



Anisotropic regularization for sparsely sampled and noise-robust Fourier ptychography

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Abstract: Fourier ptychography (FP) is a powerful computational imaging technique that provides super-resolution and quantitative phase imaging capabilities by scanning samples in Fourier space with angle-varying illuminations. However, the image reconstruction in FP is inherently ill-posed, particularly when the measurements are noisy and have insufficient data redundancy in the Fourier space. To improve FP reconstruction in high-throughput imaging scenarios, we propose a regularized FP reconstruction algorithm utilizing anisotropic total variation (TV) and Tikhonov regularizations for the object and pupil functions, respectively. To solve this regularized FP problem, we formulate a reconstruction algorithm using the alternating direction method of multipliers and show that our approach successfully recovers high-quality images with sparsely sampled and/or noisy measurements. The results are quantitatively and qualitatively compared against various FP reconstruction algorithms to analyze the effect of regularization under harsh imaging conditions. In particular, we demonstrate the effectiveness of our method on the real experimental FP microscopy images, where the TV regularizer effectively suppresses the measurement noise while maintaining the edge information in the biological specimen and helps retrieve the correct amplitude and phase images even under insufficient sampling.

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1. Introduction

In optical microscopy, the throughput of imaging is inherently constrained by the trade-off between spatial resolution and the imaging field-of-view (FOV), which are fundamentally governed by the space-bandwidth-product (SBP) of the system. Fourier ptychography (FP) is a novel computational imaging technique that increases the SBP of an imaging system by combining multiple low-resolution images illuminated from different incident angles [1–4]. In a typical Fourier ptychographic microscope (FPM) setup, a low-magnification objective lens is used for wide-field imaging, and an LED matrix is employed as an angle-varying illumination source that increases the effective numerical aperture (NA) of the system, thereby enhancing the SBP without mechanical scanning. As each low-resolution image corresponds to different regions of the sample's Fourier spectrum, the FP reconstruction process coherently fills in the Fourier spectrum of the sample using the captured low-resolution intensity images through iterative phase retrieval. With its large SBP and phase imaging capabilities, FP has the potential to revolutionize optical microscopy, with applications including digital pathology [5,6], live cell imaging [7], and 3D refractive index imaging [8–10].

Phase retrieval in FP is an inherently ill-posed problem especially when combined with resolution synthesis, as phase information cannot be directly captured by image sensors. In

FP, the missing phase information is recovered through an iterative algorithm; the estimated object function is alternatively projected to the modulus constraint of the captured images in the image plane and the support constraint of the pupil extent in the Fourier plane. Here, the convergence of the algorithm depends on the amount of data redundancy via the amount of the Fourier space overlap between adjacent measurements, as well as the complexity of the sample and the pupil function, and the fidelity of the measurement. Usually, high-signal-to-noise-ratio (SNR) measurements with more than 50% of data redundancy in the Fourier domain are generally required for stable convergence of the algorithm [3,11,12], which acts as a bottleneck for implementing practical high-speed FP systems.

Several optimization-based phase retrieval algorithms have been proposed to improve the convergence of the FP reconstruction algorithm. Initially in Zheng et al. [1], phase retrieval based on alternating-projection was used with Nesterov acceleration to improve the convergence speed, while the noise in the dark-field measurements was suppressed via synthesizing high-dynamic-range images. Yeh et al. [13] employed the Gauss-Newton method which utilizes the second-order derivative of data fidelity to accelerate the convergence of the algorithm. Maiden et al. [14] adopted the concept of momentum to further accelerate the convergence of the ptychographic iterative engine [15]. Other optimization algorithms, including Wirtinger flow [16,17], convex relaxation [18], Anscombe transform [19], adaptive denoising [20–22], and low-rank model [23] have also been demonstrated. More recently, automatic differentiation, a technique widely used in machine learning for its remarkable flexibility in automatic gradient calculation, has been applied to FP instead of using analytic gradient equation [24]. However, when there is non-negligible amount of undersampling and noise, many of these algorithms fail to converge to a reasonable solution and produce degraded images.

The ill-posedness of the FP reconstruction problem can be mitigated by applying proper regularization based on *a priori* information of the target and the system. In FP, the sparsity prior [25,26] and plug-and-play priors [27,28] have been implemented by utilizing existing image denoising algorithms such as block-matching and 3D filtering [29]. On the other hand, total variation (TV) regularization, which anisotropically promotes sparsity in the image gradient [30–32], has been effectively used in various fields of computational imaging for its edge-preserving properties [33–35]. The amplitude and phase images of typical microscopy samples also have sparse gradients, with sharp boundaries and constant backgrounds, and thus TV regularizer is expected to be well-suited for FP. However, because the TV norm is non-differentiable, it has been a challenge to implement TV regularization with existing FP reconstruction algorithms. In addition to the object regularization, the pupil reconstruction problem also can utilize regularization, based on the prior knowledge about the pupil functions of the physical imaging systems [36,37].

In this paper, we report a regularized FP reconstruction algorithm utilizing anisotropic TV regularization for the object and the Tikhonov regularization for the pupil function, which can improve the FP reconstruction quality in challenging imaging conditions (Fig. 1(a)). To efficiently solve the TV-regularized FP problem, we implement phase retrieval using the alternating direction method of multipliers (ADMM), a method that reformulates the problem as a linear constrained problem that can be split into a sum of easily optimizable objective functions [38]. To utilize the ADMM framework with the optimization problem of FP, we derived the algorithm for the retrieval of the object function, regarding the data fidelity from the Gaussian and Poisson noise models and the TV regularizer. In addition to the object regularization, we also apply Tikhonov regularization on the pupil reconstruction problem, ensuring a smooth pupil that matches the pupil aberrations of typical imaging systems (Fig. 1(b)). We compare the results to observe the effect of our regularization between conventional FP algorithms without regularization and ours using both the simulation and the experimental data with different noise levels and Fourier-space sampling ratios. The results demonstrate that our proposed framework for phase retrieval was

more robust to noisy data and reduced Fourier-space overlap between adjacent measurements, allowing for high-throughput FP (Fig. 1(c,d)). In summary, the main contributions of this paper can be summarized as follows:

- derivation of the algorithm for TV-regularized FP reconstruction algorithm via ADMM.
- validation of Tikhonov regularization on the pupil function that ensures a smooth pupil phase profile.
- demonstration of the performance of our method with extremely low measurement SNR and Fourier-space subsampling for reduced number of measurements.
- application of our algorithm using real experimental data, which shows the effectiveness of TV-regularizer for both super-resolution and phase imaging of biological specimens under challenging imaging conditions.

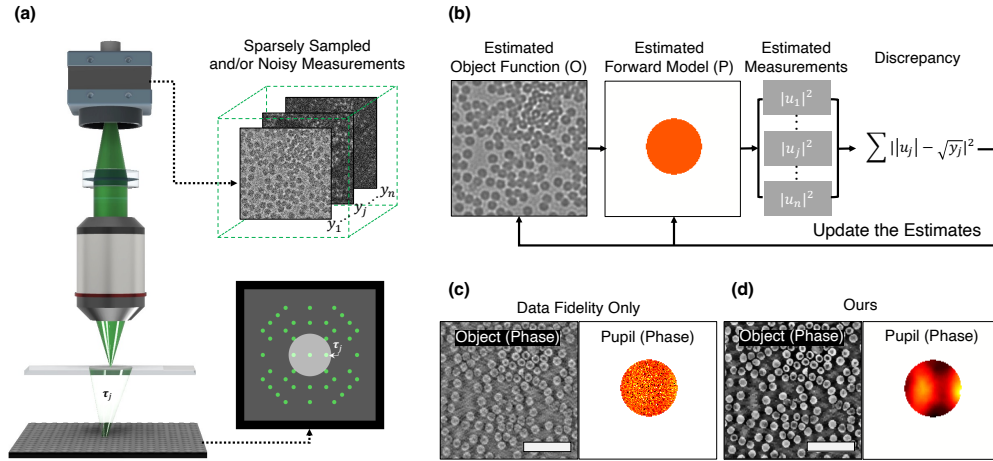


Fig. 1. Overview schematic of our work. (a) Illustration of SNR and the number of measurements in a FPM setup. (b) Reconstruction pipeline for FP, where SNR and the number of measurements mainly affect the quality of the reconstruction. $|u_j|^2$ denotes the estimated intensity from the given forward model, with the object function illuminated by the j -th wavevector (τ_j). Reconstructed phase of the object and pupil using noisy and sparsely-sampled blood smear measurements without (c) and with regularization (d). Scale bar: 50 μm .

We will report on the derivation of our optimization framework, compare the results with hyperparameter tuning with both simulated and real measurements, and discuss future direction of our approach.

2. Image model

In this section, we develop an image model for FP using multiple measurements illuminated from different angles. Based on the imaging forward model, we will derive an objective function that aims to minimize the discrepancy between observations and estimated measurements from the forward model. Here, we assume that the sample is thin enough and the imaging system has a shift-invariant pupil function. We use uppercase letters to denote sets and functions in the frequency domain and lowercase letters in the spatial domain. Let $y_j: \Omega_j \mapsto \mathbb{R}$ be a real-valued function with its spatial domain for observation associated with j -th illumination angle, and

$O: \Sigma \mapsto \mathbb{C}$ be a complex-valued function with its frequency domain for a desired reconstruction. We consider a set of observations $\{y_j\}_{j=1}^n$ associated with a set of illumination wavevectors $\{\tau_j\}_{j=1}^n$. Let $P: \Sigma \mapsto \mathbb{C}$ be a pupil function of the imaging system and we denote $u_j: \Omega_j \mapsto \mathbb{C}$, the complex field at the detector plane, as follows:

$$u_j(x) = \int_{\Sigma} O(\xi) \cdot P(\xi - \tau_j) \cdot \exp(i2\pi\xi x) d\xi, \quad (1)$$

where $x \in \Omega_j$ and $\Sigma_j \subset \Sigma$ denote a confined domain by the pupil function $P_j: \Sigma_j \mapsto \mathbb{C}$ given by $P_j(\xi) = P(\xi - \tau_j)$.

The problem of interest is to obtain a reconstruction of O from $\{y_j\}_{j=1}^n$ and by maximizing the following probability:

$$p(O|\{y_j\}_{j=1}^n, P) \propto p(\{y_j\}_{j=1}^n, P|O) \cdot p(O). \quad (2)$$

In this work, we consider both Gaussian and Poisson noise models for data fidelity from the likelihood probability, leading to the intensity and amplitude cost functions, respectively [13]. The data fidelity terms based on the intensity and the amplitude cost functions are defined as follows:

$$\sum_{j=1}^n \ell_1(O, P; y_j) = \sum_{j=1}^n \int_{\Omega_j} (y_j(x) - |u_j(x)|^2)^2 dx, \quad (3)$$

$$\sum_{j=1}^n \ell_2(O, P; y_j) = \sum_{j=1}^n \int_{\Omega_j} \left(\sqrt{y_j(x)} - |u_j(x)| \right)^2 dx. \quad (4)$$

Additionally, we consider a prior constraint imposing generic regularity on the solution of object function in the spatial domain using an anisotropic TV, which is known to be effective at preserving significant geometrical features such as edges or corners while suppressing insignificant details such as noise (Eq. (5)) [31,32,39]. Furthermore, we impose Tikhonov regularization to the pupil function in the Fourier space in order to prefer continuous and smooth solutions (Eq. (6)).

$$\ell_3(O) = \|\nabla F^* O\|_1 = \|\nabla o\|_1 := \left| \frac{\partial o}{\partial x_1} \right| + \left| \frac{\partial o}{\partial x_2} \right|, \quad (5)$$

$$\ell_4(P) = \|\nabla P\|_2^2 := \left(\frac{\partial P}{\partial x_1} \right)^2 + \left(\frac{\partial P}{\partial x_2} \right)^2, \quad (6)$$

where F denotes Fourier transform and F^* denotes its complex conjugate. o is the complex transmittance of the object in the spatial domain, whose amplitude and phase information are of interest.

We arrive at the total objective functional $\mathcal{L}$ of O and P with $\{y_j\}_{j=1}^n$ depending on the data fidelity ℓ , for either intensity (ℓ_1 , Eq. (3)) or amplitude cost function (ℓ_2 , Eq. (4)) as follows:

$$\begin{aligned} \mathcal{L}(O, P; \{y_j\}_{j=1}^n) \\ = \sum_{j=1}^n \ell(O, P; y_j) + \alpha \ell_3(O) + \beta \ell_4(P), \end{aligned} \quad (7)$$

where α and β are control parameters that determine relative weights between the terms.

3. Optimization

In this section, we present the optimization algorithm for the objective functional $\mathcal{L}$ in Eq. (7), where joint optimization is performed with respect to O and P in an alternative manner. The

desired solutions $\hat{O}$ and $\hat{P}$ are obtained through the following minimization with the cost functions:

$$\hat{O} := \arg \min_O \sum_{j=1}^n \ell(O, P; y_j) + \alpha \ell_3(O), \quad (8)$$

$$\hat{P} := \arg \min_P \sum_{j=1}^n \ell(O, P; y_j) + \beta \ell_4(P). \quad (9)$$

3.1. Optimization of object function via ADMM

We employ an optimization approach using ADMM to reconstruct the object function of FP for each unknown function in the proposed objective functional, which includes a continuous but non-differentiable TV term. ADMM decomposes the overall optimization problem into sub-problems, allowing each to be optimized separately. The ADMM approach enables us to efficiently optimize the TV term, which is not differentiable everywhere, through variable splitting and the proximal operator.

In the optimization with respect to O in Eq. (8), we apply variable splitting to form the constrained optimization problem with an auxiliary variable $V = \nabla F^* O$. The constrained objective function with the equality constraint can be reformulated using the method of Lagrange multipliers, leading to the following unconstrained optimization problem:

$$\sum_{j=1}^n \ell(O, P; y_j) + \alpha \|V\|_1 + \frac{\theta}{2} \|\nabla F^* O - V + W\|_2^2, \quad (10)$$

where $\theta > 0$ and W denotes a dual variable.

Then the alternating optimization procedure for each unknown function is given as follows:

$$O := \arg \min_O \sum_{j=1}^n \ell(O, P; y_j) + \frac{\theta}{2} \|\nabla F^* O - V + W\|_2^2, \quad (11)$$

$$V := \arg \min_V \alpha \|V\|_1 + \frac{\theta}{2} \|\nabla F^* O - V + W\|_2^2, \quad (12)$$

$$W := \arg \min_W \|\nabla F^* O - V + W\|_2^2. \quad (13)$$

In each optimization procedure, we apply a gradient descent algorithm for unknown O and V , and a gradient ascent for W . The gradient descent step for Eq. (11) yields the following:

$$O^{(t+1)} := O^{(t)} + \delta O^{(t)} + \theta F \left(-\Delta o^{(t)} + \operatorname{div} \left(V^{(t)} - W^{(t)} \right) \right), \quad (14)$$

where $o = F^* O$, div is divergence operator and t is the index for iteration. We can use $\delta O^{(t)}$ with the intensity or amplitude cost functions, as follows [13]:

$$\delta O_j^I(\xi) = \mathcal{F} \left\{ \left(u_j^* u_j - y_j \right) u_j \right\} (\xi) \cdot P^*(\xi - \tau_j), \quad (15)$$

$$\delta O_j^A(\xi) = \mathcal{F} \left\{ \left(\sqrt{u_j^* u_j} - \sqrt{y_j} \right) \frac{u_j}{|u_j|} \right\} (\xi) \cdot P^*(\xi - \tau_j). \quad (16)$$

The optimization of Eq. (12), where an L_1 -norm is involved, is achieved by solving the following equations:

$$\partial |V| + \frac{\theta}{\alpha} V = \nabla o + W, \quad (17)$$

where the gradient step of the optimization with respect to V can be obtained by the proximal operator. The solution of Eq. (17) is the form of soft-thresholding and we can get the update

function of V as follows:

$$V^{(t+1)} := \mathcal{T} \left(\nabla O^{(t+1)} + W^{(t)} \left| \frac{\alpha}{\theta} \right| \right), \quad (18)$$

where $\mathcal{T}$ denotes a soft-thresholding operator.

The dual variable W helps the balanced convergence of data fidelity and regularization and can be updated via the gradient ascent step:

$$W^{(t+1)} := W^{(t)} + \nabla F^* O^{(t+1)} - V^{(t+1)}. \quad (19)$$

3.2. Optimization of pupil function

The pupil function of the imaging system plays an important role as a complex-valued transfer function in the imaging forward model of FP. However, the exact pupil function is usually unknown, except for its Fourier extent, and needs to be jointly recovered in order to achieve the exact reconstruction of O . The pupil function can be approximated with a characteristic function χ with a radius r determined by the NA of the optical system, as follows:

$$P_j(\xi) \in \chi_j(\xi) = \begin{cases} 1 & : \|\xi - \tau_j\| \leq r \\ 0 & : \|\xi - \tau_j\| > r \end{cases}. \quad (20)$$

We adopt the concept of joint recovery of object function and pupil function, a technique also widely used in conventional FP reconstruction [40], and introduce regularization to enhance the stability of pupil function recovery. Instead of assuming the pupil as a superposition of Zernike polynomial [37], we apply a generic smoothness constraint to the solution of the pupil function by using Tikhonov regularization that aims to minimize the L_2 norm of the gradient of the complex pupil function. Since the regularization term is continuous and differentiable everywhere, the optimality condition can be obtained through a simple gradient descent step, eliminating the need for ADMM in optimization. Then, the procedure for pupil function recovery using the optimization problem in Eq. (9) can be formulated as follows:

$$P^{(t+1)} := P^{(t)} + \delta P^{(t)} - \beta \Delta P^{(t)}. \quad (21)$$

And we can use $\delta P^{(t)}$ with Eq. (22) and as an intensity cost function and Eq. (23) as an amplitude cost function as follows:

$$\delta P_j^I(\xi) = \mathcal{F} \left\{ \left(u_j^* u_j - y_j \right) u_j \right\} (\xi) \cdot O^*(\xi), \quad (22)$$

$$\delta P_j^A(\xi) = \mathcal{F} \left\{ \left(\sqrt{u_j^* u_j} - \sqrt{y_j} \right) \frac{u_j}{|u_j|} \right\} (\xi) \cdot O^*(\xi). \quad (23)$$

Here, we confine the solution space of the pupil function with a hard constraint driven by the characteristic function (Eq. (20)) during the optimization procedure of Eq. (21). Additionally, the optimization for Tikhonov regularization can be achieved by the Laplacian operator with the Neuman boundary condition.

With the derived update functions, we alternatively update the pupil function and the object function with a given set of measurements. The entire update sequence is summarized in Algorithm 1. O is initialized with the average of measurements, P is initialized with low pass filter defined by NA and V, W is initialized with random Gaussian noise. While updating the object and the pupil functions, loss from data fidelity can be updated using two methods: sequential update and global update. The choice depends on whether the updates are done for each image one at a time (sequential update) or all at once for the entire set of images (global update) in each iteration. The final reconstruction results of both methods show minimal differences, indicating that sequential procedures may suffice, thereby reducing computational requirements [13].

Algorithm 1. Regularized FP via ADMM

Input measurements ($y_j(x)$), shifted offset (τ_j), hyperparameters ($\alpha, \beta, \theta, \omega, \nu$)
Output object function (O), pupil function (P)

- 1: Initialize O, V, W, P
- 2: **repeat**
- 3: **for** $j = 1, \dots, n$ **do**
- 4: $u_j^{(t)}(x) = \int_{\Sigma} O^{(t)}(\xi) \cdot P^{(t)}(\xi - \tau_j) \cdot \exp(i2\pi\xi x) d\xi$
- 5: calculate $\delta O_j^{(t)}, \delta P_j^{(t)}$
- 6: $O^{(t)} := O^{(t)} + \delta O_j^{(t)}$
- 7: $P^{(t)} := P^{(t)} + \delta P_j^{(t)}$
- 8: **end for**
- 9: **for** $s = 1, \dots, \omega$ **do**
- 10: $O^{(t+1)} := O^{(t)} + \theta F(-\Delta o^{(t)} + \text{div}(V^{(t)} - W^{(t)}))$
- 11: $V^{(t+1)} := \mathcal{T}(\nabla O^{(t+1)} + W^{(t)} \mid \frac{\alpha}{\theta})$
- 12: $W^{(t+1)} := W^{(t)} + \nabla F^* O^{(t+1)} - V^{(t+1)}$
- 13: **end for**
- 14: **for** $r = 1, \dots, \nu$ **do**
- 15: $P^{(t+1)} := P^{(t)} - \beta \Delta P^{(t)}$
- 16: **end for**
- 17: $t := t + 1$
- 18: **until** converge

4. Experiment

In this section, we provide quantitative and qualitative evaluation of the proposed FP reconstruction algorithm under varying experimental setups. We apply our proposed FP reconstruction algorithm under two real-world imaging scenarios; with low SNR measurements and a reduced number of measurements with insufficient Fourier-space overlap, both of which have been challenging with conventional reconstruction algorithms. To validate the effectiveness of our approach, we initially conducted several simulations to compare our reconstruction result against other FP algorithms. Then, we quantitatively compare the results with and without regularization by varying the SNR and the number of measurements. Lastly, we apply our algorithm to real measurements captured with our benchtop FPM and smartphone FPM, under challenging imaging conditions. Overall, the anisotropic regularization and the pupil regularization contribute to improved reconstruction results in both simulation and real datasets, and ADMM makes the optimization more stable by effectively managing the convergence of data fidelity and regularization terms.

4.1. Comparison of FP reconstruction algorithms

To analyze the effect of regularized reconstruction in harsh imaging scenarios, we compared FP reconstruction results from various algorithms using simulated measurements. In the first scenario, we simulated FP measurements with low SNR arising from under-exposed images or low-cost cameras. We synthesized 137 raw measurements with a high level of additive white Gaussian noise, at 1dB SNR, with a factor of $4 \times$ improvement in the synthetic NA, yielding an average of 48% Fourier-space overlap (Fig. 2(a)). The second scenario mimics the use of a reduced number of measurements with insufficient Fourier-space overlap for phase retrieval. In this case, 64 simulated raw measurements were used with 33% overlap, each with 5-dB SNR. To evaluate the reconstruction performances in the above scenarios, we implemented our reconstruction algorithm described in Algorithm 1, and compared the results against several FP

reconstruction algorithms that are widely used in the field. The comparison algorithms includes Gerchberg-Saxton (GS) [40], Gauss-Newton (GN) [13], Wirtinger Flow (WF) [41], automatic differentiation (AD) [24], and the AD with the addition of TV-loss for regularization. All of the amplitude and phase reconstruction results from various algorithms have been quantitatively compared with peak signal-to-noise ratio (PSNR) performance.

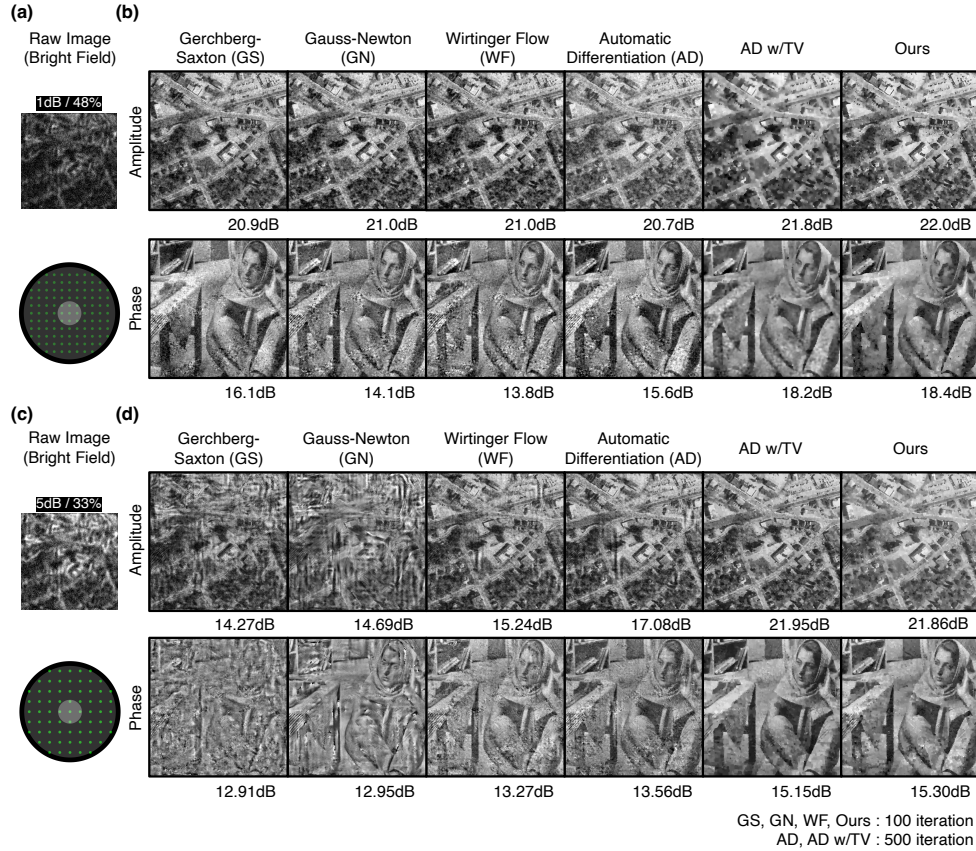


Fig. 2. Comparison of reconstruction results using various algorithms. (a, c) Raw images with plane wave illumination and illumination and SNR setting used for simulating each measurement. (b) Reconstruction results using 137 measurements at 1dB SNR (AD w/TV - $\lambda : 1e-1$, ours - $\lambda : 5e-3 / \theta : 1e-1$) (d) Reconstruction results using 69 measurements at 5dB SNR (AD w/TV - $\lambda : 5e-2$, ours - $\lambda : 5e-3 / \theta : 1e-1$)

Figure 2(b) and (d) show the reconstructed amplitude and phase with various algorithms along with the corresponding PSNR scores. In the case of noisy measurements (Fig. 2(b)), the measurement noise still prevails in the reconstruction results using GS, GN, WF and AD. When using TV regularization, the reconstruction results are effectively denoised while maintaining the edge of the image features, achieving improved PSNR scores for both amplitude and phase reconstruction. Between the two methods using TV regularizers, AD and ADMM (Ours), the reconstruction quality appears similar. However, AD-based updating methods inherently require more extensive iterative calculation (500 iterations in our experiments), which restricts their practical usage and demands a harsher computational environment along with time-consuming hyperparameter tuning. In contrast, our method using ADMM converges with significantly less number of iterations with 100 in our experiments. In cases of low-overlap measurements

(Fig. 2(d)), reconstruction qualities are generally lower when compared to the case of noisy measurements, especially for the phase with the GS, GN, WF, and AD (Fig. 2(d)). This is because the reconstruction algorithm fails to retrieve the correct phase, which leads to lower PSNR scores for both amplitude and phase. However, as in previous cases, our TV regularized FP with ADMM proves beneficial for achieving stable convergence with fewer iterations while preserving edge features within images, resulting in the highest PSNR among the algorithms.

4.2. FP reconstruction with low SNR or less numbers of measurements

Then, to evaluate extent of sub-sampling and the noise level in the measurement, for which TV-regularization is effective, we have quantitatively compared the reconstruction accuracy with and without regularization under varying SNR and sampling rates. Here, the performance of amplitude and phase reconstruction is compared separately, using an error metric as follows [13].

$$\text{Relative Error} = \frac{\|\epsilon_{\text{recover}} - \epsilon_{\text{ground truth}}\|^2}{\|\epsilon_{\text{ground truth}}\|^2}, \quad \epsilon = |O| \text{ or } \angle O \quad (24)$$

We first varied the noise level of the simulated measurements from 21 to 1 dB, while utilizing a sufficient number of measurements of 109 images, which corresponds to 46 % Fourier-space overlap in our settings. We also assumed that the pupil function of the imaging system is known, allowing us to compare the reconstruction results more accurately with respect to noise level. Figure 3(a) shows the reconstruction results using our ADMM-based FP with and without regularization, illustrating that TV regularizer produces visibly better results in low SNR measurements. The relative errors for amplitude and phase reconstructions with respect to the measurement SNR gradually increase with the noise level (Fig. 3(b)). The relative error is particularly higher for phase reconstruction, because with high levels of random noise, the raw measurements lose consistency from adjacent measurements, and the phase retrieval begins to fail.

Secondly, we varied the overlap ratios of raw measurements, ranging from 55% to 22%, while maintaining the SNR of the raw measurements at 15 dB. It is well-known that the conventional FP algorithms require sufficient constraints in the Fourier domain in order to converge, typically above 50%, although the amount depends on the noise level and the complexity of the sample [11,12]. The reconstruction results shown in Fig. 3(c) reveal significant degradation in both the amplitude and phase reconstruction under low-overlap-ratios when object regularization is not used. Artifacts in the amplitude reconstruction without TV regularization are due to errors in phase estimation below 33% overlap. On the other hand, TV-regularization effectively recovers both the amplitude and phase information of the object, aiding convergence at lower overlap ratios. The plot of relative error in Fig. 3(d) more clearly visualizes the effect of regularization, which shows that fewer constraints within FP reconstruction result in severe phase reconstruction artifacts, thereby causing high relative errors of both amplitude and phase when the overlap is below 40%.

4.3. Regularized pupil reconstruction with low SNR measurements

The pupil function represents an optical aberration of the imaging system, which, if uncorrected, degrades the achievable resolution in FP. While the joint recovery technique [40] has been effectively applied to recover the pupil function in FP, the recovered pupils may be contaminated with noise if used with noisy measurements. To improve the pupil recovery under noisy measurements, we apply two constraints to the solution of pupil function with data fidelity of pupil; hard constraints on the solution space defined by NA and Tikhonov regularization which minimize the L_2 norm of the pupil's gradient to ensure the smoothness of the pupil function within the passband. By minimizing the gradient of the pupil within the system's passband, we

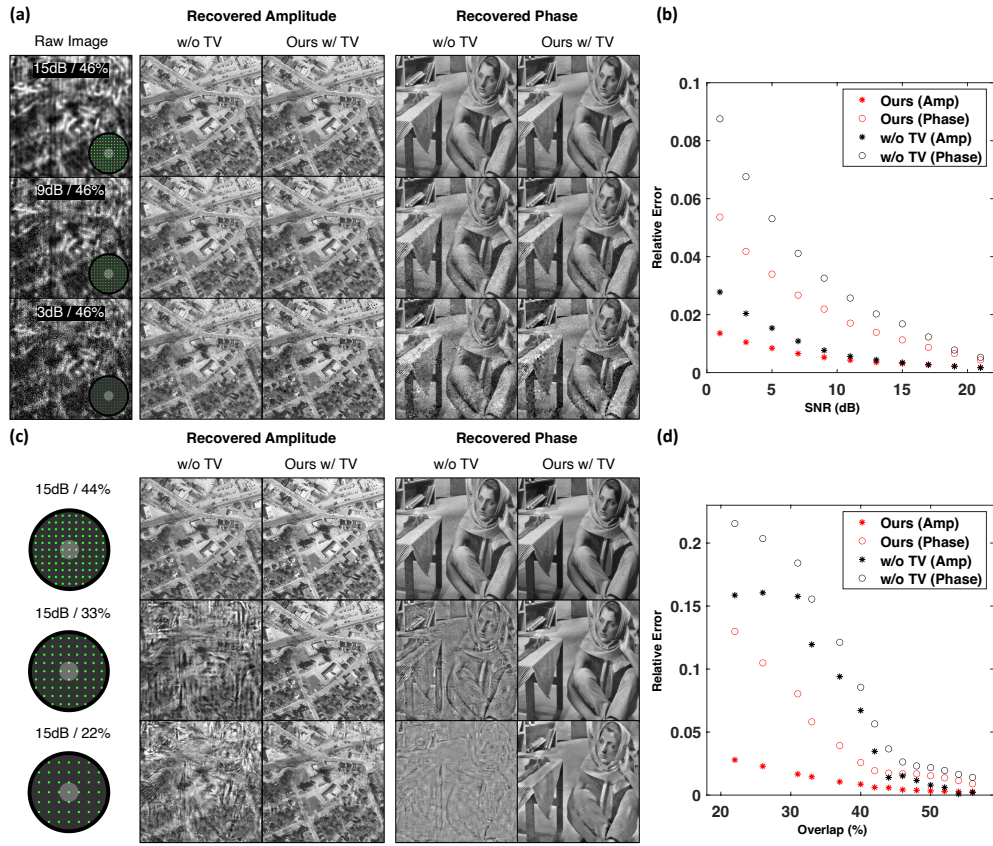


Fig. 3. Performance comparison with and without TV regularization using ADMM FP. (a) Comparison of recovered amplitude and phase with respect to the SNR of the measurements. The subset within the raw image indicates the illumination and SNR used for each simulation. (b) Comparison of relative error across different SNR levels. (c) Comparison of recovered amplitude and phase with respect to the numbers of measurements. (d) Comparison of relative error across different number of measurement.

can obtain a smooth phase profile of the pupil function even with the noisy measurements, which better match the phase aberration of physical pupils in imaging systems.

To validate this, we simulated 5 dB SNR measurements with aberrated pupils and compared the recovered pupil phase with and without pupil regularization. Figure 4 shows the recovered pupil phase for three different types of aberrations; defocus aberration (Fig. 4(a)), a combination of several major aberrations (Fig. 4(b)), and a complex aberration composed of 50 random Zernike polynomials (Fig. 4(c)), which were recovered with the complete knowledge of the object function. Without regularization, all pupil functions are heavily affected by measurement noise, while our algorithm with regularization recovers denoised pupil functions with smooth profiles. The recovered pupil was more accurate with the regularization, in that the regularized pupil fits the Zernike coefficients with higher accuracy (Fig. 4(d)). The mean squared error between the ground truth and the fitted magnitude of the Zernike coefficients was 0.04 and 0.2, with and without regularization, respectively, demonstrating the effectiveness of our approach.

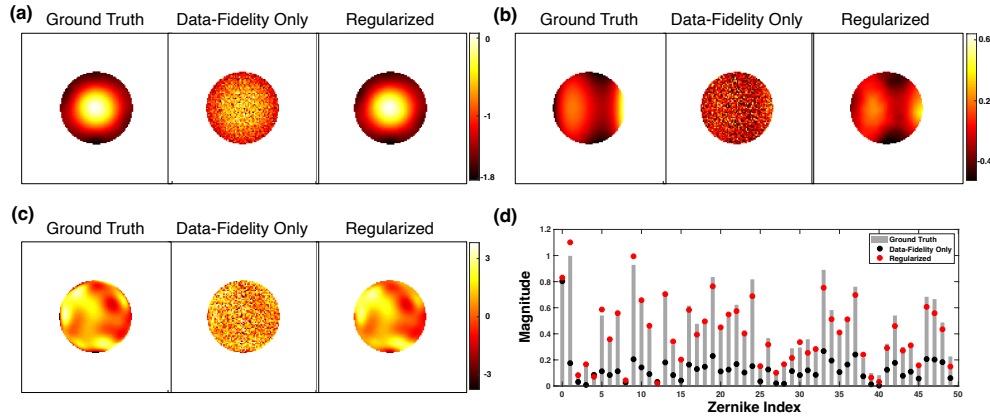


Fig. 4. Performance comparison of pupil recovery. Recovered pupil phase with (a) defocus aberration, (b) a combination of several major aberrations (defocus, spherical, coma, astigmatism, and trefoil), and (c) complex aberrations of 50 random Zernike polynomials using data fidelity only and our method with Tikhonov regularization. Pupils in a row share the same color map. (d) Fitting results with Zernike polynomial coefficient of the recovered pupils using pupil reconstruction with data fidelity only and our method.

4.4. Regularized reconstruction with real measurements

Finally, we applied our algorithm to real FP measurements from physical FP setups, particularly to noisy and sub-sampled datasets which usually yield low reconstruction quality with conventional algorithms. Noisy measurements arise from under-exposed images and/or low-cost cameras, and the Fourier-space sub-sampling yields a reduced number of measurements, which are critical factors in improving the speed and the cost-effectiveness of FP. In this context, we qualitatively compared the reconstruction quality with and without regularization to assess the effect of using low SNR and a reduced number of measurements separately and compared the reconstruction algorithm more broadly with the sparsely-sampled and noisy blood smear measurements.

To begin with, we captured images of the USAF resolution target to observe the resolution enhancement with noisy measurements (see Appendix 6.6 for detailed hardware settings). In this dataset, we used 15 $\times$ shorter camera exposure time compared to when utilizing the full bit depth of the image sensor (Fig. 5(b)), to simulate fast FP scanning. Here, we ensured sufficient overlap between adjacent measurements (70%) in the Fourier domain to better analyze the effect of measurement noise (Fig. 5(a)). In Fig. 5(c) and (d), we can observe the reconstruction outcomes using our algorithm without and with TV, respectively. Both approaches result in an enhancement of the optical resolution compared to the raw measurement shown in Fig. 5(b). However, our algorithm with anisotropic TV regularization performs better in terms of suppressing background noise and improving the sharpness of the features. Here, the features in group 9 element 2 of 870 nm can be resolved which is equivalent to a synthetic NA of 0.4. This result suggests that we can reduce the entire capture time by a factor of 15 while minimizing the degradation in reconstruction caused by measurement noise to achieve high-throughput scanning in practical FP systems.

Then, we compare the reconstruction results obtained with a smaller number of measurements, corresponding to a 40% Fourier-space overlap between measurements. 41 raw images of a histopathology slide were captured with our benchtop FP microscope with sufficient exposure, which fills the synthetic NA of 0.4. The sub-sampling pattern of the LED array is shown in Fig. 5(e), which uses 3.5 $\times$ fewer LEDs than in Fig. 5(a). In Fig. 5(f) and (g), the reconstruction without TV regularization failed to retrieve the correct phase profile of the object as well as

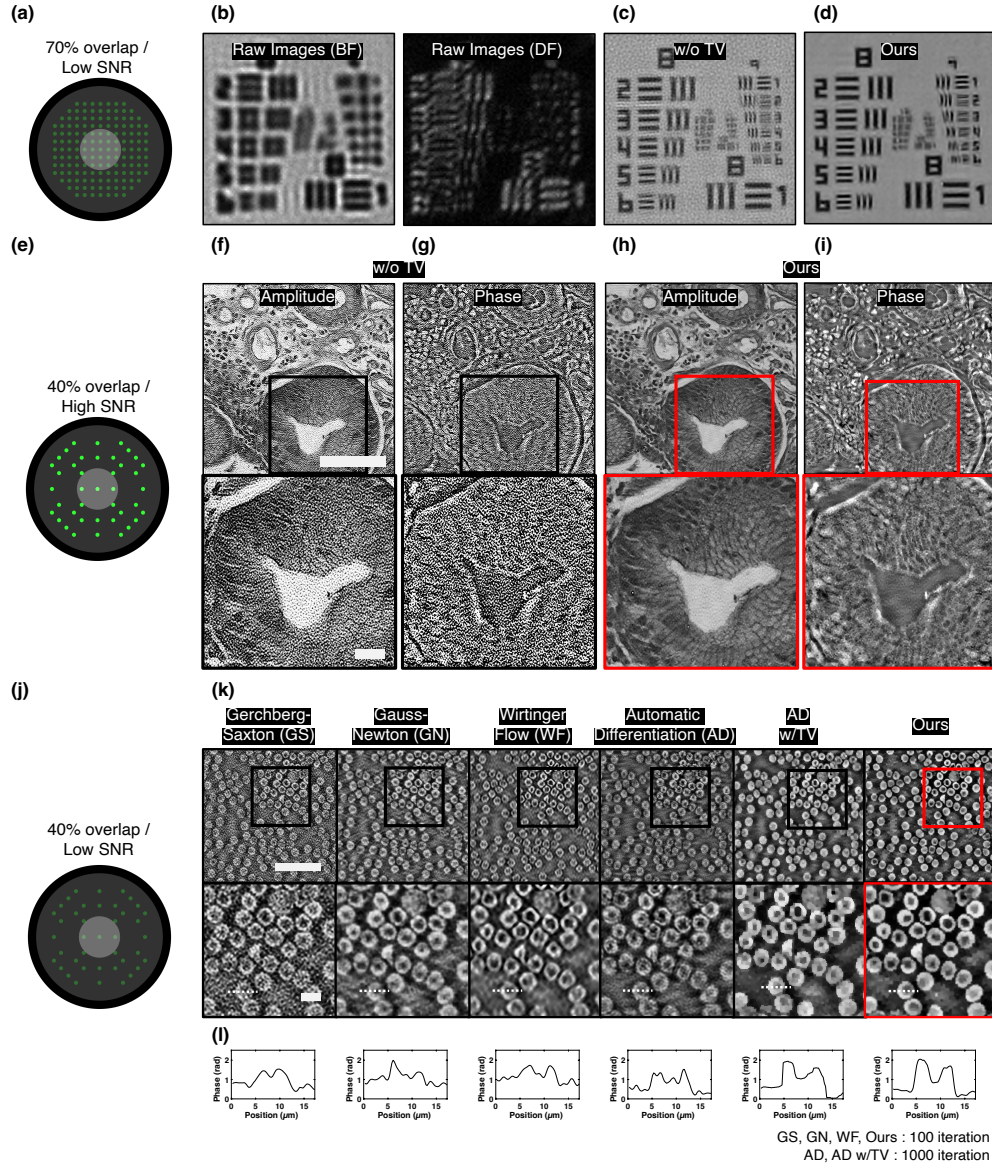


Fig. 5. Performance comparison with real experiment. (a, e, j) The illumination and corresponding SNR used for the experiment with (a) USAF resolution target, (e) pathology slide, and (j) unstained blood smear. (b) Raw image of USAF resolution target with bright-field (BF, left) and dark-field (DF, right) illumination. Reconstructed amplitude without (c) and with regularization (d) ($\lambda : 5\text{e-}3 / \theta : 1\text{e-}3$). Reconstructed amplitude (f) and phase (g) without regularization and reconstructed amplitude (h) and phase (i) with regularization ($\lambda : 5\text{e-}4 / \theta : 1\text{e-}3$). (k) Recovered phase with various reconstruction algorithms (AD w/TV - $\lambda : 1\text{e+}1$, ours - $\lambda : 1\text{e-}3 / \theta : 1\text{e-}3$). (l) Phase profile of dotted line in (k). Scale bar: 50 μm for unmagnified images and 10 μm for magnified images.

its high-resolution amplitude image. In contrast, reconstruction with regularization achieved decent amplitude and phase images of pathology samples, with sufficient denoising and improved feature contrast, resolving the boundaries between cells, as shown in Fig. 5(h) and (i).

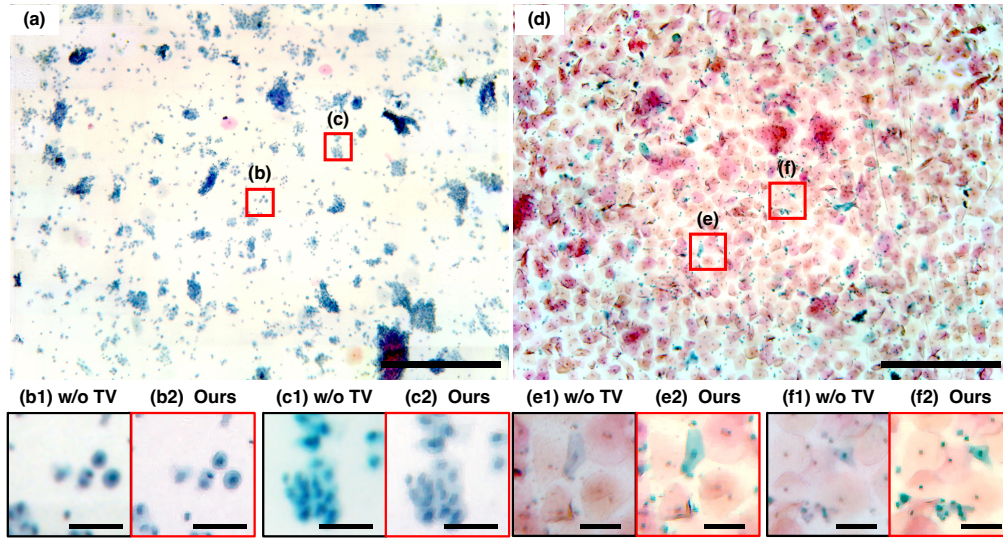


Fig. 6. The reconstructed color images of the Pap smear slides with the parabasal cells stained in blue (a-c) and the superficial cells stained in red (d-f) using smartphone FPM. Magnified results without TV regularization (b1, c1, e1, f1) and with TV regularization (b2, c2, e2, f2) ($\lambda : 3\text{e-}3 / \theta : 5\text{e-}3$ for parabasal cells and $\lambda : 1\text{e-}3 / \theta : 5\text{e-}3$ for superficial cells). Scale bar: 500 μm (a, d) and 50 μm (b, c, e, f).

Furthermore, we compare the phase recovery results of unstained blood smear under the use of low SNR and sparsely-sampled measurements (Fig. 5(j, k)). We used 41 raw measurements of blood smear slide, captured with insufficient camera exposure. Consistent with simulation results, phase reconstruction using GS, GN, WF, and AD exhibits severe reconstruction artifacts and noise (Fig. 5(k)), resulting in the loss of disc shaped features in the phase profile of the red blood cells (Fig. 5(l)). In contrast, our method presents images with clear cell boundaries and correct phase profiles, showing reduced noise in the background and requiring significantly fewer iterations when compared to AD w/TV (Fig. 5(k, l)).

Lastly, we apply our regularized reconstruction algorithm to the dataset captured by our smartphone FPM [42]. In this dataset, we utilized the smartphone's display screen as programmable illumination and captured raw images using the smartphone's built-in camera module, which results in low SNR measurements. As a result, the reconstructed images were noisy and blurry compared to typical benchtop FP images. For both Pap smear sample with sparsely distributed parabasal cells and superficial cells, our method with regularization shows sharper cell boundary edges (Fig. 6(b2, c2, e2, f2)), showing that our regularized FP reconstruction improves the image quality in various types of real FP datasets. While we reconstruct and apply TV regularization independently for each red, green, and blue channel, the results may further improve by extending the regularization to the color dimension.

5. Conclusion

In this work, we proposed an anisotropic regularized FP reconstruction algorithm via ADMM under harsh imaging condition of using sparsely and noisy measurements. We applied two different regularizations to the solution of the optimization problem: TV for the object function and Tikhonov regularization for the pupil function, considering the general properties of the microscopic objects and pupil functions as *a priori* information. Our experimental results, obtained from synthetic data under realistic imaging conditions, demonstrated the effectiveness

of the proposed objective functional with ADMM optimization when compared to the diverse FP reconstruction algorithm including GS, GN, WF, and AD. We also introduce the application of weighted step size in the gradient descent of the intensity cost function, considering implicit weights coming from the assumptions in the noise model. Furthermore, we apply our algorithm to real measurements captured with real FPM setups, demonstrating its ability to effectively suppress noise in low-SNR and low-overlap-ratio scenarios. Our findings suggest that our algorithm allows for the reconstruction of the object function using shorter exposure times and fewer measurements, achieving fast FP imaging with cost-effective hardware, which can be advantageous for many real-world applications such as diagnostic imaging, high-throughput sample screening, and imaging of dynamic objects.

While we have validated the effectiveness of our approach in two scenarios of FP, there are several potential avenues for further discussion. Firstly, as real-space ptychography and FP share a common algorithmic ground [43], we can further extend the application of our approach to more diverse imaging scenarios of both ptychography and FP where the reconstruction may yield noisy and/or faulty results, such as cases involving misalignment of illumination, aliasing in the detection, and variations in the coherence of illumination source [11,12,44,45]. Secondly, we can investigate on different types of regularization for the object and the pupil, such as different types and amounts of regularization for object's amplitude and phase. For instance, we anticipate that our algorithm can leverage the Kramer-Kronig relation [46,47] as a regularization for object updates, which exploits the physical relationship between the real and imaginary components of the refractive index distribution in the object. Lastly, our algorithm can be further extended by incorporating automated tuning of regularization parameters [48] or by using a learned ADMM approach to determine the optimal parameters for object reconstruction [49] for faster convergence.

Appendix

A.1. Data fidelity

We can derive the data fidelity term from the likelihood probability in Eq. (2). We assume the conditional independence of y_j, P given O leading to the following probability:

$$p(\{y_j, P\}_{j=1}^n | O) = \prod_{j=1}^n p(y_j | P, O) p(O | P) p(P), \quad (25)$$

where the probabilities $p(y_j | P, O)$, $p(O | P)$ and $p(P)$ are derived based on the image model and the optical properties of the imaging system.

We consider an additive noise model for an observation y_j given a local reconstruction u_j as follows:

$$y_j = |u_j|^2 + \eta_1. \quad (26)$$

We can use various noise models, such as Gaussian and Poisson noise, to model η_1 . Each noise model results in a different cost function, where the intensity cost function is used for Gaussian noise and the amplitude cost function is used for Poisson noise [13].

A.2. Weighted step size in gradient descent algorithm

Mathematically, as the standard deviation of the noise is inversely proportional to the weights in the cost function, the cost function implicitly incorporates different weights depending on the amount of noise in each measurement. However, because the loss from the bright field images is much larger than that from the dark field images, bright field images contribute more to the total cost function values given that weight is equally applied to each measurement. In other words, the intensity cost function from the Gaussian noise model, which assumes that all measurements

have the same standard deviation of the noise, places more emphasis on low spatial frequencies which induces the resulting images to lose their high spatial frequency features within images as well as in the Fourier domain [13]. On the contrary, the Poisson noise model assumes that the brighter images contain more noise, and the amplitude cost function can effectively incorporate the loss from the dark field images and be used to reconstruct the high spatial frequency of the sample. Alternatively, to compensate for these implicit weights in the cost function, we use a weighted step size for each measurement. We achieve this by utilizing the ratio of the total power of each measurement to the measurement with the center illumination as a weight for each measurement ($w_j = \sum_x y_0(x) / \sum_x y_j(x)$, $y_0(x)$ is the measurement with the center illumination). This approach enables the entire synthetic Fourier space to be updated as same as the amplitude cost function.

Figure 7 shows the comparison between the use of amplitude cost function, intensity cost function, and intensity cost function with our weighted step size scheme. These comparisons are based on the measurements of stomach cancer histology slides with high SNR and a high overlap ratio of 70%. In the resulting Fourier spectrum shown in Fig. 7(b), (d), and (f), it is evident that the intensity cost function without weighted step size contains less high-frequency information within its domain. Consequently, the boundaries of nuclei and the fiber structures in connective tissue are less clear and visible in Fig. 7(c). In contrast, when we utilize our weighted step size, we can successfully recover high-resolution features in both the resulting amplitude and phase, as illustrated in Fig. 7(e). However, the reconstruction result with the amplitude cost function is slightly more effective with respect to suppressing background noise of real measurements, we choose to use the amplitude cost function for the rest of the results.

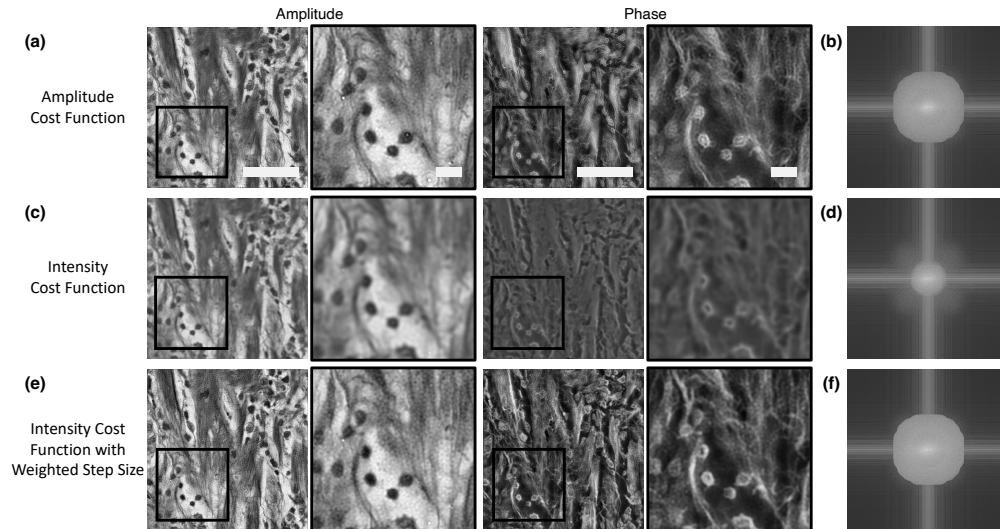


Fig. 7. Recovered amplitude (left) and phase (right) of stomach cancer histology slides with (a) amplitude cost function, (c) intensity cost function, and (e) intensity cost function with weighted step size. Corresponding magnified images are located on the right. The spatial frequency of recovered object in Fourier domain with (b) amplitude cost function, (d) intensity cost function, and (f) intensity cost function with weighted step size. Scale bar: 50 μm for unmagnified images and 10 μm for magnified images.

A.3. Regularization

The image in Fig. 8 illustrates the impact of anisotropic TV regularization with our algorithm. By leveraging prior knowledge of the image, the regularization aids in converging with the sub-sampled measurements of an overlap ratio of 33% in the Fourier domain (Fig. 8(b)). However, if the optimization process applies excessive regularization, the resulting image may lose its sharpness and flatten out its features (Fig. 8(c)).

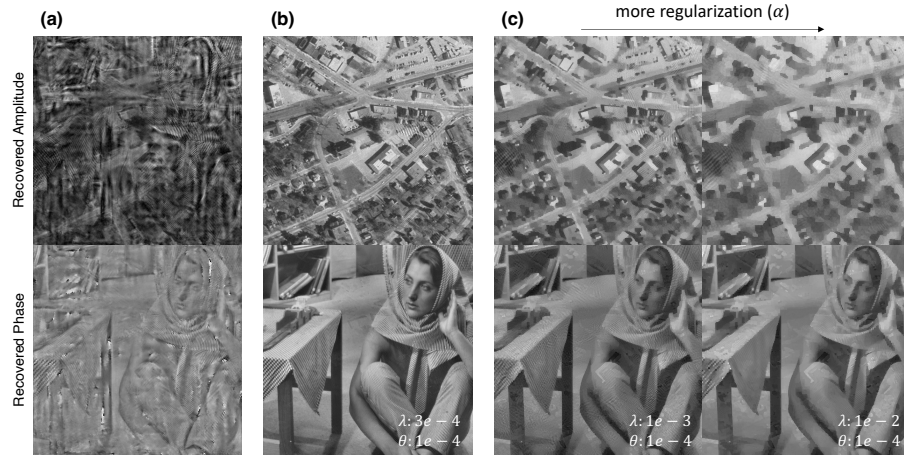


Fig. 8. Reconstruction results with varying regularizing hyperparameter. (a) Results with no regularization. (b) Results with proper regularization. (c) Results with the increase of regularization parameter from (b).

A.4. Implementation of algorithm for comparison

In our study, we have implemented algorithms that have been previously reported for the comparison, including Gerchberg-Saxton (GS) [40], Gauss-Newton (GN) [13], Wirtinger Flow (WF) [41], automatic differentiation (AD) [24], AD with TV. For fair comparison, all simulation parameters were standardized, including initialization methods and the initial step size. We utilized MATLAB to implement GS, GN, WF, and ours and the Pytorch library to implement automatic differentiation with stochastic gradient descent for optimizer. Our FP reconstruction algorithm using anisotropic regularization is available at [50]. Additionally, since both the amplitude cost function and the intensity cost function with the use of a weighted step size can be successfully used for data fidelity with comparable image quality, we opted for the amplitude cost function for simplicity. Additionally, to ensure effective regularization, we iterated the reconstruction using various combinations of hyperparameters, including the step size for data fidelity update, the penalty (θ) and the regularization parameters(α, β), and the number of sub-iterations (ω, ν) to accelerate the convergence of the regularization and dual. Then, we carefully select the combination of hyperparameters that yields the best reconstruction in terms of peak signal-to-noise ratio (PSNR). Additionally, we monitor the loss of data fidelity, regularization, and dual to estimate the algorithm's performance (see Fig. 8 to observe how the regularization parameter α affects the results). Computation time of GS, GN, WF, ours, AD is 8 s, 8 s, 8 s, 10 s, 100 s with 100 iteration for GS, GN, WF, and ours and 500 iteration for AD with a commercialized benchtop computer (AMD Ryzen 9 5900X 12-Core Processor 3.70 GHz, NVIDIA GeForce RTX 3060).

A.5. Simulation setup

To make a quantitative comparison between our algorithm with TV and various reconstruction algorithms, we generated a simulation that models FPM. The simulation employs a 4x magnification lens with a 0.1 NA, a standard 180 mm tube lens setup, and an LED-to-sample plane distance of 80 mm. Moreover, we assumed the use of consumer-grade CMOS cameras with a 2.4 μm pixel size and 12-bit depth. To ensure consistency, we assumed that all measurements were taken with the same exposure times, causing even lower SNR in the dark field measurements due to their lower signal level. For simulating SNR of the measurements, we added Poisson and Gaussian noise to the measurements, and the SNR level we described was based on the brightest images across the entire measurement set. For the experiment that involved varying numbers of measurements, we kept the synthetic NA of the FP constant and altered the LED pitch to regulate the number of measurements.

A.6. Benchtop FPM setup

Our benchtop FPM is composed of a 4x objective lens with 0.13 NA, a green LED array of sample to LED distance of 80 mm and 4 mm LED pitch, and a 12-bit and 2.4 μm CMOS camera. The overall magnification is 3.33 $\times$ which is equivalent to more than 4 $\times$ Nyquist sampling rate of the given effective pixel size. We selected a suitable step size and number of iterations and fine-tuned the hyperparameters of regularization, number of regularization sub-iterations, and penalty parameters by observing the loss graph of all optimizing terms.

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Data availability. The simulation code is available at [50]. Data may be obtained from the authors upon reasonable request.

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