

Quantum electrodynamics of photonic time crystals

Received: 17 April 2025

Accepted: 2 December 2025

Published online: 19 December 2025

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Photonic time crystals host a variety of intriguing phenomena, from wave amplification and mixing to exotic band structures, all stemming from the time-periodic modulation of optical properties. While these features have been well described classically, their quantum manifestation when coupled to an atomic electric dipole has remained elusive. Here, we introduce a quantum electrodynamical model of photonic time crystals that reveals a deeper connection between classical and quantum pictures: the classical momentum gap arises from a localization-delocalization quantum phase transition in a Floquet-photonic synthetic lattice. Leveraging an effective Hamiltonian perspective, we pinpoint the critical momenta and highlight how classical exponential field growth manifests itself as wave-packet acceleration in the quantum synthetic space. Remarkably, when a two-level atom is embedded in such a photonic time crystal, its Rabi oscillations undergo irreversible decay to a half-and-half mixed state—a previously unobserved phenomenon driven by photonic delocalization within the momentum gap, even with just a single frequency mode. Our findings establish photonic time crystals as versatile platforms for studying nonequilibrium quantum photonics and suggest new avenues for controlling light-matter interactions through time-domain engineering.

Recent advances in photonics have moved beyond spatially periodic media such as conventional photonic crystals and metamaterials^{1–8}, turning instead to structures whose optical properties are periodically modulated in time^{9–12}. Among these dynamic media are photonic time crystals (PTCs), in which parameters such as refractive index or permittivity follow a strict periodic pattern in time^{13–20}. This shift to time-domain periodicity has unveiled unusual photonic band structures and unique ways of manipulating light under time-varying conditions^{21–25}. PTCs thus offer prospects for advanced control of wave propagation, energy transfer, and frequency conversion, expanding beyond the traditional realm of static spatial lattices.

In classical descriptions, PTCs exhibit non-Hermitian behavior arising from time-periodic modulations, giving rise to gain and loss phenomena whose physical origin is distinct from that of static photonic systems^{11,18}. This non-Hermiticity manifests in distinctive band

structures marked by momentum gaps and exceptional points, and the breakdown of Floquet-mode orthogonality—quantified by the Petermann factor—heavily influences the photonic density of states near the gap edge^{13,14,18,26,27}. These changes in the photonic density of states are directly relevant for wave amplification, lasing, and frequency conversion, while also influencing atomic light-emitter interactions by altering emission and absorption pathways^{10,14,17,18,28}, thus bridging classical PTC phenomena with inherently quantum regimes.

Despite extensive classical analyses via Maxwell's equations and Floquet theory, a critical gap remains in understanding the quantum properties of PTCs. Classical models can effectively capture non-Hermitian effects (e.g., momentum gaps, exceptional points) but fall short in describing quantum dynamical phenomena unique to time-varying environments. In other words, non-Hermiticity at the classical level can obscure how photons truly behave, particularly when light-

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matter interactions such as spontaneous emission and Rabi oscillations enter the picture. For instance, the formation of momentum gaps is often attributed to pseudo-Hermiticity breaking, yet quantization of Maxwell's equations yields a Hermitian Hamiltonian, even in PTCs¹². This raises a central question: How can the non-Hermitian features observed classically be reconciled with the Hermiticity of quantum electrodynamical descriptions? Clarifying this paradox is vital for exploring purely quantum effects such as photon pair creation and annihilation, squeezing, and Floquet vacuum fluctuations, all of which exceed the scope of classical models.

The quantum mechanical aspects of photonic states within the momentum gap have been put forward recently in ref. 29, which employs the classical and quantum theories of photonic temporal interfaces^{30,31}. By viewing the time-varying medium as a scattering source of photonic states, the transition probability can be computed as a function of the squeezing parameter, which is in turn connected to the reflectivity. The exponential growth of the photon number in time from the pair generation in the PTC has the same root as the dynamical Casimir effect (DCE)^{32–35}. The amplification of vacuum fluctuations by a parametrically driven optical cavity has drawn much attention as a means to engineer the photonic vacuum and it has far-reaching applications to the physics of the expanding universe and the Hawking radiation^{36–42}. In spite of the similarity, the PTC medium is homogeneous in space and therefore right- and left-moving photonic modes ($|\pm k\rangle$) are distinct modes in contrast to the DCE. This means the photonic states in the PTC are specified by momentum. As a result, when an atomic electric dipole is coupled to the PTC, the time-varying light-atom interaction allows the exchange of photons with a total momentum quanta followed by the transition of atomic states, and unconventional quantum dynamics emerges in the momentum domain. Such quantum electrodynamics in the PTC has not been explored, and in our work we present the Floquet formalism, which provides a platform to treat the PTC time-periodically coupled to an atomic electric dipole in a unified manner in Floquet-photonic synthetic space.

Our approach reveals how classical non-Hermitian features, such as momentum gaps and time-periodicity induced gain, map onto a localization-to-delocalization quantum phase transition in a Floquet-photonic synthetic lattice. Notably, we show that Rabi oscillations in a two-level system coupled to a PTC settle into an incoherent mixed state within the momentum gap, even in single-frequency photonic modes, a phenomenon triggered by photonic delocalization under time-periodic driving. This dynamics cannot be fully captured by classical theories alone, placing PTCs at the forefront of non-equilibrium quantum photonics. Our results highlight PTCs as a powerful platform for exploring new light-matter interaction regimes and advancing quantum photonic technologies.

Results

Effective description of the quantum PTC

The PTC medium is homogeneous in space and supports a continuum set of momenta k . We focus on the quantum dynamics of photonic modes with momentum k and $-k$ because other modes $|k' \neq \pm k\rangle$ are uncoupled. The medium is time-periodically varying, as reflected in the permittivity $\varepsilon(t) = \varepsilon(t + T)$. In natural units adapted for electromagnetism ($\hbar = c = \varepsilon_0 = 1$), the Hamiltonian of the system is given by¹²

$$\hat{H}(t) = k(1 - \alpha(t))(\hat{n}_k + \hat{n}_{-k}) + k\alpha(t)(\hat{a}_k \hat{a}_{-k} + h.c.), \quad (1)$$

where $\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k$ is the photon number operator for mode k , and $\alpha(t) = \frac{1}{2}[1 - \varepsilon^{-1}(t)]$. In the photon number basis, defined by $\hat{n}_k|n_k\rangle = n_k|n_k\rangle$, the first term of the Hamiltonian corresponds to the energy contribution from the photon number. The second term describes the creation and annihilation of photon pairs, governed by the operator $\hat{a}_k \hat{a}_{-k}$, which acts as $\hat{a}_k \hat{a}_{-k}|n_k, n_{-k}\rangle = \sqrt{n_k n_{-k}}|n_k - 1, n_{-k} - 1\rangle$. This term generates off-

diagonal elements in the photon number Hilbert space, $\{|n_k, n_{-k}\rangle\}$, representing a *hopping* process between adjacent photon number states. As a result, the Hamiltonian can be interpreted as a single-particle problem on a 1-dimensional lattice, with both the onsite potential and hopping amplitudes varying periodically in time. Note that the total momentum $P = k(n_k - n_{-k})$ remains conserved under the hopping process. The Hamiltonian is expressed as

$$\hat{H}(t) = \sum_{\mathbf{n}} k\mu_{\mathbf{n}}(t)\hat{c}_{\mathbf{n}}^\dagger \hat{c}_{\mathbf{n}} + k\alpha_{\mathbf{n}}(t)(\hat{c}_{\mathbf{n}}^\dagger \hat{c}_{\mathbf{n}-1} + h.c.), \quad (2)$$

where $\mu_{\mathbf{n}}(t) = (1 - \alpha(t))(n_k + n_{-k})$, $\hat{c}_{\mathbf{n}}^\dagger|0\rangle = |n_k, n_{-k}\rangle$, and $\alpha_{\mathbf{n}}(t) = \sqrt{n_k n_{-k}}\alpha(t)$. Hereafter, we adopt the specific form $\alpha(t) = \alpha_c \cos(\Omega t)$ for the time-periodic modulation of the permittivity. The coefficients depend not only on time, but also on the *photonic site* index, $\mathbf{n} = (n_k, n_{-k})$. The lattice sites are defined only for non-negative photon numbers, $n_k, n_{-k} \geq 0$. The linearly increasing onsite potential, known as the Wannier-Stark ladder^{43–47}, leads to the localization of eigenstates. On the other hand, the hopping amplitude varies linearly across the photonic sites, which increases the quantum tunneling amplitude to photonic sites with higher numbers, providing the delocalizing effect. The setup establishes the quantum PTC as a promising new model system for investigating unexplored phenomena. Later, we show that this enables the system to undergo a sharp localization-to-delocalization transition, which occurs at the edges of the momentum gap for all photonic states when an infinitely large number of photonic number states in the Hilbert space are considered, or in the thermodynamic limit. Transformed to a time-periodic single-particle problem, the Floquet formalism is employed to describe it in the frequency domain:

$$(\hat{H}_F)_{l_f, l'_f} = -l_f \Omega \delta_{l_f, l'_f} + \frac{1}{T} \int_0^T \hat{H}(t) e^{-i\Omega(l_f - l'_f)t} dt. \quad (3)$$

The Hilbert space is spanned by the photon number basis and the Floquet basis, both of which are unbounded; see Fig. 1c. While the Floquet Hamiltonian is in the 3-dimensional synthetic space $(n_k, n_{-k}; l_f)$, it can be simplified to a 1-dimensional form by collecting Floquet-photonic sites that are relevant only for the formation of the quantum momentum gap near $k = \Omega/2$ and $E = 0$. For instance, the synthetic sites are grouped by the purple dotted line in Fig. 1c, and they are coupled by the time-periodic modulation of the permittivity. Approaching $k = \Omega/2$, the potential energies of the sites become increasingly similar, as shown in Fig. 1d. Consequently, the effective physics can be described by the following effective Floquet Hamiltonian when $\alpha_c \ll 1$:

$$\hat{H}_F^{\text{eff}} = \sum_{n=0}^{\infty} (2k - \Omega) n \hat{c}_n^\dagger \hat{c}_n + \frac{\alpha_c k}{2} n (\hat{c}_n^\dagger \hat{c}_{n-1} + h.c.), \quad (4)$$

where the Floquet-photonic basis n is introduced, with $n_k = n_{-k} = l_f = n$. Since the quantum dynamics is restricted to the photonic states sharing the same total momentum $P = k(n_k - n_{-k})$, for simplicity, we focus on the zero-total-momentum sector with $n_k = n_{-k}$. The underlying physics and the critical phenomena near the momentum gap transition for different total momentum $\delta n_k = n_k - n_{-k} \neq 0$ remain consistent and unaffected (see Supplementary Note 2). When $\alpha_c = 0$ (i.e., not driven), all (quasi-) eigenenergies of $\hat{H}_F^{\text{eff}}$ with $\delta n_k = 0$ are degenerate at zero energy at $k = \Omega/2$ (see Fig. 2a). Therefore, no matter how small the α_c -induced quantum tunneling is, a full delocalization of eigenstates in the synthetic space is expected. An intriguing question is what the nature of the transition would be: Will it occur at a single point or be a crossover? Remarkably, this degeneracy of eigenenergies persists even with a nonzero α_c , which shifts the critical momentum (i.e., the edge of the momentum gap) to $k_{c\pm} = \Omega/2 + \delta k_{c\pm}$ (see Fig. 2b). Note that eigenenergy $E_m^{\delta n_k}$ is indexed by $m = 0, 1, 2, \dots$ in ascending

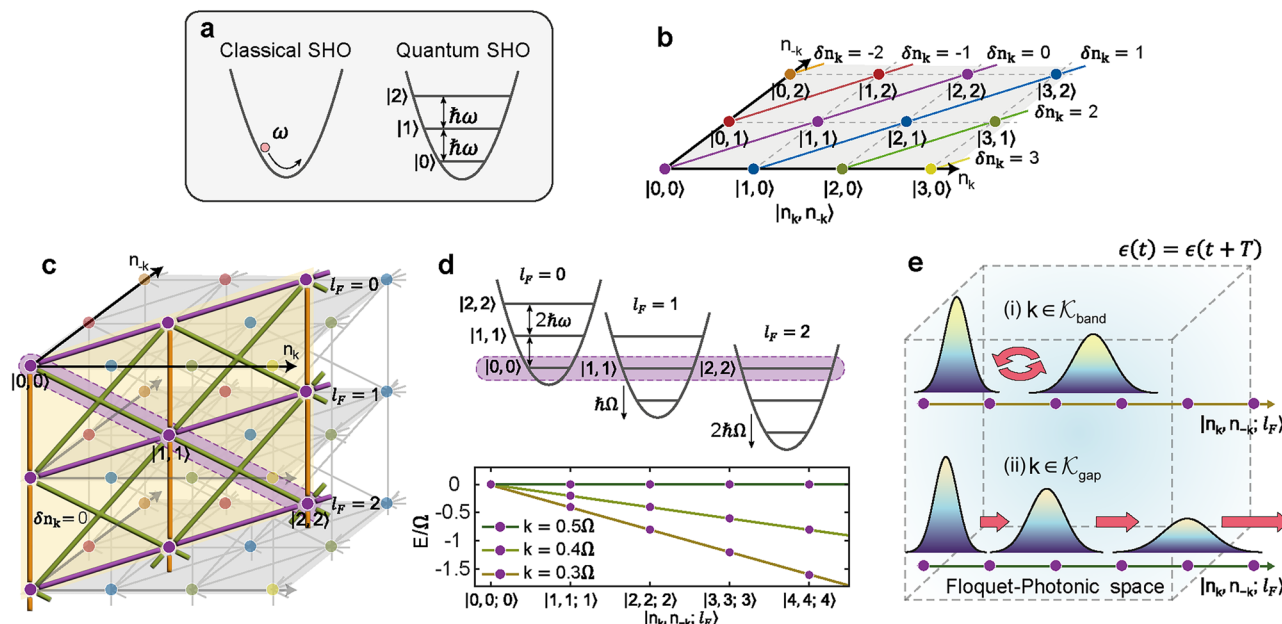


Fig. 1 | Description of the quantum PTC. **a** When a classical simple harmonic oscillator (SHO) with frequency ω is quantized, the energy spacing between neighboring eigenstates is $\hbar\omega$. A similar principle applies to the eigenfrequency spectrum of classical PTCs and the eigenenergy spectrum of quantum PTCs. **b** The 2D photon number space of a quantum PTC can be grouped with states that share the same total momentum $P = \hbar ck (n_k - n_{-k})$, which is a good quantum number. The onsite potential and hopping amplitude are time-dependent, as shown in Eq. (2). **c** By applying the Floquet formalism to the time-periodic Hamiltonian of a quantum

PTC, the photonic lattice originally described by (n_k, n_{-k}) acquires a third dimension labeled by the Floquet index $l_F = \dots, 0, 1, 2, \dots$. **d** At $k = \Omega/2$, the onsite energies of sites connected by pair creation and annihilation coincide, leading to degeneracy. Even a small coupling α_c between these sites can delocalize the eigenstates in the Floquet-photonic space. **e** A schematic depiction of wave-packet dynamics in the Floquet-photonic space: for $k \in \mathcal{K}_{\text{band}}$, the wave packet remains localized and oscillatory, whereas for $k \in \mathcal{K}_{\text{gap}}$, it becomes delocalized and accelerates in the momentum gap.

order of the photon number expectation value $\langle \hat{N} \rangle$ for each eigenstate $|\psi_m\rangle$ sharing the same total momentum δn_k .

The quantum vs classical momentum gap edge

The aforementioned effective Floquet Hamiltonian has no upper or lower bound, with its elements growing indefinitely as the photon number and Floquet index increase. When diagonalized, the eigenstates converge to a single energy as $k \rightarrow k_{c\pm} + 0^+$, thereby making it challenging to accurately estimate $k_{c\pm}$ using brute-force numerical diagonalization with truncation. We circumvent the issue by employing the transfer matrix method^{48–55} for the effective Floquet Hamiltonian.

At the critical momentum, the norms of the transfer eigenvalues become unity as the energy gap between the modes vanishes (see Methods). Using this condition, one can identify the values of the critical momenta:

$$k_{c-} = \frac{\Omega}{2 + \alpha_c}, \quad k_{c+} = \frac{\Omega}{2 - \alpha_c}, \quad (5)$$

when the permittivity is expressed as $\varepsilon^{-1} = 1 - 2\alpha_c \cos(\Omega t)$. This represents the exact result within the validity of the effective Hamiltonian, and the method can be extended to a more general time-periodic permittivity (see Supplementary Note 3). The location of the momentum gap edge in the classical PTC can also be determined to arbitrary precision using the transfer matrix approach. Since the momentum gap $\mathcal{K}_{\text{gap}} = \{k | k_{c-} < k < k_{c+}\}$ is formed by the hybridization of an infinitely many number of photonic modes sharing the same total momentum, the classical description precisely predicts the location of $k_{c\pm}$ without any quantum corrections. This is numerically verified by comparing the eigenvalues of $\hat{U}_F = T \exp(-i \int_0^T \hat{H}(t) dt)$ with those of the classical PTC. When $\hat{H}_F^{\text{eff}}$ is used, it introduces only a constant shift in the critical momenta, preserving all relative features near the critical

momentum (see Supplementary Notes 1 and 4). This implies that the physics near $k_{c\pm}$ is accurately captured by the effective Floquet Hamiltonian in Eq. (4).

The momentum gap transition in the classical PTC is reflected in the imaginary part of the *eigenfrequencies*, leading to the exponential growth of the fields. In contrast, the quantum Hamiltonian is Hermitian, ensuring that all its eigenvalues are real. The transition in the quantum PTC manifests as a localization-to-delocalization transition of eigenstates in the Floquet-photonic lattice, which is reflected in the distribution of eigenenergy level spacings. For $k < k_{c-}$, the nearest neighbor eigenenergies are equally spaced (Fig. 2a, b). In contrast, for $k \in \mathcal{K}_{\text{gap}}$, the statistics of eigenenergy level spacings follow the Wigner-Dyson distribution (orthogonal class)^{56,57} (see Fig. 2d), indicating that the eigenstates are delocalized in the Floquet-photonic space. We next examine the quantum dynamical properties using both the transfer matrix approach and the numerical time evolution of a wave packet in both regimes. This approach offers a unified perspective on both the photonic band structure and the momentum gap, framed within the context of the localization-to-delocalization transition in the synthetic space. Furthermore, it interprets the physics of the classical PTC in terms of its underlying microscopic Hamiltonian.

The localization-to-delocalization transition

The effective Floquet Hamiltonian in Eq. (4) captures two competing mechanisms that govern the dynamical behavior of a wave packet. The first term represents an onsite potential that increases (or decreases) linearly with the site index for $k < \Omega/2$ (or $k > \Omega/2$). This term induces a Wannier-Stark effect, which localizes eigenstates in systems with finite bandwidth^{44,45,58,59}. The second term describes a typical hopping Hamiltonian, but with a hopping amplitude that also increases with the site index, thereby promoting delocalization of quantum states in the Floquet-photonic space. Together, the increasing kinetic and onsite energies create a new quantum mechanical setting, where a

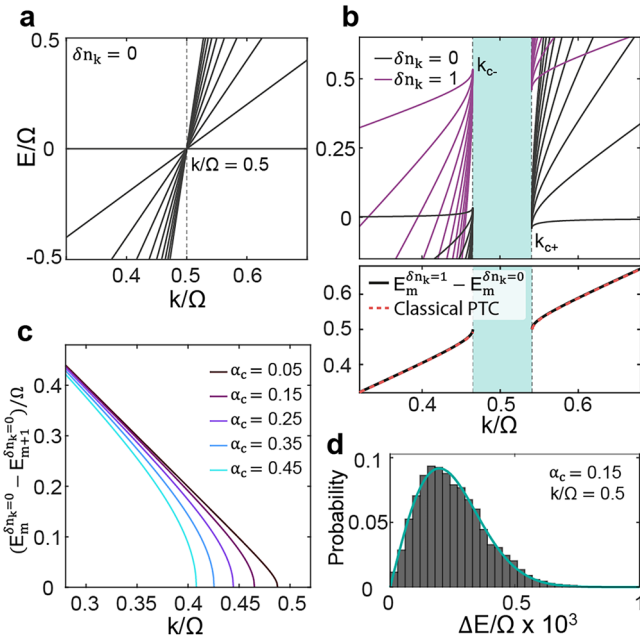


Fig. 2 | Spectral properties of the quantum PTC. **a** In the absence of driving ($\alpha_c = 0$), all Floquet-photon sites $|n_k, n_{-k}; l_F\rangle$ are decoupled, and their energies are given by $E = k(n_k + n_{-k}) - l_F\Omega$. In this plot, we show the eigenenergy spectrum for $n_k = n_{-k} = l_F = 0, 1, 2, \dots$. **b** Eigenenergies of the quantum PTC at $\alpha_c = 0.15$ for $\delta n_k = 0, 1$. The degeneracy at $k = \Omega/2$ is lifted, giving rise to $k_{c\pm} = \Omega(2\mp\alpha_c)^{-1}$. Notably, the energy spacing between eigenstates that differ by one total momentum quanta, $(E_m^{\delta n_k=1} - E_m^{\delta n_k=0})$, precisely reproduces the classical PTC spectrum $\omega_{\text{PTC}}(k)$ (see lower panel). **c** For a given k , the energy levels of eigenstates that share the same total momentum are equally spaced (see Supplementary Note 1), and this spacing converges to zero at the critical momentum k_{c-} . **d** For $k \in \mathcal{K}_{\text{gap}}$, the distribution of energy-level spacings follows Wigner-Dyson (orthogonal) statistics, indicating that the eigenstates are delocalized within the momentum gap.

localization-to-delocalization transition emerges at the critical momentum $k_{c\pm}$ in a PTC, even in the absence of quenched disorder.

An essential measure that determines the localization of quantum states is the Lyapunov exponent^{52,54,60}, defined as

$$\gamma = \lim_{N \rightarrow \infty} \frac{1}{N} \ln \left[\frac{|\Psi(n=N)|}{|\Psi(n=0)|} \right], \quad (6)$$

where the norm of the wavefunction at site n can be obtained from the multiplication of transfer matrices: $\Psi(n) = T_n T_{n-1} \dots T_1 \Psi(0)$. The Lyapunov exponent γ converges to zero for a delocalized state (indicating that the norm of the state at site $n = 0$ is equal to that of the state infinitely far away) or to a positive finite value for a localized state, $|\Psi(n)| \sim e^{-\gamma n}$. The localization length, $\xi = 1/\gamma$, is calculated both analytically and numerically for the effective Floquet Hamiltonian, Eq. (4), of the quantum PTC (see Fig. 3a upper panel). It fully characterizes the nature of the transition at the quantum momentum edge (see Methods). For $k < k_{c-}$, the localization length is given by

$$\xi^{-1} = \text{arccosh} \left(\frac{1 - \frac{\Omega}{2k}}{1 - \frac{\Omega}{2k_{c-}}} \right), \quad (7)$$

which diverges near the momentum gap edge as $\xi \sim |k - k_{c-}|^{-\nu}$ with the critical exponent $\nu = 1$ (see Supplementary Note 2). For $k \in \mathcal{K}_{\text{gap}}$, the localization length ξ diverges, $\xi \rightarrow \infty$. In Fig. 3b, the photonic onsite energy, $E_{ph}(t) = \langle \psi_t | \hat{H}_{ph} | \psi_t \rangle$ where $\hat{H}_{ph} = \sum_{n=0}^{\infty} (2kn) \hat{c}_n^\dagger \hat{c}_n$ and $|\psi_t\rangle = e^{-i\hat{H}_F^{\text{eff}} t} |0, 0; l_F = 0\rangle$, is plotted, and it provides a numerical

demonstration of this result. Note that the photonic onsite energy is related to the expectation value of the photon number by $\hat{H}_{ph} = k(\hat{n}_k + \hat{n}_{-k}) = k\hat{N}$. The height of the oscillations indicates the extent to which a wave packet can spread in the Floquet-photon space, and therefore it is proportional to the localization length. The periodicity of the oscillations can be deduced by the energy differences between neighboring eigenstates comprising the initial wave packet: $|\psi_t\rangle = \sum_m c_m e^{-iE_m t} |\psi_m\rangle$ where $c_m = \langle \psi_m | 0, 0; 0 \rangle$ and m is the eigen index. Since the energy difference approaches zero as $k \rightarrow k_{c-}$, the periodicity diverges at the critical momentum. (see Supplementary Note 6 for further details).

Within the momentum gap ($k \in \mathcal{K}_{\text{gap}}$), an initially prepared wave packet at the Floquet-photon site $n = 0$, $|\psi_{t=0}\rangle = |0, 0; l_F = 0\rangle$, not only spreads to higher nonzero photonic number states but also accelerates over time. The acceleration of the wave packet in the Floquet-photon space is associated with the rate of energy transfer from the PTC, where the total number of photons increases as energy is drawn from the source driving the PTC. The quantification of the acceleration is numerically shown in Fig. 3c and can be analytically understood by calculating the time derivative of $\langle \hat{N} \rangle = \langle \psi_t | \hat{N} | \psi_t \rangle$, i.e., the center of probability in the Floquet-photon lattice, where $|\psi_t\rangle = e^{-i\hat{H}_F^{\text{eff}} t} |\psi_{t=0}\rangle$:

$$\frac{d\langle \hat{N} \rangle}{dt} = \frac{1}{i} \langle \psi_t | [\hat{H}_F^{\text{eff}}, \hat{N}] | \psi_t \rangle = \sum_{n=0}^{\infty} n \alpha_c k \Im [\psi_t^*(n) \psi_t(n+1)], \quad (8)$$

which quantifies the photonic energy current flowing into the system. For example, a plane-wave-like wavefunction has a running $U(1)$ phase over space, $\psi(x+a) = e^{ipa/\hbar} \psi(x)$, and the current carried by the state is related to the momentum, $J(x) - \mathcal{J}[\psi^\dagger \partial_x \psi] = p/\hbar$. Likewise, in the Floquet-photon synthetic space, the similar relation $\psi(n+1) = e^{i\phi} \psi(n)$ applies as the eigenstates are delocalized ($\xi \rightarrow \infty$), and using the transfer matrix method, the analytic expression $\phi = \arccos(\frac{1-\Omega/2k}{1-\Omega/2k_{c-}})$ for $k \in \mathcal{K}_{\text{band}}$ is obtained in the large- n limit (see Supplementary Note 2). On the other hand, the average number of photons in the system is given by $\langle \hat{N} \rangle = \sum_n n |\psi_t^*(n) \psi_t(n)|$. Consequently, the exponential growth rate of photonic energy is

$$\gamma_{\text{qu}} = \frac{1}{\langle \hat{N} \rangle} \frac{d\langle \hat{N} \rangle}{dt} = 2k \sqrt{\left(\frac{\Omega}{2k_{c-}} - 1 \right)^2 - \left(\frac{\Omega}{2k} - 1 \right)^2}, \quad (9)$$

which is exactly twice the imaginary part of the (quasi-) eigenfrequency within the momentum gap in the classical PTC, $\gamma_{\text{qu}} = 2\gamma_{\text{cl}}$, the energy growth rate of classical electromagnetic waves (see Supplementary Fig. 3).

In the classical PTC, the eigenfrequencies acquire imaginary parts within the momentum gap due to the non-Hermiticity of the classical Hamiltonian. This results in exponential growth of the field strength with an exponent γ_{cl} . Within the quantum framework above, we show that the exponential growth of energy at a rate γ_{qu} associated with photons can be explained in terms of eigenstates of the quantum Hamiltonian with nonzero momentum in the synthetic lattice. That is, the probability sum of the wave packet is normalized at all times (because the quantum Hamiltonian is Hermitian), but its transport motion toward higher photon number states through pair generating hopping processes leads to the amplification of photonic energy. Our analysis of the quantum PTC offers not only a complementary view of the classical description of the non-Hermitian physics; by decomposing into Floquet-photon quantum states, it provides a photon Fock state resolved understanding of the energy amplification. From a physical perspective, the kinetic energy of a quantum state is proportional to the bandwidth of a lattice model, and our effective Hamiltonian (4) has an increasing hopping strength (or local bandwidth) with synthetic site n . As a result, as a quantum state moves

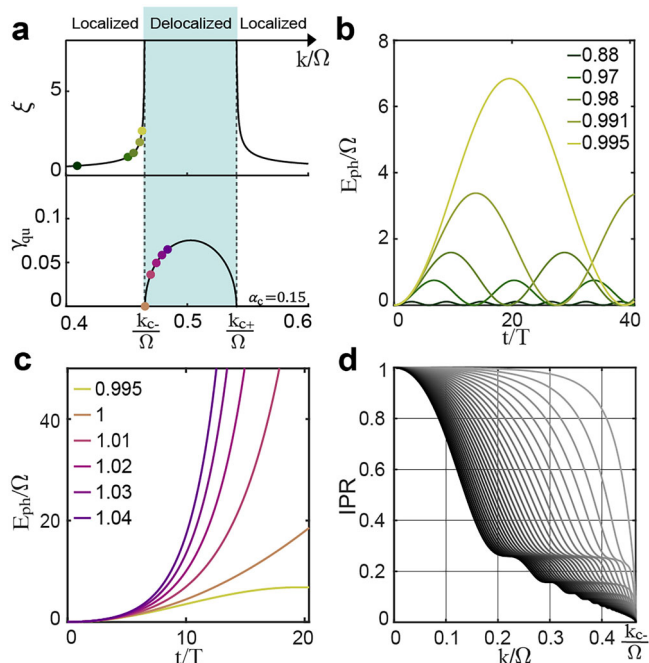


Fig. 3 | Localization-to-delocalization transition in the Floquet-photonic synthetic space. **a** The localization length ξ [Eq. (7)] in the band (upper panel) and the photonic-energy growth rate [Eq. (9)] in the momentum gap (lower panel). **b** Oscillatory photonic energy $E_{ph} = k(\hat{N})$ over time for $k < k_{c-}$, where the initial wave packet is prepared at $n_k = n_{-k} = l_f = 0$. Colors in the legend indicate the ratio k/k_{c-} . **c** For $k \in \mathcal{K}_{gap}$, the photonic energy of the same initial wave packet grows exponentially over time. **d** The inverse participation ratio of eigenstate $|\psi_m\rangle$ ($m = 0, \dots, 50$) with $\delta n_k = 0$.

toward a higher n , the effective group velocity increases and this is the source of photon generation at an exponential rate within the momentum gap. It is worth noting that the energy required for the acceleration is supplied by the external driving force of the quantum PTC, contained in $\hat{H}_{Floq} = \sum_{n=0}^{\infty} (-\Omega) n \hat{c}_n^\dagger \hat{c}_n$ in (4).

Lastly, the inverse participation ratio of the eigenstate $|\psi_m\rangle$, $IPR_{\psi_m} = \sum_{n=0}^N |\psi_m(n)|^4$, another essential measure of the localization-to-delocalization transition, in the Floquet-photonic space is plotted in Fig. 3d. As the system approaches the critical momentum ($k \rightarrow k_{c-}$), the IPR converges to zero in the thermodynamic limit ($N \rightarrow \infty$), which is consistent with the diverging localization length. The plateaus of the IPR, one of the unique features in the quantum PTC, arise in the process of delocalization of eigenstates toward $n = 0$ direction (see Supplementary Note 5 for details). Next, we present Rabi oscillations in the quantum PTC, a distinguishing feature of purely quantum behavior.

The dynamics of the quantum Rabi model in the PTC

When a two-level atom with an electric dipole moment is placed inside a PTC, its interaction with the quantized electromagnetic field is governed by the well-known quantum Rabi model^{61–63}. This model predicts coherent oscillations of the ground- and excited-state populations, driven by their coupling to the single frequency photon field. The dynamics of the two-level system is described by the Hamiltonian $\hat{H}_{at} = \Delta(\hat{I} + \hat{\sigma}_z)/2$, expressed in the eigenbasis of the ground and excited states. The ground (excited) state is set to $E_g = 0$ ($E_e = \Delta$). The corresponding electric dipole moment operator is $\hat{d} = d_0 \hat{\sigma}_x$. Focusing on $|k\rangle$ and $|-k\rangle$ modes in the quantum PTC, a quantized electric field is coupled to the electric dipole moment of a two-level atom. The interaction Hamiltonian is given by⁶⁴:

$$\hat{H}_{int} = ig\sqrt{k}\epsilon^{-1}\hat{\sigma}_x \otimes (\hat{a}_k^\dagger - \hat{a}_{-k}) + h.c., \quad (10)$$

where the coupling depends on the time-periodic permittivity $\epsilon(t)$ and is proportional to $\sqrt{k} \sim 1/\sqrt{\lambda}$. The coupling constant g includes additional factors that are independent of both time and momentum. The interaction term facilitates coupling between Floquet-photonic states differing by one total momentum unit and the atomic states. Previously, the Hilbert space of the quantum PTC was effectively described by a 1-dimensional wire in the Floquet-photonic space at a fixed total momentum. When coupled to atomic states, the Hilbert space accessible to a wave packet expands to an array of coupled wires corresponding to different total momenta $P = k(n_k - n_{-k})$ (see Fig. 4a). When the energy of the excited atomic state is tuned to $\Delta = k$, the Floquet-photonic sites combined with the atomic state (shown in the left column of Fig. 4a) all share the same onsite energy ($E_{on} = k(n_k + n_{-k}) - l_f\Omega + E_{at}$) at $k = \Omega/2$, indicating that the effective physics near $E = \Omega/2$ is well captured by those states when $\alpha_c, g \ll 1$. Neighboring sites are then coupled by α_c (green), g (purple), and α_g (blue), and consequently all eigenstates can become fully delocalized in the depicted two-dimensional Hilbert space. When $k < \Omega/2$, a set of sites still shares the same onsite energy and is grouped by the same highlighting color in Fig. 4a. Therefore, here again the competition between the Wannier-Stark effect in the 2-dimensional synthetic lattice and the increasing quantum tunneling with site $|n|$ takes place, which gives rise to a possible quantum dynamical phase transition in the atom-quantum PTC hybrid system. Note that for $\alpha_c \neq 0, g = 0$, the energy spacing between states that differ by a total momentum quanta, $\omega_{CPTC} = E_m^{\delta n_k=1} - E_m^{\delta n_k=0}$, departs from k as shown in Fig. 2b (lower panel), and the above choice of the excited atomic energy $\Delta = k$ does not make a resonant transition. See Supplementary Note 8 for the choice of resonant transition energy, $\Delta = \omega_{CPTC}$.

For $k < k_{c-}$, the eigenstates are well localized, and their coupling to atomic states results in conventional Rabi oscillations (Fig. 4b): the initial quantum state $|0, 0; 0\rangle \otimes |e\rangle$ hops back and forth between the nearest states $|1, 0; 0\rangle \otimes |g\rangle$ and $|0, 1; 0\rangle \otimes |g\rangle$ with an oscillation periodicity determined by the inverse of the light-atom coupling strength, $1/g\sqrt{k}$. Near the edge k_{c-} and within the momentum gap, their behavior deviates significantly. The spreading of an initially localized wave packet leads to irreversible changes in the atomic states, resulting in dissipation phenomena, as shown in Fig. 4c,d (upper panel) as the localization length $\xi \rightarrow \infty$, despite the system being limited to $|k\rangle$ and $|-k\rangle$ photonic modes and a single atom. The initially excited atomic state, $\rho_{at}(t=0) = |e\rangle\langle e|$, exhibits underdamped oscillations before settling into the steady-mixed state reflecting dissipation caused by coupling to a number of Floquet-photonic states within a distance ξ from the origin $\mathbf{n} = \mathbf{0}$ in the synthetic space, $\rho_{at}(t \rightarrow \infty) = \frac{1}{2}(|g\rangle\langle g| + |e\rangle\langle e|)$. Interestingly, the steady state that emerges is not the atomic ground state but a half-and-half mixed state (HHMS). This occurs because the eigenstates of the atom-quantum PTC hybrid system are delocalized along both the vertical and horizontal axes in Fig. 4a in the depicted Hilbert space where the atomic states $|e\rangle$ and $|g\rangle$ are equally populated. The decay rate from the initial state (either excited or ground) to the HHMS can be approximately determined using the Fermi-Golden Rule, provided that the coupling strength between the atom and the quantum PTC is much larger than the energy-level spacing between photonic states⁶⁵. This condition is satisfied within the momentum gap when a large enough photonic and Floquet spaces ($n \rightarrow \infty$) are considered. Furthermore, it is worth noting that the atomic states are entangled with the total momentum parity of the Floquet-photonic states: the excited atomic state is entangled with photonic states of even total momentum ($|\delta n_k| = 0, 2, 4, \dots$), while the ground atomic state is entangled with photonic states of odd total momentum ($|\delta n_k| = 1, 3, 5, \dots$). Therefore, measuring the atomic state simultaneously projects the Floquet-photonic states. These phenomena are unique to quantum PTCs and have no direct analogs in classical PTCs. Specifically, we propose that atomic-state dissipation, a distinct feature of quantum PTCs, can be systematically observed within the

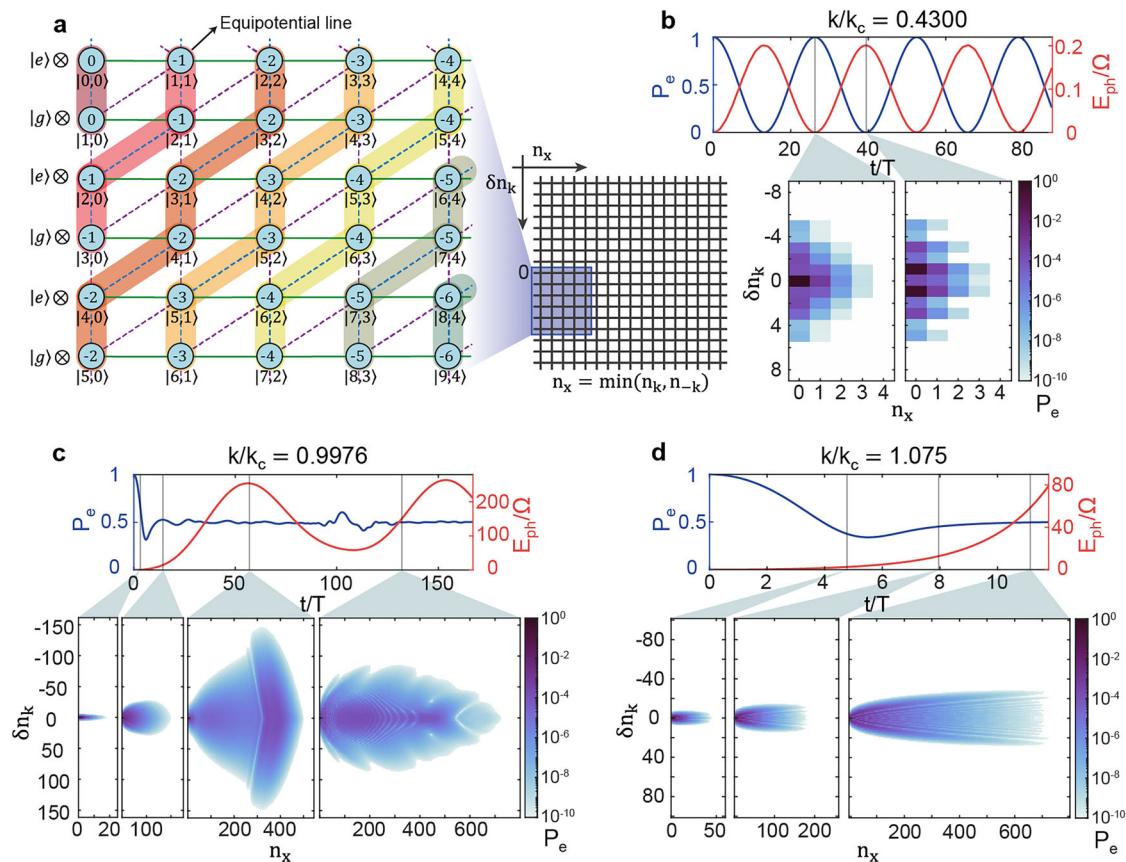


Fig. 4 | Dynamics of the quantum Rabi model in the PTC. **a** Schematic of the expanded Floquet-photon lattice $\mathbf{n}$ combined with atomic states, indicated on the left end of each line. The Floquet index l_F and the photon number state $|n_k, n_{-k}\rangle$ are marked within and next to each site, respectively. When the atomic energy is set to $\Delta = k$, all onsite energies among the illustrated lattice sites coincide at $k = \Omega/2$. At $k \neq \Omega/2$, sites with identical onsite energies are grouped together by color. Neighboring sites are coupled by α_c (green dotted lines), g (blue dotted lines), and α_g (purple dotted lines). **b** At $k \ll k_c$, both the excited-atomic-state population and the photonic energy oscillate coherently, akin to the conventional quantum Rabi model. In the lower panel, $|\psi_t(\mathbf{n})|^2$ on the lattice shown in (a) is plotted (log scale).

The initial wave packet at $t = 0$ is prepared in $|0, 0; 0\rangle \otimes |e\rangle$, positioned at $\mathbf{n} = (0, 0)$. **c** At $k/k_c = 0.9976$, the initial atomic state $\rho_{at}(t=0) = |e\rangle\langle e|$ decays into the mixed state $\rho_{at} = \frac{1}{2}(|g\rangle\langle g| + |e\rangle\langle e|)$ because the wave packet spreads over many Floquet-photon sites with a large localization length ($\xi \gg 1$), where $\rho_{at}(t) = \text{Tr}[\psi_t \psi_t^\dagger]$ and the trace is taken over the Floquet-photon space. Nonetheless, the photonic energy retains its oscillatory behavior. **d** For $k \in \mathcal{K}_{gap}$, the photonic energy diverges exponentially as the atomic state decays. The lower panel illustrates continuous propagation of the quantum states, showing no sign of localization. We set $\alpha_c = 0.15$ and $g = 0.03$ for all three simulations.

momentum gap as a function of coupling strength and permittivity modulation.

Discussion

Spontaneous emission of a photon, accompanied by the de-excitation of an atomic state, occurs when a continuum photonic spectrum is available. That is, the initial quantum state $|n_k = 0\rangle \otimes |e\rangle$ transitions to $|n_k = 1\rangle \otimes |g\rangle$ when $|\hbar k - \Delta| \lesssim g$. The entropy associated with a single photon in free space is significantly greater than that of no photon. Consequently, spontaneous emission decay is an irreversible process that dissipates the energy of the excited atomic state. This energy is irretrievably transferred to the vast continuum of photonic modes. In the quantum PTC, a similar dissipative process occurs but is confined to single-frequency photonic modes with a time-periodic permittivity. Near the critical momentum and within the momentum gap, the large, effectively unbounded set of Floquet-photon states acts as a continuum, allowing the atom to relax irreversibly into a mixed state. It is also worthwhile to note that because of the delocalization of eigenstates in the atomic-photon two-dimensional domain (Fig. 4a), a transition to the mixed state can occur irrespective of the specific form of the initially prepared atomic state in $|e\rangle$ or $|g\rangle$. Namely, with energy transferred from the driven permittivity to photonic energy, another remarkable phenomenon arises in a nearly symmetric manner: a

quantum state $|n_k = 0, n_{-k} = 0\rangle \otimes |g\rangle$, initially prepared in the atomic ground state, undergoes irreversible excitation to the same mixed state (see Supplementary Note 7). Following the same steps described in the main text, we depict the Hilbert space relevant to the photonic vacuum and the ground atomic state in Supplementary Fig. 7b, c (lower panels) for two different choices of atomic energy. Crucially, there is no direct coupling by $\hat{H}_{int}$ between the two Hilbert spaces associated with the quantum dynamics of $|\Phi_0^g\rangle = |0, 0; 0\rangle \otimes |g\rangle$ and $|\Phi_0^e\rangle = |0, 0; 0\rangle \otimes |e\rangle$. With no interference, for an arbitrary atomic initial state $|\Phi\rangle_{t=0} = \sqrt{r}|\Phi_0^e\rangle + \sqrt{1-r}|\Phi_0^g\rangle$, the time evolution of the atomic population is simply given by $\hat{\rho}_{at}(t) = r\hat{\rho}_{at}^e(t) + (1-r)\hat{\rho}_{at}^g(t)$, where $\hat{\rho}_{at}^{e/g}(t)$ is independently obtained from the time evolution of $|\Phi_0^{e/g}\rangle$ in the corresponding Hilbert space (see Supplementary Fig. 8).

In this work, we provide a comprehensive understanding of the classical PTC near the critical momentum and within the momentum gap, leveraging the framework of quantum electrodynamics. While our analysis has focused on weak coupling and a closed quantum system with a single atomic light emitter, future work could explore regimes where stronger coupling or multiple atoms introduce collective phenomena analogous to superradiance. Additional complexity may arise if the system is allowed to exchange energy with an external reservoir—potentially stabilizing different steady states or enabling new non-equilibrium phase transitions. We anticipate that circuit QED

platforms and rapidly tunable photonic cavities could offer practical testbeds, where time-domain engineering of the optical properties would systematically probe the momentum gap and measure the emerging half-and-half mixed atomic state.

Methods

Effective Hamiltonian in the Floquet-photonic synthetic lattice

When the energy quanta from the driving frequency of permittivity equals to the energy of two photons, $\hbar\Omega = 2\hbar ck$, the onsite potential energy of Floquet-Photonic synthetic space $-l_F\hbar\Omega + (n_k + n_{-k})\hbar ck = (-2l_F + n_k + n_{-k})\hbar\Omega/2$. For example, the series of synthetic sites grouped in Fig. 1(c,d) shares the same potential energy and therefore no Wannier-Stark localization. At the same time the sites are coupled due to the time periodic coefficient of photon pair creation and annihilation term in Eq. (1), which generates off-diagonal terms in Floquet index, and it leads the delocalization of eigenstates within the grouped sites. The Floquet Hamiltonian elements are

$$(\hat{H}_F^{\text{eff}})_{l_F, l_F} = -l_F\hbar\Omega + \hbar k(\hat{n}_k + \hat{n}_{-k}), \quad (11)$$

$$(\hat{H}_F^{\text{eff}})_{l_F, l_F+1} = \frac{1}{2}k\alpha_c(\hat{a}_k\hat{a}_{-k} + h.c.), \quad (12)$$

and $(\hat{H}_F^{\text{eff}})_{l_F, l_F-1} = (\hat{H}_F^{\text{eff}})_{l_F, l_F+1}$. There are collections of similarly coupled sites in (1, 1, 1)-direction in the Floquet-Photonic lattice (n_k, n_{-k}, l_F) but at differently shared onsite potential energy. This is the basis for the construction of our effective Hamiltonian in Eq. (4), specifically for $n_k = n_{-k} = l_F = n$ case. As a result, the original 3-dimensional Hilbert space in (n_k, n_{-k}, l_F) is reduced to the collection of 1-dimensional Hilbert spaces decoupled in energy and total momentum. When a two-level atom with energy spacing Δ is coupled to the quantum PTC, we again collect sites sharing the same onsite potential energy including the atomic states. Due to the light-matter interaction not conserve the total momentum of photonic states, the effective Hilbert space is extended to 2-dimension as described in Fig. 4a.

Diagonalization of time periodic Hamiltonian in the photon number space

The spectral properties of the quantum PTC in Fig. 2a-c are obtained from the diagonalization of effective Hamiltonian $\hat{H}_F^{\text{eff}}$ in Eq. (4). The convergence of eigenvalues sharing the same total momentum as approaching to the critical momentum is verified from the effective Hamiltonian with the size of the synthetic space up to $N = 10,000$. The dynamical properties shown in Fig. 3 are from the eigenstates of $\hat{H}_F^{\text{eff}}$ and their time evolution as explain in the main text. The Wigner-Dyson distribution observed in the momentum gap for the energy level spacing is computed from $\hat{U}_F$ to take into account all eigenvalues within the quasi-energy domain. The Floquet unitary operator obtained from Eq. (1):

$$\hat{U}_F = \mathcal{T} \exp\left(-i \int_0^T \hat{H}(t) dt\right) \simeq \mathcal{T} \prod_{j=1}^{j=M} e^{-i\hat{H}(t_j)\Delta t}, \quad (13)$$

where $\Delta t = T/M$ and $t_j = (j-1)T/M$. We take $M = 1000$, and the instantaneous Hamiltonian is represented in Photonic number space up to $N = 4000$.

Transfer matrix method in the Floquet-Photonic lattice

The Schrodinger equation in Eq. (4) can be expressed in a recursive form in terms of the amplitude $\psi(n) = \langle n, n; n|\psi \rangle$ at site n :

$$\psi(n+1) = \frac{2}{\alpha_c k} \frac{E - (2k - \Omega)n}{n+1} \psi(n) - \frac{n}{n+1} \psi(n-1). \quad (14)$$

where the relation holds for $n \geq 0$, since the photon number cannot be a negative integer. The relation is then cast as

$$\begin{bmatrix} \psi(n+1) \\ \psi(n) \end{bmatrix} = \begin{bmatrix} \frac{2}{\alpha_c k} \frac{E - (2k - \Omega)n}{n+1} & -\frac{n}{n+1} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi(n) \\ \psi(n-1) \end{bmatrix}. \quad (15)$$

The first matrix on the right-hand side is the transfer matrix that provides a relationship between consecutive quantum states in the Floquet-photonic space: $\Psi(n) = T_n \Psi(n-1)$, where $\Psi(n) = [\psi(n+1), \psi(n)]^T$. In the large n limit, the transfer matrix becomes independent of site n and energy E , which crucially indicates that the momentum gap transition at $k = k_{c\pm}$ depends only on α_c and Ω (see Supplementary Note 2 for nonzero total momentum case). This robustly defines the quantum momentum gap:

$$\lim_{n \rightarrow \infty} T_n = T = \begin{bmatrix} \frac{2}{\alpha_c} \left(\frac{\Omega}{k} - 2\right) & -1 \\ 1 & 0 \end{bmatrix} \quad (16)$$

Its eigenvalues are $\lambda_{\pm} = \frac{Z \pm \sqrt{Z^2 - 4}}{2}$, where $Z = \frac{2}{\alpha_c} \left(\frac{\Omega}{k} - 2\right)$. The Lyapunov exponent in terms of the transfer matrix is expressed as:

$$\begin{aligned} \gamma &= \lim_{N \rightarrow \infty} \frac{1}{N} \ln \left[\frac{|\Psi(N)|}{|\Psi(0)|} \right], \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N} \ln \left[\frac{\Psi^\dagger(0) T_1^\dagger \cdots T_N^\dagger T_N \cdots T_1 \Psi(0)}{\Psi^\dagger(0) \Psi(0)} \right]. \end{aligned} \quad (17)$$

where $|\Psi(n)| = \sqrt{|\psi(n+1)|^2 + |\psi(n)|^2}$. The Lyapunov exponent γ is positive nonzero for localized phase. It is zero for delocalized phase. The Lyapunov exponent is $\gamma = 0$ for $Z^2 \leq 4$, and for $Z^2 \geq 4$ (see Supplementary Note 2 for details)

$$\gamma = \ln \lambda_+ = \text{arcosh}(|Z|/2), \quad (18)$$

The phase transition takes place at $Z^2 = 4$, which provides the locations of critical momenta:

$$k_{c-} = \frac{\Omega}{2 + \alpha_c}, \quad k_{c+} = \frac{\Omega}{2 - \alpha_c}. \quad (19)$$

The localization length ξ is the inverse of the Lyapunov exponent γ . Specifically for $k < k_{c-}$ and $k > k_{c+}$,

$$\xi^{-1} = \gamma = \text{arcosh}\left(\left|\frac{1 - \Omega/2k}{1 - \Omega/2k_{c\pm}}\right|\right). \quad (20)$$

Within the momentum gap ($k_{c-} < k < k_{c+}$), eigenstates are delocalized ($\gamma = 0$) and they come with a momentum ϕ that characterizes the speed of phase rotation in the Floquet-Photonic synthetic lattice, $\psi(n+1) = e^{i\phi} \psi(n)$, where in the $n \rightarrow \infty$ limit (See Supplementary Note 2 for details)

$$\phi = \arccos\left(\frac{1 - \Omega/2k}{1 - \Omega/2k_{c-}}\right). \quad (21)$$

This information is not captured in the Lyapunov exponent which takes the magnitude of wave function. As elaborated in the main text, the exponential growth rate of photon energy (or number) is analytically expressed in Eq. (9).

Data availability

This work is purely theoretical and produced no experimental or observational datasets. All numerical results can be reproduced from the equations and parameters provided in the manuscript and Supplementary Information. MATLAB/Python scripts used to generate the

figures are available from the corresponding authors for non-commercial academic use.

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Acknowledgements

J.H.B. and K.W.K. acknowledge financial support from the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (no. RS-2025-00521598) and the Korean Government (MSIT) (no. 2020R1A5A1016518). K.L. and B.M. are supported by the National Research Foundation of Korea (NRF) through the government of Korea (NRF-2022R1A2C301335313) and the Samsung Science and Technology Foundation (SSTF-BA2402-02).

Author contributions

J.H.B. and K.L. contributed equally. J.H.B. performed analytical derivations and numerical calculations and prepared figures; K.L. performed numerical calculations and prepared figures. K.W.K. developed the theoretical framework; K.W.K. and B.M. conceived and supervised the project, provided theoretical guidance, verified derivations, and refined the presentation. All authors contributed to writing and editing and approved the final manuscript.

Competing interests

The authors declare no competing interests.

Additional information

Supplementary information The online version contains supplementary material available at <https://doi.org/10.1038/s41467-025-67572-0>.

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Peer review information *Nature Communications* thanks the anonymous reviewer(s) for their contribution to the peer review of this work. A peer review file is available.

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