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Technical Note

THEORETICAL STUDY OF MOTION OF SMALL SPHERICAL AIR BUBBLES IN A UNIFORM SHEAR FLOW OF WATER

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ABSTRACT

A simple Couette flow velocity profile with an appropriate correlation for the free terminal rise velocity of a single bubble in a quiescent liquid can produce reliable results for the trajectories of small spherical air bubbles in a low-viscosity liquid (water) provided the liquid remains under uniform shear flow. Comparison of the model adopted in this paper with published results has been accomplished. Based on this study it has also been found that the lift coefficient in water is higher than its typical value in a high-viscosity liquid and therefore a modified correlation for the lift coefficient in a uniform shear flow of water within the regime of the Eötvös number $0.305 \leq Eo \leq 1.22$ is also presented.

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1. Introduction

Water, being the most important working fluid in industrial processes, bears a great importance. Air–water two-phase flow systems are of interest for instance in chemical reactors, sparging, and mixing processes, heat transfer phenomena in a power plant's auxiliary systems, and nuclear power plants. It has been reported that the motion of small air bubbles inside flow channels is responsible for the reduction and/or increase of wall friction [1]. Therefore a substantial amount of research (although mostly related with high viscosity liquids) has been carried out into the behavior of air bubbles in quiescent or moving liquids [2–6]. These studies were based on either extensive computer computations or else complicated experiments.

It was reported by Celata et al. [7] that when an air bubble starts to rise inside a quiescent liquid its motion may fall into the viscous regime, the surface tension dominated regime or the inertia dominated regime depending on the dimensionless numbers that govern the physics. One such number is the Eötvös number that highlights the relative effect of buoyancy and surface tension forces. According to Clift et al. [8], at least for air–water systems the surface tension dominated regime holds when $0.25 < Eo < 40$. Many

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correlations depending upon the degree of contamination of the system exist in the literature that can handle all of the above regimes simultaneously [2,4,9,10]. However, since most of these correlations were developed for viscous liquids under stagnant conditions they need some tuning parameters in order to match the calculated trajectories with the experimental or simulated data if the problem involves a low viscosity liquid and a liquid velocity field as well. This particular fact has also been mentioned by Tomiyama et al. [4,11].

Therefore in this study we present a simple one step method (noniterative) that can be used to generate the bubble trajectories for small spherical bubbles (1.5 mm $\leq$ d $\leq$ mm) in a steady, laminar, fully developed, and uniform shear flow of water. It is shown in this study that a simple Couette flow velocity profile in combination with a general correlation for the free terminal rise velocity of the bubble in any stagnant Newtonian liquid proposed by Rodrigue [6] is sufficient to produce reliable results for the trajectories of small air bubbles. The properties of water and other relevant nondimensional numbers used in this study are shown in Table 1.

In this study we have considered a uniform shear rate of 6.2 s^{-1} for the sake of comparison and also to keep the flow under the laminar regime. It is well known that compared to other common liquids water has the highest surface tension coefficient with air and therefore within the regime of the Eötvös number considered in this study it is anticipated that there exist negligible changes in the shape of the bubbles. Therefore the bubbles are considered to remain spherical during their motion [3]. Another interesting and distinguishing feature of sheared liquids is that the motion of air bubbles having a diameter of up to 5.2 mm were observed to follow a trajectory without any zigzag motion [3,11,12]. This characteristic of a sheared liquid remains favorable for this work since any zigzag motion is avoided. Therefore, only those bubble diameters that can maintain straight trajectories without any significant size effects, which ultimately circumvent the use of the modified Eötvös number throughout this study, are considered. Compared to the above discussion, the trajectories of small spherical air bubbles > 1.5 mm in diameter were observed to follow a zigzag pattern in the case of stagnant liquids [13–15].

2. Mathematical formulations

In order to proceed, we first need to establish the expressions for the components of the bubble relative velocity in the horizontal and vertical directions. It is to be noted that in this study the bubble motion remains in two dimensions, mainly due to the range of the Eötvös number considered here. The geometry of the problem is depicted in Fig. 1, which represents the schematic of a simple Couette flow. The liquid (water) between the moving and the stationary plate remains under steady, laminar, fully developed, and uniform shear flow. The width between the plates is W = 30 mm and the origin of the coordinate system (L) coincides with the point of injection of the bubble and can be adjusted to arbitrary locations. Finally, U is the velocity of the moving plate. The general equation of motion of a bubble inside any geometrical domain can be expressed as Newton's second law of motion in Lagrangian coordinates by Equation 1 [3,4].

$$\left(\rho_g + 0.5\rho_l\right)\frac{DV_b}{Dt} = -\frac{3}{4}\frac{C_D}{d}\rho_l\left|V_r\right|V_r - C_L\rho_l V_r \times \nabla \times V_r + \left(\rho_l - \rho_g\right)g \quad (1)$$

In Equation 1 V_r is the bubble relative velocity, V_l is the liquid velocity, V_b is the bubble velocity, ρ_g and ρ_l represent the density of the gaseous phase (air) and liquid phase (water), respectively, d is the diameter of the bubble, C_D and C_L are the drag and lift coefficients respectively, and g is the acceleration due to gravity. The first term on the right hand side of Equation 1 represents the drag force, the second term represents the lift force, and the last term represents the effect of gravity, the so-called buoyancy force. It must be noted that the wall effects can be considered to be negligible in this study. Therefore following Rahba and Buwa [3] and Bothe et al. [5] Equation 1 can be split into its horizontal and vertical components as follows.

2.1. Horizontal component of bubble equation of motion

$$0.5\frac{dV_{rx}}{dt} = -\frac{3}{4}\frac{C_D}{d}\left|V_r\right|V_{rx} + C_L\xi V_{ry} \quad (2)$$

The horizontal component of Equation 1 is therefore given by Equation 2. In Equation 2 V_{rx} is the relative bubble velocity in the lateral or horizontal direction, V_{ry} is the relative bubble velocity in the vertical direction, and ξ is the magnitude of the

Table 1 – Fluid properties and dimensionless numbers used in this study.

Properties/ dimensionless no.	Water	Air bubble
$\rho(kg/m^3)$	998	1.2
$\mu(Pa.s)$	0.00098	—
$\sigma(N/m)$ for air–water interface	0.072	—
Eo	0.305–1.22	—
M	2.4252×10^{11}	—

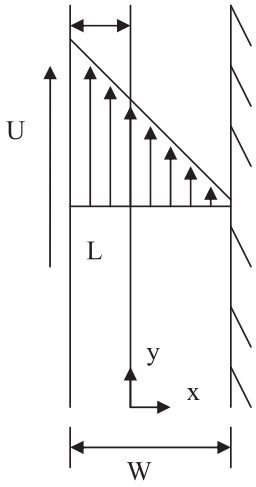


Fig. 1 – Geometry and coordinate system for the problem domain.

shear rate (or liquid velocity gradient), which is constant (6.2 s^{-1}) in this study (since the velocity profile in Fig. 1 is linear). Since the bubble diameters remain small in this study, steady state conditions for the bubble can be assumed [3]. Therefore, neglecting the acceleration effects Equation 2 can now be written as Equation 3.

$$V_{rx} = \frac{4C_L \xi V_{ry} d}{3C_D |V_r|} \tag{3}$$

Later it will be shown from the basic order of magnitude analysis that the order of V_{rx} remains smaller than the order of V_{ry} Therefore, we can neglect the term V_{rx}^2 from the definition of magnitude of the relative velocity of the bubble and obtain Equation 4.

$$|V_r| = \sqrt{V_{rx}^2 + V_{ry}^2} \approx V_{ry}. \tag{4}$$

Introducing Equation 4 into Equation 3 we can finally solve for V_{rx} as Equation 5.

$$V_{rx} = \frac{4C_L \xi d}{3C_D} \tag{5}$$

The analytical expression for the liquid velocity profile in a Couette flow for the coordinate system mentioned in Fig. 1 is given by Equation 6.

$$V_{ly}(x_b) = \frac{U}{W}(W - L - x_b). \tag{6}$$

In Equation 6 V_{ly} (x_b) is the local liquid velocity at a horizontal bubble position ofx_b. Here it must be noted that the knowledge of local liquid velocity is important in order to calculate the bubble velocity $[V_b(x_b) = V_{ry} + V_{ly}(x_b)]$ in the vertical direction.

2.2. *Vertical component of bubble equation of motion*

$$0.5\left(\frac{dV_{ry}}{dt} - \xi V_{rx}\right) = -\frac{3}{4}\frac{C_D}{d}|V_r|V_{ry} - C_L \xi V_{rx} + g. \tag{7}$$

Again, assuming steady state conditions for the bubble and substituting values of Equations 4 and 5 in Equation 7 we can obtain the expression for V_{ry} as Equation 8.

$$V_{ry} = \frac{1}{3C_D}\sqrt{12C_D dg + 8\xi^2 d^2 C_L - 16C_L^2 \xi^2 d^2}. \tag{8}$$

3. Order of magnitude analysis

This section presents a simple order of magnitude analysis for Equations 5 and 8 which will also validate the assumptions regarding V_{rx} used in the derivations in the *Mathematical formulations* section of this paper. We proceed with the analysis by representing V_{rx} and V_{ry} in nondimensional forms as follows:

From Equation 5 we know that:

$$V_{rx} = \frac{4C_L \xi d}{3C_D}.$$

Dividing the above expression by a reference velocity scale U we get:

$$V_{rx}^* = \frac{V_{rx}}{U} = \frac{4C_L \xi d}{3C_D U}.$$

Also, since $\xi = \left|\frac{dV_{ly}}{dx}\right|$, a suitable scale for the horizontal coordinate can be the width between the plates, i.e., (x W). Since the velocity scale has already been selected as U, we have the following expression for the order of nondimensional relative velocity of the bubble in the horizontal direction:

$$O(V_{rx}^*): \quad O\left(\frac{4C_L d}{3C_D W}\right)$$

Since the drag and lift coefficients in the present study (i.e., $1.5 \le d \le 3$ mm) usually remain within the same order of magnitude (10^{-1}), it can be seen that the order of magnitude of V_{rx}^* depends on the nondimensional parameter ($d \setminus W$). Usually the bubble diameter d is smaller than the width W between the plates, therefore the order of V_{rx}^* would remain less than unity.

Similarly in the case of V_{ry} using Equation 8 and the same scales for the liquid velocity and the horizontal coordinate, we get the following result for the nondimensional vertical relative velocity of the bubble:

$$(V_{rx}^*)^2 = \left(\frac{V_{ry}}{U}\right)^2 = \left(\frac{4}{3C_D}\right)\left(\frac{dg}{U^2}\right) + \left(\frac{8C_L}{9C_D^2}\right)\left(\frac{d}{W}\right)^2 - \frac{16}{9}\left(\frac{C_L}{C_D}\right)\left(\frac{d}{W}\right)^2.$$

or

$$O\left(V_{ry}^*\right)^2 : O\left(\left(\frac{4}{3C_D}\right)\left(\frac{dg}{U^2}\right) + \left(\frac{8C_L}{9C_D^2}\right)\left(\frac{d}{W}\right)^2 - \frac{16}{9}\left(\frac{C_L}{C_D}\right)^2\left(\frac{d}{W}\right)^2\right).$$

After interpreting the correct orders of magnitudes for various terms involved in the above expression we can finally solve for the order of V_{ry}^* as:

$$O\left(V_{ry}^*\right)^2 : O\left(\left(\frac{10}{Fr}\right) + 10\left(\frac{d}{W}\right)^2 - \left(\frac{d}{W}\right)^2\right).$$

Note that in the above expression the second and the third term remain small since (dW) and can be neglected. This means that actually there is no effect of the shear rate ξ on the vertical relative velocity of the bubble (at least in the case of the laminar regime). Also note that the term $\left(\frac{dg}{U^2}\right)$ represents the inverse of the Froude number i.e., $\left(\frac{1}{Fr} = \frac{dg}{U^2}\right)$ It can be noticed that as long as the inertial effect remains smaller than the effect of gravity the Froude number $\left(Fr = \frac{U^2}{dg}\right)$ remains less than unity; this actually corresponds to the case of laminar flow regime. Therefore we have for (1.1)V_{ry}^*:

$$O(V_{rz}^*): O\left(\frac{1}{\sqrt{Fr}}\right).$$

It can be noticed that as long as the inertial effects remain smaller (laminar flow) or comparable to the gravity effects then Fr remains less than or equal to unity and therefore $O(V_{ry}^*) > O(V_{rx}^*)$ is valid. However, for the sake of comparison purposes it is observed that for $\xi = 6.2\ s^{-1}$ the order of V_{ry}^*

remains 10^{-1} in this study whereas the order of V^*_{rx} is 10^{-2}, hence $O(V^*_{ry}) > O(V^*_{rx})$ is valid. Therefore the assumptions we made earlier in this paper are also validated through this order of magnitude analysis.

4. Lift coefficient in water

Tomiyama et al. [4] carried out experiments using high viscosity liquids (water and glycerol solutions). Their experimental setup was designed to generate a simple uniform shear flow between a moving belt and a stationary wall (similar to Fig. 1). In their experiments, for all the values of the shear rate ξ the liquid flow field was assumed to remain steady and laminar. They proposed a set of correlations for the lift coefficient in such kinds of flows and claimed that when a bubble remains small ($d \leq 4.4$ mm) the lift coefficient is only a function of the bubble Reynolds number and it tends to reach a constant positive value of 0.288 when $Re_b \geq 25$. They further mentioned that the lift coefficient for a constant diameter bubble, i.e., for small spherical bubbles, is not affected by the liquid velocity gradient or the value of the shear rate and thus remains constant with the applied shear rate. However since their correlation was originally developed for uniform shear flows of high viscosity liquids, in principle it should not be used for low viscosity liquids. Nevertheless, they mentioned that their correlation gives suitable results for low viscosity systems as well.

It was mentioned earlier in this article, and confirmed by the experiments of Habermann and Morton [16], that in the case of water and for the Eötvös number regime considered in this study the problem lies in the surface tension controlled regime. This argument can also be confirmed by Fig. 2, which indicates the rise and fall behavior of the terminal rise velocity with increasing bubble diameters, which is only a characteristic of the surface tension dominated regime. Because of this, although the bubble diameters remains small in this study ($d \leq 4.4$ mm) it was found that the correlation for the lift coefficient based on the Eötvös number (Equation 9) as suggested in [4] for a 2.84 mm diameter bubble in a uniform shear flow of water gives a value for the lift coefficient that is 19.62% lower than the value estimated from the simulation results of Rabha and Buwa [3].

$$C_L = 0.00105Eo_d^3 - 0.0159Eo_d^2 - 0.0204Eo_d + 0.474 \quad (4 \leq Eo_d \leq 10.7) \tag{9}$$

This trend suggests that the lift coefficient in low viscosity liquid (water) under uniform shear flow within the regime of the Eötvös number range studied here ($0.305 \leq Eo \leq 1.22$), i.e., the surface tension dominated regime, might be easily and well correlated with the Eötvös number rather than the bubble Reynolds number. In Equation 9 Eo_d is the Eötvös number based on the long axis characteristic dimension for large distorted bubbles as suggested by Wellek et al. [17]. However in the case of water and when the bubble diameters are such that the range of Eötvös numbers is $(0.305 \leq Eo \leq 1.22)$ the distortion can be neglected and we can simply use Eo instead of Eo_d in Equation 9. Based on these arguments a modified correlation for the lift coefficient is presented as Equation 10.

$$C_L = 0.001307Eo^3 - 0.01979Eo^2 - 0.0254Eo + 0.5901 \quad (0.305 \leq Eo \leq 1.22). \tag{10}$$

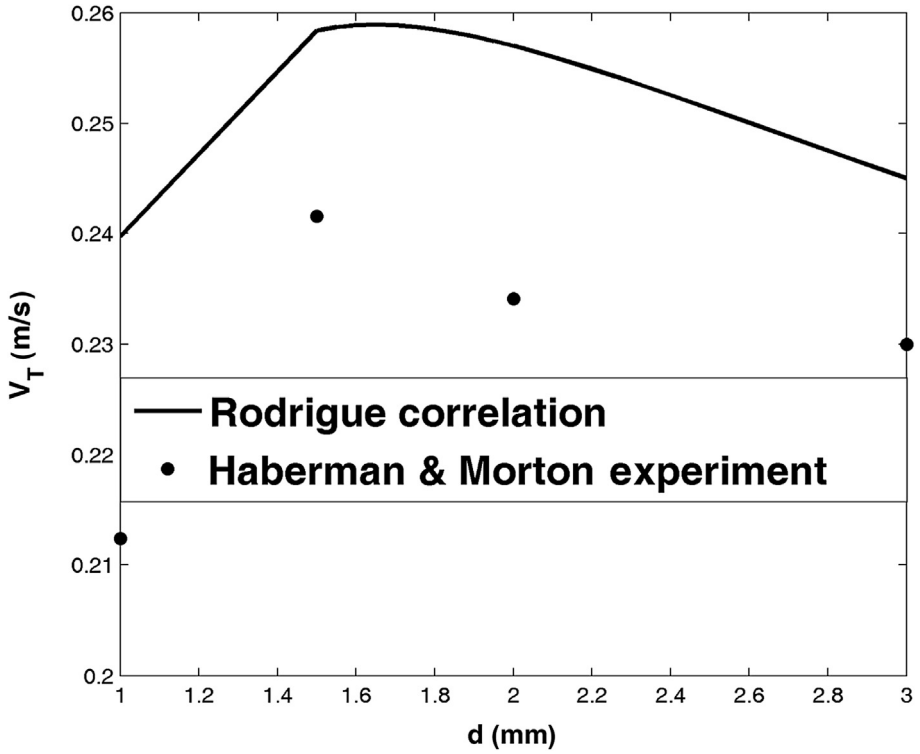


Fig. 2 – Comparison of Rodrigue [6] correlation with experiments of Haberman and Morton [16] for stagnant water.

Equation 10 is valid for water under uniform shear flow and for the Eötvös number range of ($0.305 \leq Eo \leq 1.22$). As already mentioned, Equation 9 tends to give lower values of the lift coefficient for the bubbles in water, therefore, Equation 10 was obtained by simply adjusting the coefficients of Equation 9 to match the simulation results of Rabha and Buwa [3] for small diameter bubbles.

5. Trajectory calculations

It was highlighted at the start of this paper that when calculating the trajectories of single bubbles in a sheared liquid, one usually needs to tune the values of the drag and/or the lift coefficients. However, the formulations presented in this section do not require any tuning if the working regime lies within the range specified in this paper. In the *Order of magnitude analysis* section it was pointed out that within the laminar regime of the liquid flow the second and third terms in Equation 8 have negligible effects on the magnitude of the vertical relative velocity which means that we can simply write Equation 8 as Equation 11.

$$V_{ry} = \sqrt{\frac{4dg}{3C_D}}. \tag{11}$$

Equation 11 states that under such circumstances the relative velocity of the bubble in the vertical direction simply becomes equal to the terminal rise velocity of the bubble in a stagnant or quiescent liquid i.e., $V_{ry} = V_T$. Hence we can write Equation 11 as:

$$V_T = \sqrt{\frac{4dg}{3C_D}}. \tag{12}$$

Therefore, if we know the terminal rise velocity of a bubble in stagnant water at a given temperature we can estimate the drag coefficient from Equation 12. From the literature survey it was found that a general correlation for the free rise velocity of a bubble in any Newtonian liquid can be explicitly calculated by the correlation proposed by Rodrigue [6]. The correlation is given as Equations 13–16 and its comparison with the experimental results of Haberman and Morton [16] is shown in Fig. 2. It can be seen that the correlation is able to predict the rise velocities with an error of approximately 10%.

$$M = \frac{g\left(\rho_l - \rho_g\right)\mu_l^2}{\rho_l^2\sigma^3} \text{ (The Morton number)}. \tag{13}$$

$$F = g\left[\frac{\rho_l^5 d^8}{\sigma\mu_l^4}\right]^{1/3} \text{ (The flow number)}. \tag{14}$$

$$V = \frac{F}{12}\left[\frac{\left(1 + 1.31\times 10^{-5}M^{11/20}F^{73/33}\right)^{21/176}}{\left(1 + 0.020F^{10/11}\right)^{10/11}}\right] \times (\text{The velocity number}). \tag{15}$$

And finally

$$V_T = V\left[\frac{\sigma\mu_l}{\rho_l^2 d^2}\right]^{1/3} = V_{ry}. \tag{16}$$

For a fixed equivalent diameter and gas–liquid properties the Morton number and the flow number can be readily calculated from Equations 13 and 14 respectively, then from Equation 15 the velocity number can be calculated and finally from Equation 16 we can explicitly calculate the value of the terminal rise velocity of the bubble. Once the terminal rise velocity is known using Equation 12, we can calculate the drag coefficient of the bubble. Since from Equation 10 we can easily find the lift coefficient, once the lift and the drag coefficients are known we can explicitly calculate the bubble relative velocity in the horizontal direction from Equation 5 as shown below.

$$V_{rx} = \frac{4C_L\xi d}{3C_D}.$$

However, in order to generate the bubble trajectories we must know the bubble velocity in the vertical and horizontal directions as well. The bubble velocity is defined as a vector sum of the relative velocity and the local liquid velocity as:

$$V_b = V_r + V_l. \tag{17}$$

Since the knowledge of bubble velocity requires information about the local liquid velocity, Equation 6 (Couette flow velocity profile) can be easily used at this stage.

$$V_{ly}(x_b) = \frac{U}{W}(W - L - x_b).$$

In the above expression, W and L are arbitrary geometrical dimensions (in this study W = 30 mm). In order to use the above expression the distance L from the moving wall must coincide with the experimental or simulated point of injection of the bubble. It has been mentioned previously that since the applied shear rate is equal to the velocity gradient of the above expression, it can be readily established that $\xi = \left|\frac{U}{W}\right|$. Since W is always fixed for a given problem we can adjust the plate velocity to obtain the desired value of the shear rate or vice versa (in this study $\xi = 6.2\ s^{-1}$). Since the flow field is unidirectional (only the vertical component of the liquid velocity exists). Thus we have Equations 18 and 19.

$$V_{bx} \approx V_{rx}. \tag{18}$$

and

$$V_{by}(x_b) = V_{ry} + V_{ly}(x_b). \tag{19}$$

Equation 18 indicates that in a unidirectional uniform shear flow of a liquid the bubble velocity in the horizontal direction remains constant, whereas Equation 19 indicates that the bubble velocity in the vertical direction can vary with the horizontal positions the bubble may acquire during its motion. Once the bubble velocity is known we can calculate the trajectories from Equations 20 and 21.

$$x_b = V_{bx} \times \Delta t. \tag{20}$$

And

$$y_b = V_{bx} \times \Delta t. \tag{21}$$

In Equations 20 and 21 Δt is the time step (usually of the order of 10^{-3} s or lower), and x_b and y_b are the bubble's horizontal and vertical positions, respectively.

6. Results and discussion

In this section we will present the comparison of the above model with the simulation results of Rabha and Buwa [3] along with two other lift models available in the literature i.e., the models of Dijkhuizen et al. [18] and Hibiki and Ishii [19] and the experimental results of Tomiyama et al. [4]. The comparative result in case of water is shown in Fig. 3. In Fig. 3 X^* and Y^* are the nondimensional horizontal and vertical distances respectively defined as $X^* = \frac{x_b}{w}$ and $Y^* = \frac{y_b}{w}$. It is to be noted that even though the correlation of Rodrigue [6] predicts the terminal rise velocities with an error of approximately 10%, the calculated trajectory matches well with the simulated results of Rabha and Buwa [3] (maximum deviations of 10^{-1} mm) as compared to the other two models. Here it is important to mention that the models of Dijkhuizen et al. [18] and Hibiki and Ishii [19] includes the effect of liquid viscosity and bubble deformation as well.

With the formulations adopted here (in case of water), no tuning or adjustments are required for the lift and/or drag coefficients and the procedure remains explicit and non-iterative. Hence it can be claimed that the proposed model is well suited for low viscosity liquids (in this case water for the regime of Eötvös number considered in this study). Trajectories for different bubble diameters in water using the present model are shown in Fig. 4. From Fig. 4 it can be seen that due to the presence of the liquid velocity field the trajectories of the bubbles no longer remain straight and as a bubble migrates laterally towards the stationary wall, the deviation from the straight trajectory increases. However, since their slope changes by a small amount (1.1° for 1.5 mm diameter bubble) in case of uniform shear flows the trajectories can be considered as straight lines. It can also be observed from Fig. 5 that the smallest bubble made the maximum lateral migration. This is mainly due to the characteristics of the surface tension dominated regime as it can be noticed from Fig. 2 that when the bubble diameter decreases the terminal velocity tends to increase which means that the drag coefficient decreases. Also from Equation 10, as the bubble diameter reduces the lift coefficient increases. These effects tend to increase the relative velocity of small bubbles in the horizontal direction in case of water (a low viscosity liquid). It can also be noticed that as the bubble migrates laterally closer to the stationary wall the local liquid velocity keeps on dropping which tends to reduce the bubble velocity with the lateral or horizontal positions acquired by the bubble thus causing changes in the slope of the trajectories.

By contrast, in the case of a viscous liquid, for instance the experimental liquid used by Tomiyama et al. (water–glycerol solution) [4], the correlation mentioned here was not able to give good results unless some tuning for the terminal velocity calculated via Equation 15 was performed. In such a case Equation 15 can be written as Equation 22.

$$V_T = \beta V \left[\frac{\sigma \mu_L}{\rho_L^2 d^2}\right]^{1/3}. \tag{22}$$

In Equation 22 β is the tuning parameter. The value of the tuning parameter must be adjusted using a trial and error method until the calculated terminal velocity matches the measured one. The comparison of the tuned result with the experimental result of Tomiyama et al. [4] is shown in Fig. 5. It must be noted that in this case only the value of the terminal velocity was tuned. It is to be noted that the value of the lift coefficient was not adjusted at all and was fixed as 0.288.

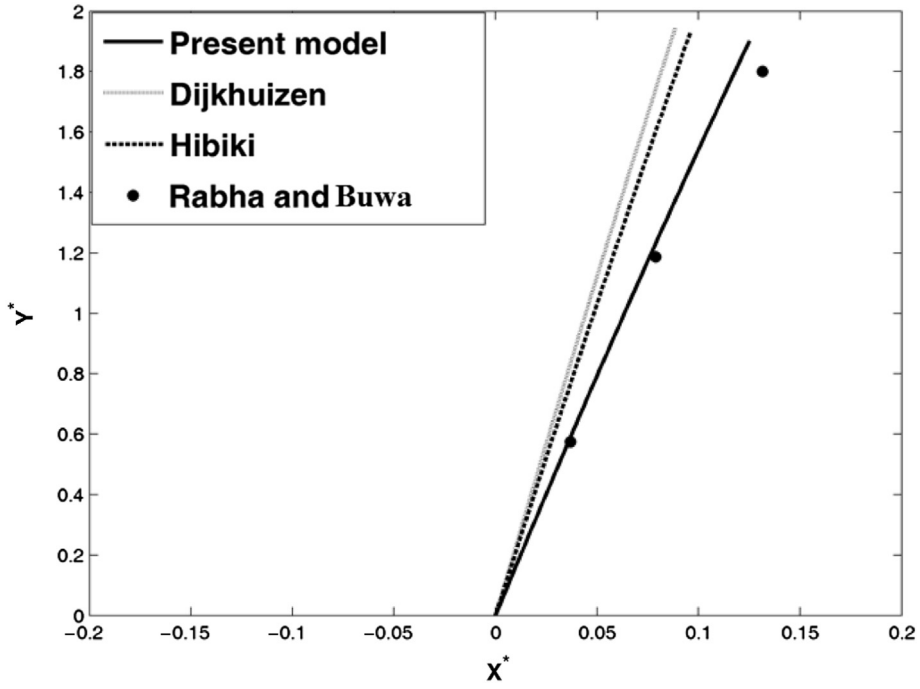


Fig. 3 – Comparison of present model with simulations of Rabha and Buwa [3] and models of Dijkhuizen et al. [18] and Hibiki and Ishii [19].

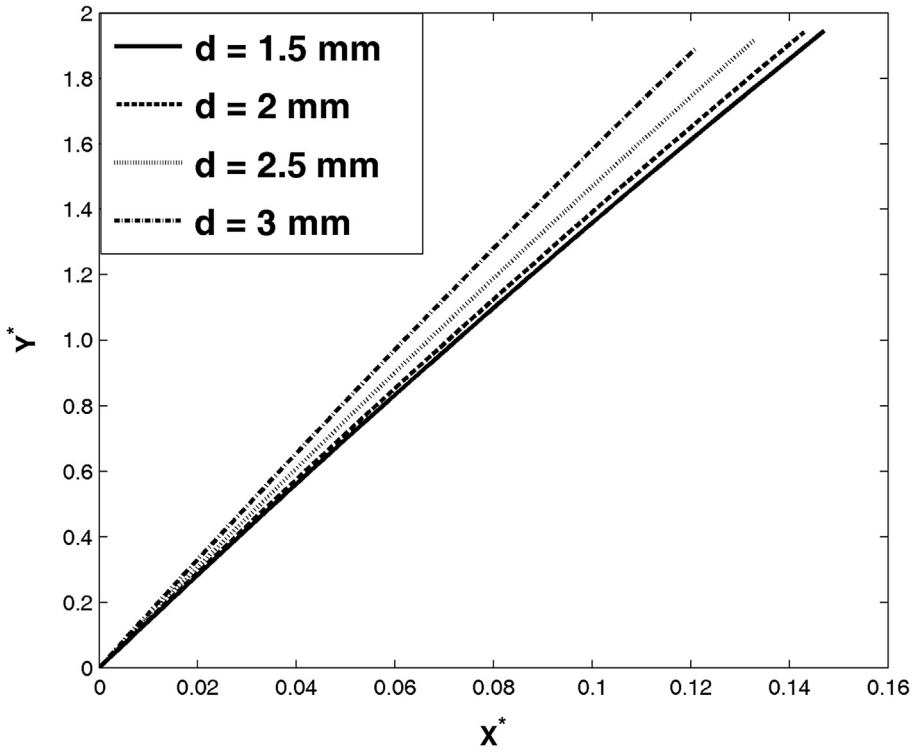


Fig. 4 – Trajectories for different bubbles in water using the present model.

In conclusion, in this paper we showed that for small spherical bubbles ($0.305 \leq Eo \leq 1.22$) in a uniform shear flow of water, a simple Couette flow velocity profile along with an explicit correlation for the free terminal rise velocity can provide satisfactory comparison without any tuning requirements. However, the procedure was not able to give appropriate results in case of high viscosity liquids and thus requires some tuning of the calculated terminal rise velocity. It was concluded on the basis of past studies and experiments that in the case of water (low-viscosity liquid) even though the diameters remain small the lift coefficient can be related to the Eötvös number mainly because the regime falls into the

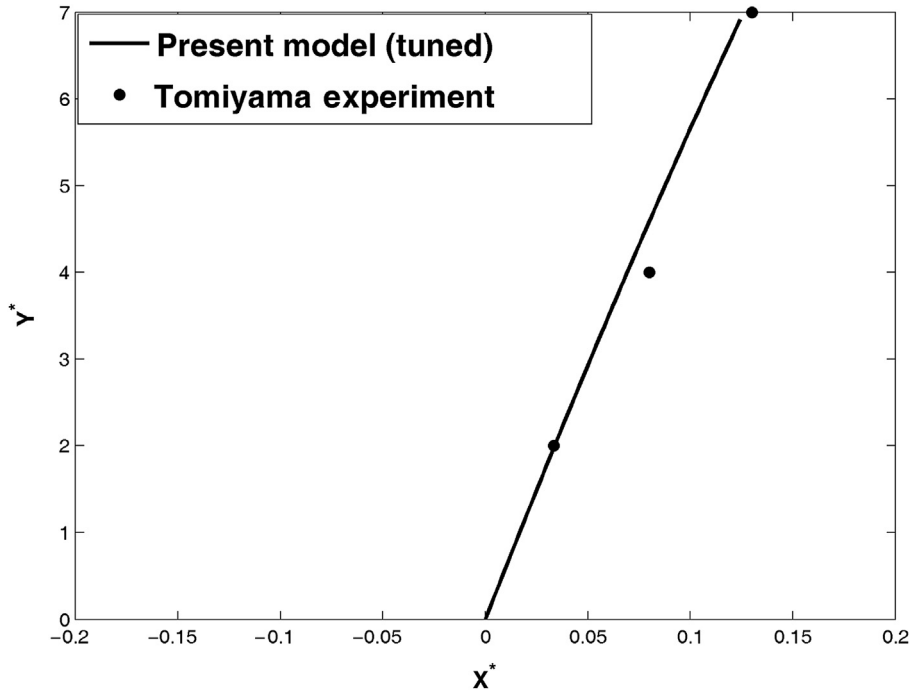


Fig. 5 – Comparison of the present model with experiments of Tomiyama et al. [4].

surface tension dominated region ($0.305 \leq Eo \leq 1.22$) and the value of the lift coefficient turns out to be higher than the typical value it would take under high viscosity liquids. A modified version of the correlation for the lift coefficient based on the Eötvös number was also proposed for water under uniform shear flow valid for $0.305 \leq Eo \leq 1.22$. This suggests that in the case of low viscosity liquids, especially water, the lift coefficient can be correlated with the Eötvös number even when the bubble diameters are < 4.4 mm. Based on these encouraging results the proposed method can be used to study the motion of small air bubbles in the shear flow of water prior to any experimental and or expensive simulations. As the procedure remains general in approach it can also be used for other combinations of gas bubbles and low viscosity liquids.

Conflicts of interest

All contributing authors declare no conflicts of interest.

Acknowledgments

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Appendix

Nomenclature

C_D	drag coefficient, dimensionless
C_L	lift coefficient, dimensionless
d	diameter of the bubble, mm
D/D_t	material derivative
Eo	Eötvös number based on bubble diameter ($= g(\rho_l - \rho_g)d^2/\sigma$), dimensionless
Eo_d	Eötvös number based on characteristic dimension of bubble along long axis ($= g(\rho_l - \rho_g)d_H^2/\sigma$), dimensionless
F	flow number ($= g(\rho_l^5 d^8/\sigma\mu_l^4)^{1/3}$), dimensionless
Fr	Froude number ($= U^2/dg$), dimensionless
g	gravitational constant (9.806 m/s^2)
L	point of injection of the bubble, horizontal distance from the moving plate or origin of the coordinate system, mm
M	Morton number ($= g(\rho_l - \rho_g)\mu_l^4/\rho_l^2\sigma^3$), dimensionless
Δt	time step, s
U	velocity of the moving plate, m/s
V	velocity number $\left(= \frac{F}{12}\left[\frac{(1+1.31\times10^{-5}M^{11/20}F^{73/33})^{21/176}}{(1+0.020F^{10/11})^{10/11}}\right]\right)$, dimensionless
V_b	bubble velocity, m/s
V_b (x_b)	bubble rise velocity at a particular horizontal or lateral bubble position, m/s
V_{bx}	horizontal component of the bubble velocity, m/s
V_{by}	vertical component of the bubble velocity, m/s
V_{by} (x_b)	Vertical component of the bubble velocity at a particular horizontal or lateral bubble position, m/s
V_l	local liquid velocity, m/s
V_{ly} (x_b)	vertical liquid flow velocity at a particular horizontal or lateral bubble position, m/s
V_r	bubble relative velocity, m/s
V_{rx}	horizontal or lateral component of the relative velocity of bubble, m/s
V_{ry}	vertical component of the relative velocity of bubble, m/s
$\lvert V_r \rvert$	magnitude of the relative velocity of bubble, m/s
V_{rx}^*	horizontal or lateral component of the relative velocity of bubble, dimensionless
V_{ry}^*	vertical component of the relative velocity of bubble, dimensionless
V_T	free terminal rise velocity of the bubble in stagnant liquids, m/s
W	width between the plates, mm
x_b	horizontal or lateral position of the bubble, mm
X^*	horizontal or lateral position of bubble, nondimensional
y_b	vertical position of the bubble, mm
Y^*	vertical position of the bubble, nondimensional

Greek letters

ρ	density, kg/m^3
μ	absolute viscosity, $Pa.s$
ξ	magnitude of the shear rate, 1/s
σ	coefficient of surface tension, N/m
β	tuning parameter

Subscripts

b	bubble
bx	bubble horizontal
by	bubble vertical
l	liquid
ly	liquid vertical
r	relative
rx	relative horizontal
ry	relative vertical
T	terminal

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