2 \theta_s, -\cos^2 \theta_a, -\sin^2 \theta_a)$ and $m_a \equiv (m_{H_1}, m_{H_2}, m_{A_1}, m_{A_2})$. Thus, in the limit of decoupling the singlet scalar S with $m_S \gg \kappa', \hat{m}_S$, the mixing angles for dark scalars become $|\theta_s|, |\theta_a| \ll 1$ and the mass splittings between dark scalars become small. In this case, from eqs. (50) and (51), we obtain the mass splitting between the neutral inert doublet-like scalars as

$$\begin{aligned} m_{H_1}^2 - m_{A_1}^2 &\simeq \frac{(\kappa + \kappa')^2 v_H^2}{m_{H_0}^2 - m_s^2} - \frac{(\kappa - \kappa')^2 v_H^2}{m_{H_0}^2 - m_a^2} \\ &= \frac{2(\kappa^2 + \kappa'^2) \hat{m}_S^2 v_H^2 + 4\kappa\kappa' (m_{H_0}^2 - m_{s,0}^2) v_H^2}{(m_{H_0}^2 - m_s^2)(m_{H_0}^2 - m_a^2)}, \end{aligned} \quad (54)$$

with $m_{s,0}^2 \equiv m_s^2 - \hat{m}_S^2$. As $m_s^2, m_a^2 \simeq m_S^2 \gg m_{H_0}^2$, the above result becomes further approximated to

$$m_{H_1}^2 - m_{A_1}^2 \simeq \frac{2(\kappa^2 + \kappa'^2) \hat{m}_S^2 v_H^2}{m_S^4} - \frac{4\kappa\kappa' v_H^2}{m_S^2}, \quad (55)$$

which is equal to $\lambda_{5,\text{eff}} v_H^2$, being consistent with the effective $\lambda_{5,\text{eff}}$ term in eq. (20), obtained after the singlet scalar is integrated out. Therefore, in the decoupling limit of the singlet scalar S , the neutral inert doublet-like scalars contribute dominantly to the one-loop neutrino masses as

$$(\mathcal{M}_\nu)_{ij} \simeq \frac{\lambda_{5,\text{eff}} v_H^2}{16\pi^2} \sum_k \frac{y_{N,ik} y_{N,jk} M_{N,k}}{m_0^2 - M_{N,k}^2} \left[1 - \frac{M_{N,k}^2}{m_0^2 - M_{N,k}^2} \ln \frac{m_0^2}{M_{N,k}^2} \right], \quad (56)$$

where $m_0^2 = (m_{H_1}^2 + m_{A_1}^2)/2 \simeq m_{H_0}^2$ and $\lambda_{5,\text{eff}}$ is given by eq. (20). We remark that, without accidental cancellations in $\lambda_{5,\text{eff}}$, the decoupling limit for the singlet scalar S is desirable for small neutrino masses. On the other hand, for sizable dark mixing angles for which the singlet scalar S is not decoupled, the mass splittings between dark scalars become large, so small neutrino masses require small Yukawa couplings or large masses for the RH neutrinos.

Dark matter density

When A_0 is the lightest neutral Z_2 -odd particle, we need to determine the relic density by solving the Boltzmann equations for the number density of A_0 . But, when the other scalars in the inert doublet have comparable masses to the one for A_0 , it is important to include the co-annihilation processes for them too.

Suppose that the singlet Z_2 -odd scalar S is decoupled but the excitation of the singlet scalar φ is kept in the low energy. Ignoring the Higgs mixing, we obtain the Boltzmann equation governing the number density for A_0 , n_{A_0} , as

$$\begin{aligned} \frac{dn_{A_0}}{dt} + 3Hn_{A_0} = & -\langle\sigma v\rangle_{2,\text{eff}}(n_{A_0}^2 - (n_{A_0}^{\text{eq}})^2) \\ & -\langle\sigma v\rangle_{A_0H_0 \rightarrow f\bar{f}}(n_{A_0}n_{H_0} - n_{A_0}^{\text{eq}}n_{H_0}^{\text{eq}}) \\ & -\langle\sigma v\rangle_{A_0H^\pm \rightarrow f\bar{f}'}(n_{A_0}n_{H^\pm} - n_{A_0}^{\text{eq}}n_{H^\pm}^{\text{eq}}) \\ & -\langle\sigma v\rangle_{A_0H^\pm \rightarrow f'\bar{f}}(n_{A_0}n_{H^\pm} - n_{A_0}^{\text{eq}}n_{H^\pm}^{\text{eq}}) \end{aligned} \quad (57)$$

where

$$\begin{aligned} \langle\sigma v\rangle_{2,\text{eff}} = & \langle\sigma v\rangle_{A_0A_0 \rightarrow hh} + \langle\sigma v\rangle_{A_0A_0 \rightarrow h\rho} \\ & + \langle\sigma v\rangle_{A_0A_0 \rightarrow \rho\rho} + \langle\sigma v\rangle_{A_0A_0 \rightarrow W W} \\ & + \langle\sigma v\rangle_{A_0A_0 \rightarrow ZZ} + \langle\sigma v\rangle_{A_0A_0 \rightarrow f\bar{f}}. \end{aligned} \quad (58)$$

Here, we note that there is a factor of $1/2$ included in each thermal averaged cross section for two identical particles, which cancels a factor of 2 in each annihilation rate for two identical particles. There are similar Boltzmann equations for the heavier scalars, $H_0, H^\pm$, with the corresponding annihilation cross sections.

In the presence of particles with similar masses, participating in the co-annihilation processes, the heavier scalars, $H_0, H^\pm$, that survive annihilation, eventually decay into A_0 . Thus, we consider the Boltzmann equation for the total number density of $A_0, H_0, H^\pm$, participating in the (co-)annihilation processes, i.e. $n_{\text{DM}} = n_{A_0} + n_{H_0} + n_{H^\pm}$, in the following form,

$$\frac{dn_{\text{DM}}}{dt} + 3Hn_{\text{DM}} = -\sum_{a,b} \langle\sigma v\rangle_{ab}(n_a n_b - n_a^{\text{eq}} n_b^{\text{eq}}), \quad (59)$$

with $a, b = A_0, H_0, H^\pm$ being annihilating particles. Making an approximation for each component in the total number density by $n_a/n_{\text{DM}} \approx n_a^{\text{eq}}/n_{\text{DM}}^{\text{eq}} \equiv w_a/w$, with

$$w_a = g_a(1 + \Delta_a)^{3/2} e^{-x\Delta_a}, \quad \Delta_a \equiv \frac{m_a - m_{A_0}}{m_{A_0}}, \quad (60)$$

and $w = \sum_a w_a$, we can rewrite the above Boltzmann equation as

$$\frac{dn_{\text{DM}}}{dt} + 3Hn_{\text{DM}} = -\langle\sigma v\rangle_{\text{eff}}(n_{\text{DM}}^2 - (n_{\text{DM}}^{\text{eq}})^2) \quad (61)$$

where the effective annihilation cross section is given by

$$\langle\sigma v\rangle_{\text{eff}} = \frac{1}{w^2} \sum_{a,b} \langle\sigma v\rangle_{ab} w_a w_b. \quad (62)$$

We note that $g_a = 1$ for $a = A_0, H_0, H^\pm, H^-$.

Then, the relic abundance for dark matter is determined by

$$\Omega_{\text{DM}} h^2 = 0.2744 \left(\frac{Y_{\text{DM}}}{10^{-11}} \right) \left(\frac{m_{A_0}}{100 \text{ GeV}} \right) \quad (63)$$

where $Y_{\text{DM}} = n_{\text{DM}}/s$ is the present abundance for dark matter, given by

$$Y_{\text{DM}} = \sqrt{\frac{45g_*}{8\pi^2 g_{*s}^2}} \frac{x_f}{m_{A_0} M_P (I_s + 3I_p/x_f)}, \quad (64)$$

where we made a velocity expansion by $(\sigma v)_{ij} = a_{ij} + b_{ij}v^2$ and $(\sigma v)_{\text{eff}} = a_{\text{eff}} + b_{\text{eff}}v^2$, and

$$I_s = x_f \int_{x_f}^{\infty} x^{-2} a_{\text{eff}} dx, \quad I_p = 2x_f^2 \int_{x_f}^{\infty} x^{-3} b_{\text{eff}} dx. \quad (65)$$

Here, we note that

$$x_f = \ln y_f - \frac{1}{2} \ln \ln y_f, \quad y_f = \sqrt{\frac{45}{4\pi^5 g_*}} m_{A_0} M_P \langle\sigma v\rangle_{\text{eff}}, \quad (66)$$

and M_P is the reduced Planck scale, given by $M_P = 2.435 \times 10^{18} \text{ GeV}$, $g_* = 3.36$ and $g_{*s} = 3.91$.

If $\Delta_a \gtrsim 1/x_f$ for $a = H_0, H^\pm$, we can ignore the (co-)annihilation processes for the heavier scalars, $H_0, H^\pm$, because of the Boltzmann suppression of the number densities for the heavier scalars. In this case, $a_{\text{eff}} \simeq a_{A_0A_0}$ and $b_{\text{eff}} \simeq b_{A_0A_0}$, so $I_s = I_p = 1$.

If $\Delta_{H^\pm} \gtrsim 1/x_f$ but $\Delta_{H_0} \lesssim 1/x_f$, we need to keep the heavier scalar H_0 in the total number density for dark matter, so we get

$$a_{\text{eff}} \simeq \frac{1}{4}(a_{A_0A_0} + 2a_{A_0H_0} + a_{H_0H_0}) \quad (67)$$

and b_{eff} with $a_{ij} \rightarrow b_{ij}$. In this case, we obtain $I_s = \frac{1}{4}(a_{A_0A_0} + 2a_{A_0H_0} + a_{H_0H_0})/a_{A_0A_0}$ and $I_p = \frac{1}{4}(b_{A_0A_0} + 2b_{A_0H_0} + b_{H_0H_0})/b_{A_0A_0}$.

If $\Delta_{H_0} \lesssim \Delta_{H^\pm} \lesssim 1/x_f$, we need to include the (co-)annihilation processes for the heavier scalars in determining the total number density for dark matter, so we get

$$\begin{aligned} a_{\text{eff}} \simeq & \frac{1}{4(1+w_{H^\pm})^2} (a_{A_0A_0} + 2a_{A_0H_0} + a_{H_0H_0} \\ & + 4w_{H^\pm} a_{A_0H^\pm} + 4w_{H^\pm} a_{H_0H^\pm} + 2w_{H^\pm}^2 a_{H^\pm H^\pm}) \end{aligned} \quad (68)$$

and b_{eff} with $a_{ij} \rightarrow b_{ij}$.

Data availability

No data was used for the research described in the article.

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