



Original Article

Application of Chernoff bound to passive system reliability evaluation for probabilistic safety assessment of nuclear power plants

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ARTICLE INFO

Article history:

Received 29 June 2021

Received in revised form

8 February 2022

Accepted 8 March 2022

Available online 17 March 2022

Keywords:

Probabilistic safety assessment

Nuclear power plant

Chernoff bound

Passive system

Micro modular reactor

ABSTRACT

There is an increasing interest in passive safety systems to minimize the need for operator intervention or external power sources in nuclear power plants. Because a passive system has a weak driving force, there is greater uncertainty in the performance compared with an active system. In previous studies, several methods have been suggested to evaluate passive system reliability, and many of them estimated the failure probability using thermal–hydraulic analyses and the Monte Carlo method. However, if the functional failure of a passive system is rare, it is difficult to estimate the failure probability using conventional methods owing to their high computational time. In this paper, a procedure for the application of the Chernoff bound to the evaluation of passive system reliability is proposed. A feasibility study of the procedure was conducted on a passive decay heat removal system of a micro modular reactor in its conceptual design phase, and it was demonstrated that the passive system reliability can be evaluated without performing a large number of thermal–hydraulic analyses or Monte Carlo simulations when the system has a small failure probability. Accordingly, the advantages and constraints of applying the Chernoff bound for passive system reliability evaluation are discussed in this paper.

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1. Introduction

The passive safety concept has recently become one of the most popular features of the nuclear industry. Many advanced reactors under development, particularly small modular reactors (SMRs), incorporate passive safety systems for accident mitigation. Passive safety systems can be utilized in nuclear power plants (NPPs) to enhance safety and minimize the possibility of severe accidents [1]. This is because passive systems are designed to largely rely on physical phenomena (e.g., natural circulation) and reduce human intervention, external energy sources, and signal input [2,3].

Although passive systems are expected to have higher reliability, the uncertainty could be larger than that in active systems because of the randomness and lack of data on physical phenomena [3]. More specifically, physical phenomena can be influenced by small changes in operating conditions owing to the weak driving force [4]. Thus, the uncertainty of physical phenomena is expected to have a substantial effect on the safety of advanced reactors. For a more reasonable risk assessment of advanced reactors, the

reliability of passive safety systems needs to be carefully analyzed considering the uncertainty in physical phenomena.

In deterministic safety analysis (DSA), Regulatory Guide 1.157 [5] allowed the best-estimate calculations of emergency core cooling system performance during a loss-of-coolant accident and required uncertainty analysis. To support the regulatory guide, several best-estimate plus uncertainty (BEPU) methods were developed to relax the conservative restrictions [6]. Code scaling, applicability, and uncertainty (CSAU) [7], a BEPU approach, was developed by the U.S. Nuclear Regulatory Commission and contractors. CSAU utilizes the Monte Carlo method to determine the 95/95 tolerance limit of safety measures, which is a tolerance limit with a 95% probability level and 95% confidence level. However, the Monte Carlo method is not practical for complex systems because numerous simulations are required [8]. Thus, the nonparametric method [9] based on Wilks' formula [10] is widely used to calculate the 95/95 tolerance limit in BEPU methods [6,11].

In the probabilistic safety assessment (PSA) of conventional power reactors, the failure of mechanical components has been mainly considered. As many advanced reactors are expected to rely on passive systems, the functional reliability concept was introduced in passive systems by adopting a stress-strength model based on the structural reliability theory [12]. It has been suggested

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that passive systems can fail even if all the mechanical components are successfully operated [3]. In the functional reliability concept, the failure probability of a passive system is defined as the probability that an operating condition (e.g., mass flow rate or peak cladding temperature) exceeds the allowable range expressed in a probability density function or constant value.

Based on the functional reliability concept, systematic procedures have been proposed for the reliability evaluation of passive safety systems (REPAS) [13] and reliability methods for passive safety functions (RMPS) [14]. In REPAS and RMPS, deviation of an operating condition from its nominal value is modeled using critical plant parameters (e.g., incondensable gas mass fraction), and Monte Carlo sampling is employed to calculate the failure probability.

Because the Monte Carlo method requires a long computation time for small failure probabilities, efficient simulation techniques such as subset simulation and line sampling have been proposed to reduce the number of simulations [3,15,16]. Meanwhile, the simulation results can be approximately obtained using a surrogate model derived from fewer thermal–hydraulic analyses. In previous studies [17,18], Monte Carlo simulations were performed on nonlinear regression models to calculate the failure probability of passive systems.

Even though the Monte Carlo method is a flexible technique that can be used in complex models, it does not bind the failure probability, which ensures that the probability does not exceed a certain value [19]. If the probability is required to be less than a certain limit, rather than calculating the exact probability, probability bounds can be useful [20]. For example, in the PSA framework, probability bounds can be utilized instead of the Monte Carlo methods if it is only required to show that the functional failure probability of a passive system is less than a target value.

Because a passive safety system is highly reliable, the estimation of its failure probability requires a huge number of simulations such as thermal–hydraulic analyses with Monte Carlo samples for environmental parameters. If PSA revealed that it will be sufficient for the passive safety system to demonstrate its failure probability to be less than some level, normally higher than its actual failure probability, mathematical inequalities like Chernoff bound can be useful by avoiding the need to perform a huge number of thermal–hydraulic analyses to estimate the extremely low failure probability of it. For example, if PSA revealed that it is sufficient to demonstrate that the failure probability of a passive safety system is less than $1\text{E-}4$ while its actual failure probability is $1\text{E-}6$, the number of necessary thermal–hydraulic analyses can be significantly reduced. In such a case, the failure probability of the passive safety system will be conservatively assumed to be $1\text{E-}4$ in the PSA model. It is very important to reduce the number of thermal–hydraulic analyses when the upper limit of the failure probability of a highly-reliable passive safety system is sufficient for PSA purposes.

In the functional reliability concept with a single failure criterion, a small failure probability can be interpreted as the tail probability of the probability distribution of the operating condition. The Chernoff bound, an inequality based on the large deviation theory [21], can provide an exponentially decreasing bound on the tail probability [22]. For example, the Chernoff bound was applied to an electrical power system to evaluate the upper bound of power system unreliability [23,24].

In this study, the Chernoff bound was applied to passive system reliability evaluation to show that the functional failure probability is less than the target value when the failure probability is small. To facilitate the use of the Chernoff bound in passive system reliability evaluation, a systematic procedure was developed and is provided in Section 2. Following this procedure, the failure probability of an example system, specifically, the passive decay heat removal

(PDHR) system in a micro modular reactor (MMR), was evaluated, as described in Section 3. In the same section, the proposed method is compared with the existing methods, and the advantages and limitations of applying the Chernoff bound in passive system reliability evaluation are discussed. Finally, a summary is provided in Section 4.

2. Quantification method of passive system reliability

2.1. Failure probability estimation of a passive system

As mentioned in Section 1, uncertainty in the plant and environmental parameters of a passive system may significantly impact its performance owing to the weak driving force. For example, in the case of an NPP with an air-cooling system, the uncertainty in the ambient air temperature may be reflected as a variation in the peak cladding temperature (PCT) or core exit temperature (CET).

In this study, the plant and environmental parameters (e.g., ambient air temperature) and performance indicators (e.g., PCT and CET) are referred to as explanatory variables and response variables, respectively. For simplicity, n explanatory variables and one response variable are considered in this section. The exact relationship between the explanatory and response variables can be expressed as a function g , given in Eq. (1) as

$$Y = g(X_1, \dots, X_n) \quad (1)$$

where X_i ($i = 1, \dots, n$) is the explanatory variable and Y is the response variable.

In the functional failure probability concept with a constant failure criterion, the successful performance of the passive system can be determined by whether or not the response variable exceeds the failure limit. Thus, the failure probability of a passive system can be expressed by Eq. (2), using the cumulative distribution function of the response variable, as follows:

$$\Pr(Y \geq k) = \int_k^{\infty} f_Y(y) dy = 1 - F_Y(k) \quad (2)$$

where k is the failure limit of a response variable Y , $f_Y(y)$ is the failure probability density function, and $F_Y(k)$ is the cumulative distribution function of the response variable.

In previous studies [13,14], the failure probability in Eq. (2) was estimated as the ratio of the number of samples exceeding the failure limit to the total number of samples, as shown in Eq. (3):

$$\Pr(Y \geq k) \approx \frac{N_F}{N_T} \quad (3)$$

where N_F and N_T are the number of samples exceeding the failure limit and the total number of samples, respectively.

Eq. (3) is one of the most intuitive ways to determine the failure probability of a passive system. Even though Eq. (3) provides a convenient and easy-to-use method for computing the failure probability, it requires high computational time and effort when the passive system is highly reliable.

To reduce the computational time and effort, the response variable can be computed using an approximation $g_A(X_1, \dots, X_n)$ called a surrogate model, which can be developed using the exact analysis results obtained by $g(X_1, \dots, X_n)$. If it can be assumed that the explanatory variables are independent and the relationship between the explanatory variables and the response variable is linear, the response variable can be represented by the linear combination of the explanatory variables, as shown in Eq. (4):

$$Y = g(X_1, \dots, X_n) \approx g_A(X_1, \dots, X_n) = c_0 + \sum_{i=1}^n c_i X_i + \varepsilon = c_0 + S_n + \varepsilon \quad (4)$$

where c_0 is an intercept term, c_i ($i = 1, \dots, n$) is the regression coefficient of the explanatory variable, and ε is an error term. The intercept term and regression coefficients of the explanatory variables can be determined using multiple linear regression. The goodness of fit of the model can be estimated using the adjusted coefficient of determination in Eq. (5) [25]:

$$R_a^2 = 1 - \frac{m-1}{m-p-1} (1 - R^2) = 1 - \frac{(m-1) \sum (y_i - \hat{y}_i)^2}{(m-p-1) \sum (y_i - \bar{y})^2} \quad (5)$$

where m is the sample size, p is the total number of explanatory variables, y_i is the actual value of the response variable, $\hat{y}_i$ is the predicted value of the response variable, and $\bar{y}$ is the mean value of the response variable.

By assuming that the explanatory variables are independent and the error term is negligible, the failure probability density function of the response variable can be calculated using the convolution of the failure probability density functions of the explanatory variables, as shown in Eq. (6):

$$\begin{aligned} f_Y(y) dy &\approx (f_{c_1 X_1} * \dots * f_{c_n X_n})(y) = \int \dots \\ &\int f_{c_n X_n}(x - c_{n-1} x_{n-1} - \dots - c_1 x_1) f_{c_{n-1} X_{n-1}}(c_{n-1} x_{n-1}) \dots \\ &f_{c_1 X_1}(c_1 x_1) d(c_1 x_1) \dots d(c_{n-1} x_{n-1}) \\ &= \frac{1}{|c_1 \dots c_n|} \int \dots \int f_{X_n}(x - c_{n-1} x_{n-1} - \dots \\ &- c_1 x_1) f_{X_{n-1}}(x_{n-1}) \dots f_{X_1}(x_1) dx_1 \dots dx_{n-1} \end{aligned} \quad (6)$$

where $f_{X_i}(x_i)$ ($i = 1, \dots, n$) is the probability density function of the explanatory variable, and $*$ is the convolution operator.

The right-hand side of Eq. (6) can be derived using Eq. (7),

$$\begin{aligned} f_{c_i X_i}(c_i x_i) &= \frac{d}{d(c_i x_i)} \Pr(c_i X_i \leq c_i x_i) \\ &= \begin{cases} \frac{1}{c_i} \frac{d}{dx_i} \Pr(X_i \leq x_i), & c_i > 0 \\ \frac{1}{c_i} \frac{d}{dx_i} \Pr(X_i \geq x_i), & c_i < 0 \end{cases} \\ &= \begin{cases} \frac{1}{c_i} \frac{d}{dx_i} F_{X_i}(x_i), & c_i > 0 \\ \frac{1}{c_i} \frac{d}{dx_i} (1 - F_{X_i}(x_i)), & c_i < 0 \end{cases} \\ &= \begin{cases} \frac{1}{c_i} f_{X_i}(x_i), & c_i > 0 \\ -\frac{1}{c_i} f_{X_i}(x_i), & c_i < 0 \end{cases} = \frac{1}{|c_i|} f_{X_i}(x_i) \end{aligned} \quad (7)$$

where c_i ($i = 1, \dots, n$) is a nonzero constant.

The convolution in Eq. (6) gives the analytic solution of the failure probability of the passive system under the assumption of a linear relationship between the explanatory variables and the response variable. However, the computational time of convolution increases exponentially as the number of explanatory variables increases.

For complex models where the analytic approach is not applicable, Monte Carlo simulations can be utilized [26,27]. In this case, a Monte Carlo simulation can be performed on the surrogate model presented in Eq. (4). Then, the failure probability can be determined by counting the number of failed samples, whose results exceeded the failure limit, as expressed in Eq. (3).

2.2. Chernoff bounds

According to the large deviation theory, the tail probability of the sequence of random variables exhibits an asymptotic exponentially decaying behavior [21]. One of the probability bounds based on the large deviation theory, the Chernoff bound [28], can provide probability bounds on the sum of random variables [22]. Chernoff bound provides a probability upper bound on the tail of the sum of independent random variables. When a probability bound, instead of the statistical estimation on a very low probability, is sufficient for a basic event in PSA, the number of thermal-hydraulic analysis can be significantly reduced while obtaining slightly conservative quantification results.

The Chernoff bound can be derived using Markov's inequality and moment-generating functions. For a monotonically increasing function $\phi(x)$, the Markov's inequality is given by Eq. (8) [22]:

$$\Pr(Y \geq k) = \Pr(\phi(Y) \geq \phi(k)) \leq \frac{E[\phi(Y)]}{\phi(k)} \quad (8)$$

If $\phi(y)$ is substituted with an exponential function, the Markov's inequality is given using the moment generating functions, as presented in Eq. (9) [22], for any $t > 0$:

$$\Pr(Y \geq k) = \Pr(S_n \geq k - c_0) = \Pr(e^{tS_n} \geq e^{t(k-c_0)}) \leq \frac{E[e^{tS_n}]}{e^{t(k-c_0)}} = \frac{M_{S_n}(t)}{e^{t(k-c_0)}} \quad (9)$$

where $M_{S_n}(t)$ is the moment generating function of S_n . This type of probability bound derived using this approach is widely known as the Chernoff bound [22]. In particular, if the explanatory variables are assumed to be independent, $M_{S_n}(t)$ is given as a multiplication of the moment generating functions of the explanatory variables, as shown in Eq. (10).

$$\Pr(Y \geq k) \leq \min_{t > 0} \frac{M_{S_n}(t)}{e^{t(k-c_0)}} = \min_{t > 0} \frac{\prod_{i=1}^n M_{X_i}(c_i t)}{e^{t(k-c_0)}} \quad (10)$$

Using the Chernoff bound, the probability bounds of the failure probability of a passive system can be calculated.

2.3. Procedure for reliability analysis of a passive system

To facilitate the use of the Chernoff bound in passive system reliability evaluation, a systematic procedure was proposed, as shown in Fig. 1. The proposed procedure intends to increase the usability of the Chernoff bound. Furthermore, using the Chernoff bound, it is expected that the failure probability of a passive system can be estimated with proper computational time and effort, even if the passive system is highly reliable and therefore the failure of it is rare.

The first step of the procedure is to define the required function and system mission of the target system. Active systems usually operate with mechanical components using external power sources, which can create a constant driving force; thus, the success or failure of the active system can be determined by the operation of mechanical components for a given mission time. However, passive system reliability cannot be defined as for an active system because

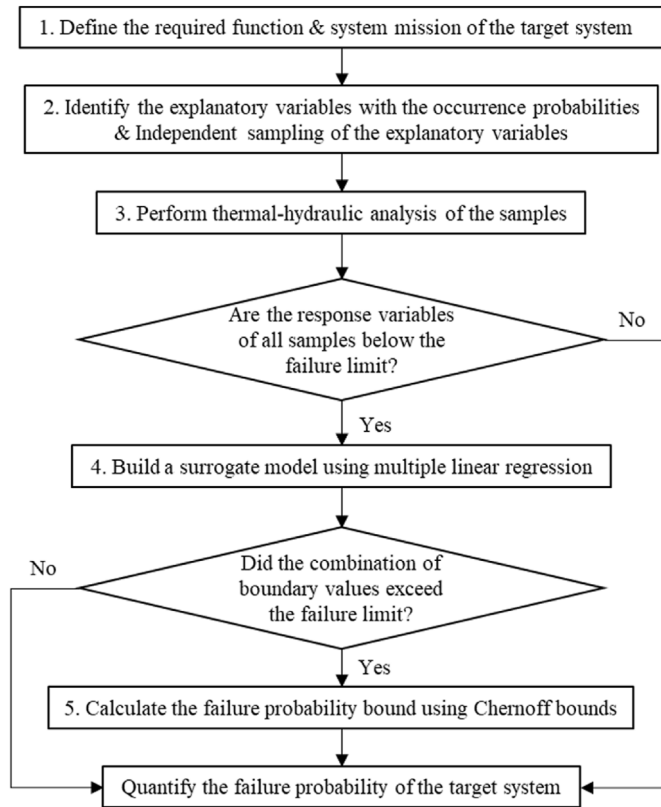


Fig. 1. Procedure for passive system reliability evaluation using the Chernoff bound.

the passive system usually does not have mechanical components. Instead, passive system reliability can be defined as the probability of performing a specified function for a given mission time [2]. Thus, to calculate the failure probability precisely, the required function and mission of the target system should be clearly defined. For example, in a previous study [13], the required function was defined as removing the decay heat, and the system mission was determined to be the removal of the decay heat within 20% of the nominal cooling rate.

The second step of the procedure is to identify the explanatory variables with their probability density functions and uncertainty ranges to propagate the uncertainty of the operating conditions. In DSA, the uncertainty ranges and probability distributions are usually assigned based on experimental results or expert judgments [18,29]. The explanatory variables can be selected via sensitivity analysis, as described in a previous study [4]. In this study, probability distributions with moment generating functions are assigned to the explanatory variables for the use of the Chernoff bound. Then, the explanatory variables should be sampled based on their probability distributions.

In the third step, simulations (e.g., thermal–hydraulic analysis) of the samples are performed. If one or more samples exceed the failure criteria, the failure probability can be estimated using Eq. (3). However, if none of the samples exceed the failure criteria, it does not ensure that the failure probability is zero, because the sample size is not sufficient to identify a failed sample. In the case of such rare events for which all the thermal–hydraulic analysis results indicate success, the failure probability can be estimated by developing an approximate relation as described below.

The fourth step of the procedure is to build a surrogate model. In this study, to utilize the Chernoff bound, multiple linear regression was performed to represent the response variable using a linear combination of explanatory variables. By representing the response

variable with the surrogate model, the boundary values that the response variable can have were obtained. For example, if the response variable Y is represented by Eq. (11), as follows:

$$Y = c_1 X_1 + c_2 X_2 \quad (11)$$

where $X_i \sim U(l_i, u_i)$, c_1 and c_2 are positive coefficients, and l_i and u_i are the lower and upper boundary values, respectively, it would have a value in the range $[c_1 l_1 + c_2 l_2, c_1 u_1 + c_2 u_2]$.

To facilitate the use of the Chernoff bound, if the upper boundary value $c_1 u_1 + c_2 u_2$ of the response variable has a lower value than the failure limit, the failure probability can be neglected. This is because if the upper boundary value is below the failure limit, the probability calculated by applying the Chernoff bound or Monte Carlo method is expected to be extremely small and close to zero.

The fifth step of the procedure involves calculating the failure probability bound by applying the Chernoff bound to the surrogate model. If the upper boundary value of the response variable is above the failure limit, the failure probability of the passive system cannot be ignored. The Chernoff bound is calculated using Eq. (10) to provide the bounds for the failure probability.

In summary, the failure probability of the target system can be quantified using three methods: thermal–hydraulic analysis, considering upper boundary values of the linear regression equation, and application of the Chernoff bound. When any of the samples exceeds the failure limit in the thermal–hydraulic analysis, the failure probability can be estimated by counting the number of failed samples exceeding the failure limit, as represented in Eq. (3). If none of the samples exceeds the failure limit, surrogate models can be utilized to estimate the small failure probability that cannot be computed using Eq. (3). To facilitate the use of the Chernoff bound, if the upper boundary value of the response variable has a lower value than the failure criterion, the failure probability can be assumed to be zero. If not, the Chernoff bound can be utilized to produce approximate probability bounds for the response variables. The proposed procedure is expected to help provide a failure probability estimate or the bounds for the failure probability of a passive system without significantly increasing the number of thermal–hydraulic analyses on a highly reliable passive system.

3. Application to an example system

The proposed procedure was applied to the PDHR system of an MMR, as shown in Fig. 2 [30]. The MMR aims for autonomous

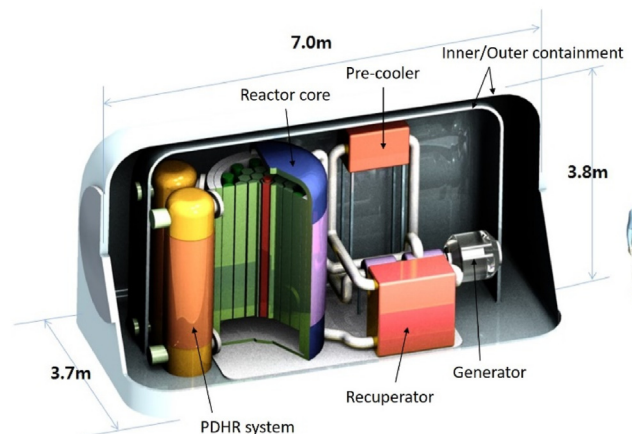


Fig. 2. Layout of MMR (concept design) [30].

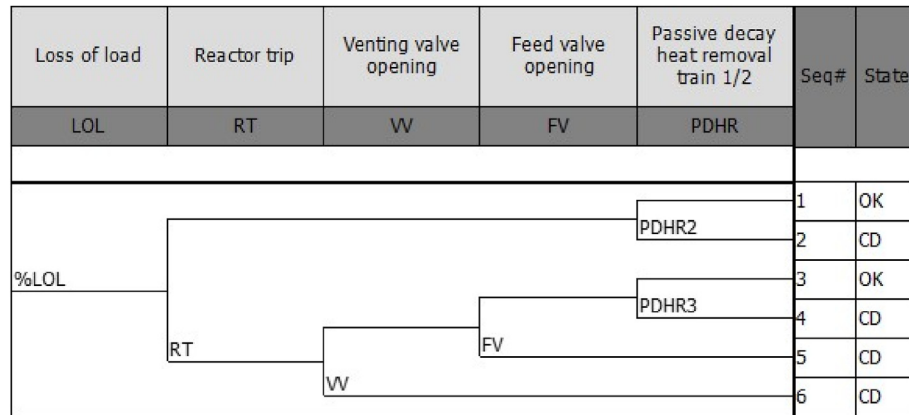


Fig. 3. Event tree for loss-of-load event [33].

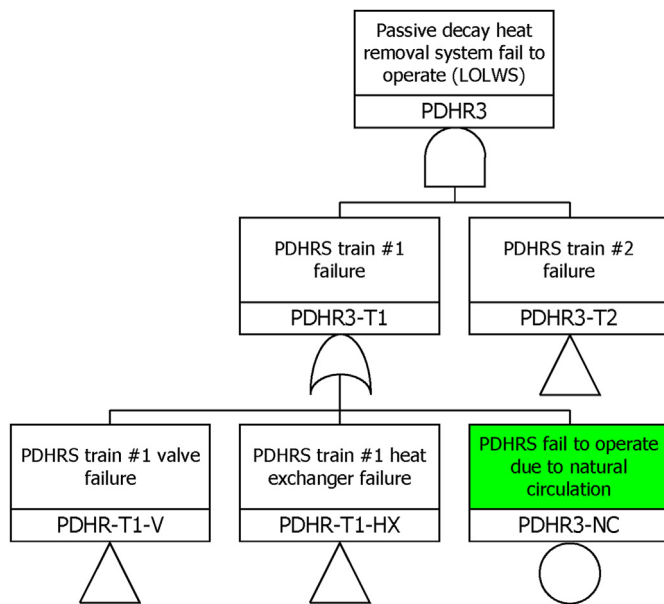


Fig. 4. Fault tree for PDHR system [33].

operation while minimizing operator intervention and external power sources. The conceptual design of the MMR adopts the PDHR system, which operates by natural circulation. Each of the two redundant trains of the PDHR system is capable of removing 100% of the decay heat after full-power operation. In this work, a case study was conducted on the loss-of-load event, in which the external power grids are abruptly disconnected from the generator [31]. The event tree is shown in Fig. 3 [33].

In a report by the International Nuclear Safety Advisory Group (INSAG-12) [34], it was suggested that the core damage frequency (CDF) should not exceed 1.0E-5/reactor-year for future NPPs. In this

study, one tenth of it, i.e., 1.0E-6/reactor-year, was set as a target value for the CDF from loss-of-load event to satisfy the safety goal. Based on the conceptual design, fault trees were developed, including mechanical and functional failures, as shown in Fig. 4. The minimal cutsets relevant to mechanical and functional failures are presented in Tables 1 and 2, respectively.

The CDF from the loss-of-load event can be calculated as the sum of the CDFs relevant to mechanical failure and functional failure. As the CDF owing to the mechanical failure was calculated as 1.38E-7 using the generic data for pressurized water reactors [35,36], the CDF relevant to the functional failure shown in Table 2 should be less than 8.62E-7. To demonstrate this, the functional failure probabilities, $PDHR2 - NC$ and $PDHR3 - NC$, were calculated using the procedure presented in Fig. 1.

The first step of the passive system reliability evaluation is to define the required function and system mission of the PDHR system. The required function of the PDHR system is to cool the reactor, and the system mission is defined as maintaining PCT and CET below the safety limits of 1200 and 676 °C, respectively [31].

The explanatory variables and their probability density functions are presented in Table 3. To identify the explanatory variables, plant and environmental parameters were collected from previous studies [18,29,32], and then, the explanatory variables were selected depending on the applicability to the input model [33]. The probability density functions of the plant parameters, from X_1 to X_6 , were assumed such that 95% of the probability values existed in the range of $\pm 20\%$ from their nominal values. The probability density functions of the environmental parameters related to the ambient air, from X_7 to X_9 , were assumed to follow uniform distributions. For use in thermal–hydraulic analysis, 100 samples were generated via MModule for SAMpling Input and QUantifying Estimator (MOSAQUE) [37] code using the Latin hypercube sampling (LHS) [38] method.

In the third step, a thermal–hydraulic analysis was performed to calculate the response variables for the given explanatory variables. Thermal–hydraulic analysis was performed using the

Table 1
Top five minimal cutsets for mechanical failure in loss-of-load event.

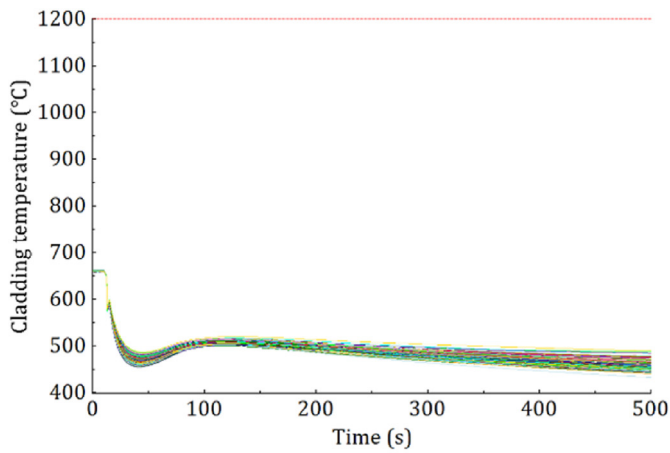
Value	Fussell–Vesely importance	Accumulated Fussell–Vesely importance	Basic event #1	Sequence	Description
5.742E-8	0.415543	0.415543	PDHR-V1-CCF	#LOL-2	PDHR valve #1 CCF
5.742E-8	0.415543	0.831086	PDHR-V2-CCF	#LOL-2	PDHR valve #2 CCF
4.092E-9	0.029611	0.860698	BAT-FTO	#LOL-2	PDHR valve failure due to battery
2.765E-9	0.020006	0.880706	PDHR-HX1-CCF	#LOL-2	PDHR heat exchanger #1 CCF
2.765E-9	0.020006	0.900709	PDHR-HX2-CCF	#LOL-2	PDHR heat exchanger #2 CCF

Table 2
Minimal cutsets relevant to functional failure in loss-of-load event.

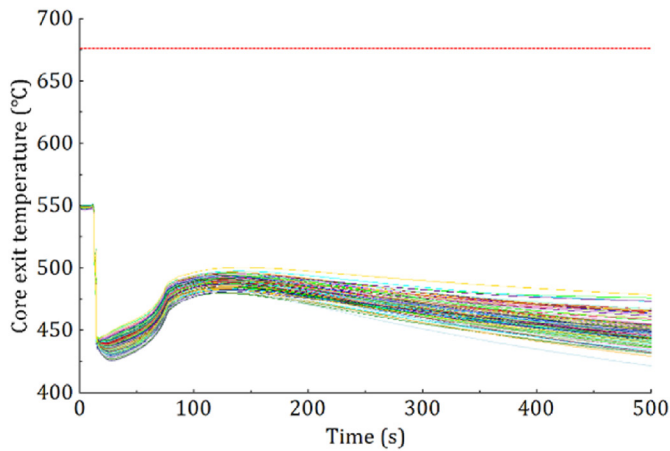
Basic event #1	Basic event #2	Basic event #3	Sequence	Description
PDHR2-NC			#LOL-2	Functional failure of PDHR after reactor trip
PDHR3-NC	RT-DM-FTOP	RT-RD-FTOP	#LOL-4	Functional failure of PDHR after reactor trip failure

Table 3
Explanatory variables for PDHR system [33].

	Uncertainty parameter	Current value	Distribution	Nominal value	Range	Standard deviation
X_1	Fuel thermal conductivity	dependent on T	Truncated normal	1.0	0.8–1.2	0.102043
X_2	Fuel heat capacity	dependent on T	Truncated normal	1.0	0.8–1.2	0.102043
X_3	Cladding thermal conductivity	dependent on T	Truncated normal	1.0	0.8–1.2	0.102043
X_4	Cladding heat capacity	dependent on T	Truncated normal	1.0	0.8–1.2	0.102043
X_5	Fuel temperature coefficient	−0.457 pcm/K, BOL	Truncated normal	1.0	0.8–1.2	0.102043
X_6	Coolant density coefficient	2.063 pcm/(kg/m ³)	Truncated normal	1.0	0.8–1.2	0.102043
X_7	PDHR inlet air temperature	25 °C	Uniform	25.0	5.0–45.0	
X_8	PDHR inlet air pressure	102.4 kPa	Uniform	102.4	102.35–102.45	
X_9	PDHR outlet air pressure	102.3 kPa	Uniform	102.3	102.25–102.35	

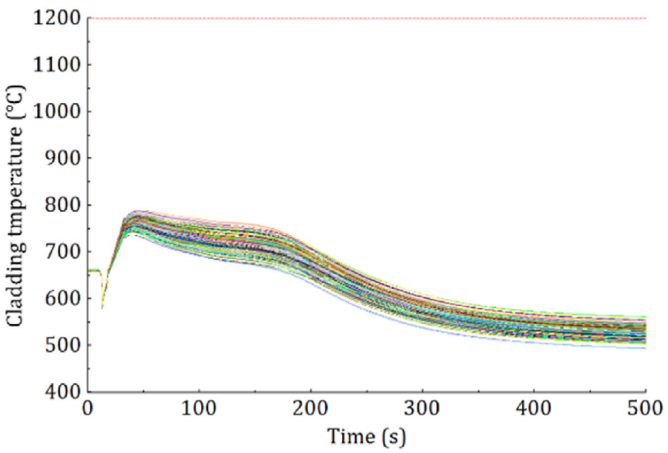


(a) PCT

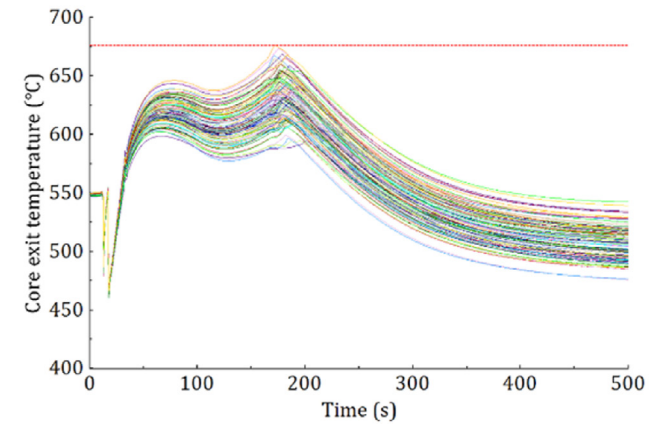


(b) CET

Fig. 5. PCT and CET results for 100 samples of LOL event.



(a) PCT



(b) CET

Fig. 6. PCT and CET results for 100 samples of LOLWS event.

GAMMA + code [39,40], a thermal–hydraulic analysis code developed for very-high-temperature reactors, for 100 samples generated in the second step. For loss of load after the reactor trip, given as sequence 2 in Fig. 3, the PCT and CET exhibited lower values than the operating conditions, as shown in Fig. 5. Thus, the functional failure probability $PDHR2 - NC$ can be assumed to be insignificant. For the loss of load without scram (LOLWS), given as sequence 4 in Fig. 3, one of the 100 samples exceeded the safety limit of the CET, as shown in Fig. 6. Different lines in Figs. 5 and 6 represent different results of the thermal–hydraulic analysis with 100 samples. According to the process displayed in Fig. 1, the failure probability can be quantified as 0.01; however, subsequent procedures were performed for the LOLWS to demonstrate the entire process.

In the fourth step, multiple linear regression was performed for the response variables (PCT and CET). Because there are large differences in the explanatory variables, as shown in Table 3, the explanatory variables were standardized as $\frac{x-\mu}{\sigma}$, where μ and σ are the mean and standard deviation of each explanatory variable, respectively. Thus, the impact of the magnitude of the explanatory variables on the regression coefficients can be reduced. Then, the entire sample was randomly split into two groups: 80 samples for the training set and 20 samples for the test set. The training set includes those data used to calculate the coefficients for multiple linear regression while the test set includes those data used to test the validity of the multiple linear regression. Multiple linear regression was performed for the training set using Python code, and the model suitability was evaluated using the test set. As a result of the multiple linear regression, the response variable can be represented as a linear combination of the explanatory variables, as shown in Eq. (4).

The linear regression analysis results are presented in Fig. 7, where the blue dots indicate the 20 samples of the test set. The regression coefficients show that the explanatory variable X_6 , the coolant density coefficient, has the greatest influence on the CET in the LOLWS event. The model suitability was evaluated as 0.974 using the adjusted coefficient of determination in Eq. (5).

The upper bounds of PCT and CET were calculated as 793.6 and 687.7 °C, respectively. As the safety limit for the PCT is 1200 °C, the failure probability owing to the PCT was regarded as insignificant compared with that owing to the CET. The upper boundary value of CET has a higher value than its safety limit of 676 °C; thus, the Chernoff bound would be applied for CET in the next step.

The fifth step involves computing the failure probability bound of the PDHR system using the Chernoff bound. For the explanatory

variables given in Table 3, the right-hand side of Eq. (10) can be written as Eq. (12):

$$\frac{\prod_{i=1}^n M_{X_i}(c_i t)}{e^{t(k-c_0)}} = e^{-t(k-c_0)} \left[\prod_{i=1}^6 e^{\mu_i c_i t + \frac{(\sigma_i c_i t)^2}{2}} \left(\frac{\text{erf}\left(\frac{\beta_i - \sigma_i c_i t}{\sqrt{2}}\right) - \text{erf}\left(\frac{\alpha_i - \sigma_i c_i t}{\sqrt{2}}\right)}{\text{erf}\left(\frac{\beta_i}{\sqrt{2}}\right) - \text{erf}\left(\frac{\alpha_i}{\sqrt{2}}\right)} \right) \right] \left[\prod_{i=7}^9 \frac{e^{c_i t b_i} - e^{c_i t a_i}}{c_i t (b_i - a_i)} \right] \quad (12)$$

where μ_i is the mean; σ_i is the standard deviation; α_i and β_i are the standardized lower and upper bounds, respectively, for the random variables following a truncated normal distribution; and a_i and b_i are the lower and upper bounds, respectively, for the uniformly distributed random variables. By differentiating Eq. (12), the t value, at which the Chernoff bound is minimum, can be calculated. Alternatively, it might be practical to calculate Eq. (12) for numerous t values and find the minimum.

By calculating Eq. (12), the failure probability bound for $PDHR3 - NC$ was identified, as shown in Fig. 8, as approximately $5.95E-3$ for the CET when the safety limit was 676 °C in the LOLWS. The component reliability data $RT - DM - FTOP$ and $RT - RD - FTOP$ were both assumed to be $1.54E-7$. Therefore, the CDF relevant to the functional failure probability is identified as $1.55E-18$, which is significantly less than the required value.

Fig. 9 shows a comparison of the failure probability depending on the safety limit calculated from three different methods: thermal–hydraulic analysis, Monte Carlo simulation, and the Chernoff bound. The LHS method was used to generate the samples of the explanatory variables, and the red triangles in the figure indicate the failure probability calculated using the proportion of the failed samples in the thermal–hydraulic analysis results, as shown in Eq. (3). One sample exhibited the highest value among the 100 thermal–hydraulic analysis results, between 675 and 680 °C; thus, the failure probability was calculated as 0.01 at the safety limit of 675 °C.

Monte Carlo simulation was performed with the sample size of $1.0E+8$ using the multiple linear regression model developed in the fourth step, and the results are indicated by the black squares in Fig. 9. For convenience, a multiple linear regression model was developed for the same sample using thermal–hydraulic analysis

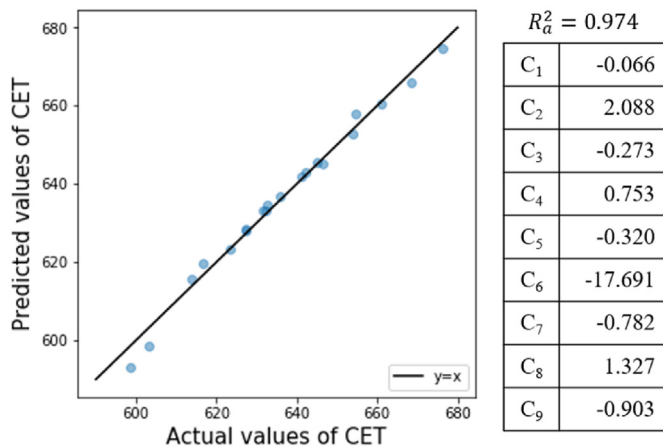


Fig. 7. Linear regression analysis result for CET in LOLWS.

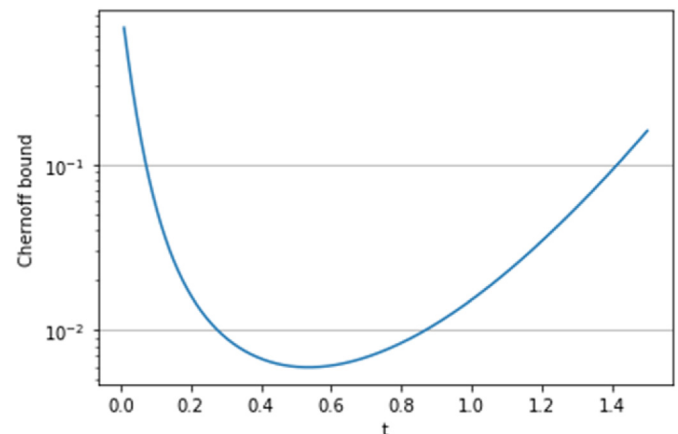


Fig. 8. Chernoff bound for safety limit of 676 °C in LOLWS.

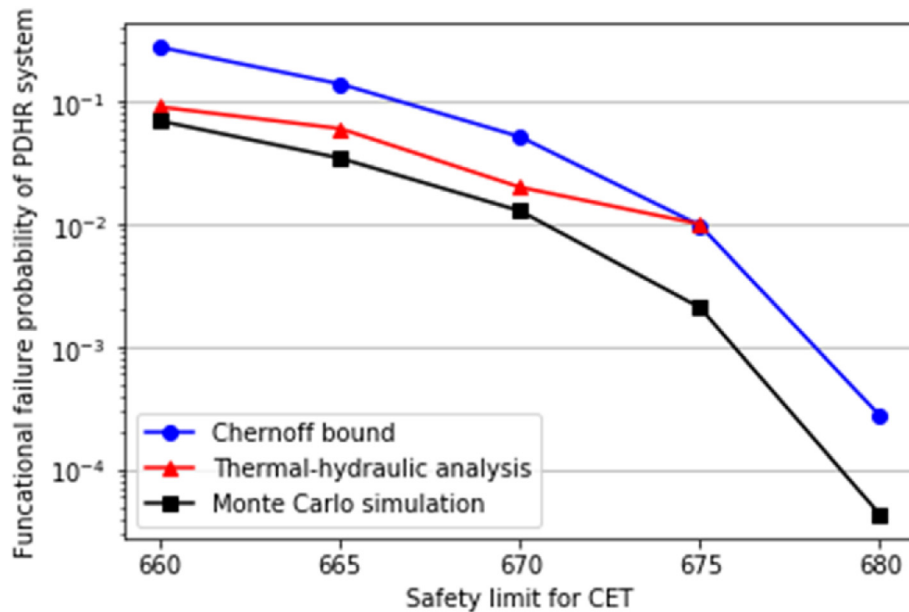


Fig. 9. Comparison of failure probability in LOLWS.

results, which was generated from the probability distributions of the explanatory variables. Hence, the generated sample data points of the plant parameters, assigned a normal distribution, are concentrated at the center. Because of the scarcity of samples near the failure limit of 675 °C, the linear regression model may not represent the thermal–hydraulic analysis results. To improve the representativeness of the regression model over the entire range of the explanatory variables, other sampling techniques in response surface methodology [41] can be utilized instead of the LHS method.

Meanwhile, the failure probabilities calculated using the Chernoff bound are indicated by the blue circles in Fig. 9. Higher values can be seen compared with the thermal–hydraulic results; however, the Chernoff bound may not always provide the upper bound for the failure probability. The applicability of the Chernoff bound varies depending on the representativeness of the surrogate model. It can be useful when: (1) a linear relationship is confirmed between the response and explanatory variables, and (2) the regression model represents the response variables near the failure limit. The accuracy of Eq. (10) would increase if (1) there is a strong linear relationship and (2) representativeness of the model can be increased using other efficient sampling techniques, as mentioned in the previous paragraph.

There are several constraints on the application of the Chernoff bound. As mentioned in Section 2, the probability distributions assigned to explanatory variables should have moment generating functions for $t > 0$. For this reason, the Chernoff bound will not be applicable when a lognormal distribution is assigned to an explanatory variable. Here, the distribution refers to those for exploratory variables, the plant and environmental parameters (e.g., ambient air temperature), and performance indicators (e.g., PCT and CET), not the probability distribution for the failure of systems or components. Those exploratory variables are widely modeled with normal distribution. In addition, the usefulness of the upper bound calculated from the Chernoff bound is dependent on the safety limit. As shown in Fig. 9, the functional failure probability was calculated to be approximately 0.275 for the safety limit of 660 °C, whereas the Monte Carlo simulation results produced approximately 6.91E-2. More specifically, the Chernoff bound may

provide overly conservative upper bounds that will not be relevant in PSA, depending on the safety limit.

Despite these constraints, the Chernoff bound may provide a meaningful upper bound for the failure probability of a passive safety system in a PSA. For the Monte Carlo simulation method at the safety limit of 685 °C, none of the samples exceeded the safety limit, indicating that a sample size larger than 1.0E+8 should be provided to calculate the failure probability. However, the Chernoff bound can provide the upper bound for the failure probability at the safety limit of 685 °C as about 1.005E-8, without numerous simulations. In addition, the Chernoff bound can be useful if it is required to ensure that the probability is less than a given value [20]. In this respect, the evaluation of probability bounds using the Chernoff bound can be useful for passive systems with a small failure probability by showing that the failure probability is less than the target value, which is derived from the quantification results of PSA.

4. Conclusions

The importance of passive system reliability evaluation is increasingly being emphasized as many advanced reactors incorporate the passive safety concept. To quantify the functional failure probability of passive systems efficiently, many previous methodologies have focused on the Monte Carlo simulation on the response surface. However, the failure probability calculated using the Monte Carlo method requires considerable time and effort. Instead, the Chernoff bound can be useful for estimating the upper bound of the failure probability of a safety system. The use of Chernoff bound is not for replacing the existing PSA base on event trees and fault trees, but to provide a conservative failure probability of a basic event in a fault tree when the basic event is associated with the failure of a highly-reliable passive safety system.

In this paper, a systematic procedure is provided with an example to apply the Chernoff bound in passive system reliability evaluation, and the advantages and disadvantages are discussed. The proposed procedure could provide reasonable results when the linear regression model properly represented the thermal–hydraulic analysis results across a range of explanatory variables. Nevertheless, there

are limitations in the use of the Chernoff bound: it is applicable for probability distributions with moment generating functions, and the usefulness of the upper bound for the failure probability in a PSA depends on the safety limit. If the requirements are satisfied, the Chernoff bound can be utilized to estimate the failure probability of passive systems following the procedure. A linear regression model was developed for passive systems with a small failure probability, and then, the Chernoff bound was calculated for the model without numerous Monte Carlo simulations. As a result, the failure probability can be bounded by a fixed boundary value that is not affected by the randomness of the Monte Carlo sampling, which is useful for highly reliable passive systems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This research was supported by grants from the Nuclear Safety Research Program of the Korea Foundation of Nuclear Safety, with financial resources from the Multi-Unit Risk Research Group (MURRG) and funding from the Korean government's Nuclear Safety and Security Commission (Grant Code: 1705001), and the Nuclear Research & Development Program of the National Research Foundation of Korea, with funding from the Korean government's Ministry of Science and ICT (Grant number NRF-2017M2B2B1071973).

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