



Emergency response vehicle routing allowing lane straddling in congested traffic conditions under connected and autonomous vehicle environment

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Abstract

Routing an emergency response vehicle (ERV) is essential to sustain the society and save people in high risk. ERV must have top priority on highways to pass through without interference with other non-ERVs. Congested road traffic conditions, however, might not allow for such a priority. When non-ERVs fully occupy all lanes in a road segment, in principle, an ERV cannot pass through the segment until the congestion releases. On the other hand, the situation is often observed in the field that non-ERVs sidestep to make way for an ERV, and the ERV straddles lanes and runs along a lane delineation line. The present study devised an optimization model to allow such a situation. Whereas most ERV routing problems have been formulated for uncongested traffic conditions, the present study presents a robust formulation that can work for both congested and uncongested traffic conditions. The connected and autonomous vehicle (CAV) environment is prerequisite for the present modeling framework. The proposed optimization model is robust to derive an optimal trajectory for ERV in road segments of diverse traffic and geometric conditions.

1 | INTRODUCTION

Ensuring the optimal passage of emergency response vehicle (ERV) is crucial to saving lives at risk and sustaining a safe society. Prioritizing the ERV movement can be broken down into two categories. First, the preemption of traffic signal for ERV at an intersection is relatively easier than optimizing an ERV's route through a road segment. The signal grants the right of way to an approaching ERV, allowing it to pass through the intersection without stopping. Many researchers have studied signal preemption for ERVs (Nellore & Hancke, 2016; Cao et al., 2022) and evidenced considerable enhancement of ERV passage (Wang et al., 2013; Li et al., 2018). Vehicle-to-vehicle (V2V) and

vehicle-to-everything (V2X) technologies are inevitable for the traffic signal preemption for ERVs.

Second, the facilitation of ERV movement in a road stretch has been also dealt with under the assumption that vehicles are fully connected and controlled. Various approaches to optimizing ERV movement in road segments can be found in academia, ranging from trajectory-based models (Sarkar et al., 2022; Hannoun et al., 2019) to station-level routing strategies (Saha & Hajibabai, 2018). It is observed that drivers yield to ERVs on the road, but such voluntary cooperation is not enough to secure the safe passage of ERV. Researchers have attempted to optimize the trajectory of ERV, thereby non-ERVs sidestep to make way for the safe passage of the ERV. However, it is not the case

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FIGURE 1 Non-ERVs making way for ERVs in congested traffic conditions.

in a severely congested traffic condition where non-ERVs occupy all lanes laterally in a road stretch. The present study proposes a methodology to find a solution for such a situation.

It is frequently observed that an ERV runs straddling lanes, while non-ERVs sidestep laterally on a voluntary basis, as shown in Figure 1. The present study concentrates on this situation where traffic jams hinder an ERV's passage along a normal lane. To the best of our knowledge, optimizing an ERV's trajectory allowing for the lane straddling has not yet been found in literature. For this consideration non-ERVs' maneuvers are assumed to be fully controllable in a centralized manner under the connected and autonomous vehicle (CAV) environment. The present study sets up a novel formulation to optimize both an ERV's trajectory and non-ERVs' yielding behavior on a congested road segment, allowing vehicles to position themselves along shoulders or lane delineation lines.

The proposed problem belongs to the category of integer linear programming (ILP). The initial stage, where the location and speed of vehicles are given, should be input for the problem. The proposed ILP is set up for the treatment of a single ERV entering a road segment and then extended to consider multiple ERVs at the same time. Based on an initial stage, maximizing an ERV's cumulative speed along its trajectory determines the optimal trajectory of ERV and final yielding locations of non-ERVs at the final stage. It should be noted that the present study in this set-up does not provide a time-varying solution, thereby regarding the problem as a static optimization problem. More concretely, on the entry of an ERV, to make way for the ERV, each non-ERV must stop in an optimal yield-

ing position within its possible stopping distance without infringing positioning constraints.

Transitioning from the initial to final stages does not occur instantaneously in field applications. Since road segments are consecutively linked, deriving a solution for the next segment and transitioning non-ERVs according to the solution can be done while an ERV is passing the previous road segment. This preemptive implementation makes the static method to act as a dynamic application.

Such a static problem was first proposed by Hannoun et al. (2019), and the present study extends their model to cover the lane straddling for congested traffic conditions. The proposed optimization model is applicable to both congested and uncongested traffic conditions. Figure 2 illustrates three examples of the initial and final stages in order of congestion level. The final stage in the bottom shows that an ERV runs between lanes to overcome vehicle blocking. For this case, other previous models that only allow driving along normal lanes cannot lead to a feasible solution at this congested level.

The proposed ILP is tested against diverse cases altering road geometric and traffic conditions. The experiments show that the proposed model can derive the optimal routing of ERV in severely congested situations, which have never been provided so far in academia. In addition, through sensitivity analysis, the present study provides guidance on how to apply the proposed model to various conditions. The sensitivity analysis with respect to parameters of the model is also conducted. Several parameters turn out to have a great influence on the property of solutions, which will be helpful for further researchers.

This paper is organized as follows. The next section introduces related works and describes some previous studies while focusing on the context and history of ERV routing studies. The third section presents a modeling framework that encompasses the formulation. How to synthesize a testbed and set up scenarios is explained in the fourth section. The fifth section presents the results from applying the proposed model into diverse scenarios established in the previous section. The results from the sensitivity analysis are also discussed in this section. The last section draws conclusions and discusses further extensions.

2 | RELATED WORK

2.1 | Outline of previous studies

As technologies in the intelligent transport system (ITS) advance, various innovative ideas have emerged such as incidence detection and work zone traffic management (Adeli & Karim, 2005; Hooshdar & Adeli, 2004; Jang &

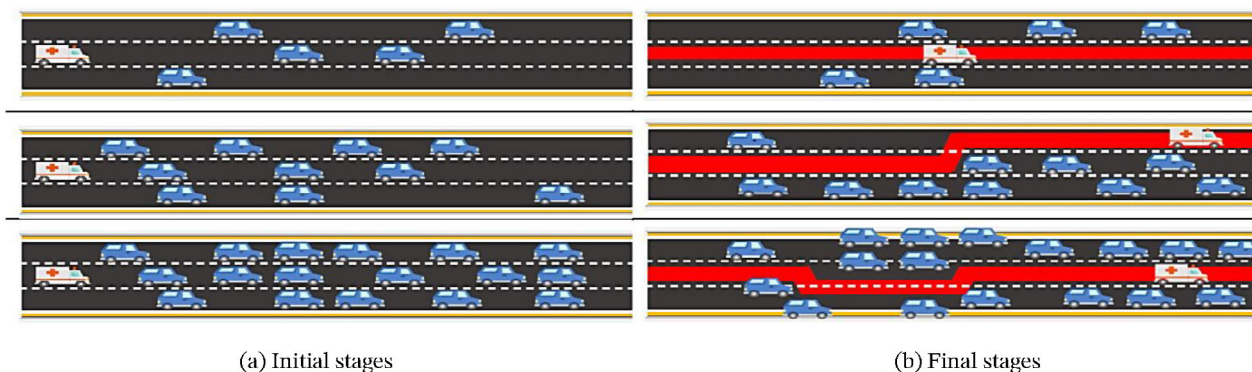


FIGURE 2 Non-ERV's making way for the temporary passage of ERV.

Adeli, 2004a; Karim & Adeli, 2003; Samant & Adeli, 2001; and Jang & Adeli, 2004b). In this context, the challenge of ensuring the rapid and reliable movement of ERVs through urban road networks has also been widely studied in ITS domains. In general, existing studies for ERV prioritization can be classified into three main directions—signal-level control at intersections to prioritize ERV movement, segment-level routing and coordination to optimize travel paths between intersections, and integrated strategies that combine signal control and routing within a unified framework. This section reviews representative work in each category and highlights the specific gap the present study aims to bridge.

On the other hand, three representative methodologies have been adopted to deal with ERV prioritization. The static optimization, to which the present study belongs, derives an ERV's optimal trajectory and non-ERVs' optimal yielding positions based on integer programming. Dynamic programming is to derive the optimal path of an ERV assuming that an environment model is known. The model in an environment means vehicle moving mechanisms and interactions. Under this assumption, the optimal path can be recursively found according to traffic conditions. Reinforcement learning can also be interpreted as dynamic programming, but a big difference is that the model is unknown. Thus model-free reinforcement learning finds an optimal policy by sampling or simulation on a trial-and-error basis.

Figure 3 outlines previous study streams and methodologies for ERV prioritization. For summary, their list is in Table A.1 in the Appendix according to the topic, method, and contribution criteria.

2.2 | Traffic signal preemption

Signal-level control represents one of the earliest and most extensively studied approaches for prioritizing ERVs.

These methods reduce the ERV delay by adjusting signal phases upon detection of approaching ERVs. Early studies addressed communication delays in V2X-based systems; for instance, Li et al. (2018) proposed a compensation distance strategy to mitigate network-induced latency, improving the accuracy of preemption (Li et al., 2018).

Subsequent research shifted toward quantifying travel time benefits. Wang et al. (2013) categorized preemption strategies—path, intersection, and segment preemption—and developed models that better reflect their operational differences. More recent approaches incorporate real-time traffic data, enabling dynamic signal adjustments based on real-time traffic states such as queue length and discharge rates.

In areas without V2X infrastructure, vision-based alternatives have been explored. Nellore & Hancke (2016) utilized visual sensors and wireless nodes to detect ERVs and adjust traffic signals, accordingly, extending the applicability to less-connected environments. Additionally, reinforcement learning (RL) has emerged as a promising tool. Cao et al. (2022) applied a deep Q-network to learn adaptive signal policies for approaching ERVs, demonstrating scalability and reduced need for manual tuning.

Despite these advancements, most systems remain intersection-focused and fail to account for the cumulative impact of preemption along entire routes, limiting their effectiveness in complex urban road networks.

2.3 | ERV routing in road segment

While intersection-level preemption is important, ERVs often traverse multiple intersections and road segments during response operations. Accordingly, research has increasingly focused on segment-level routing to improve overall response time and route feasibility under real-world conditions. Traditional methods employ shortest-path algorithms such as Dijkstra, A*, or heuristic

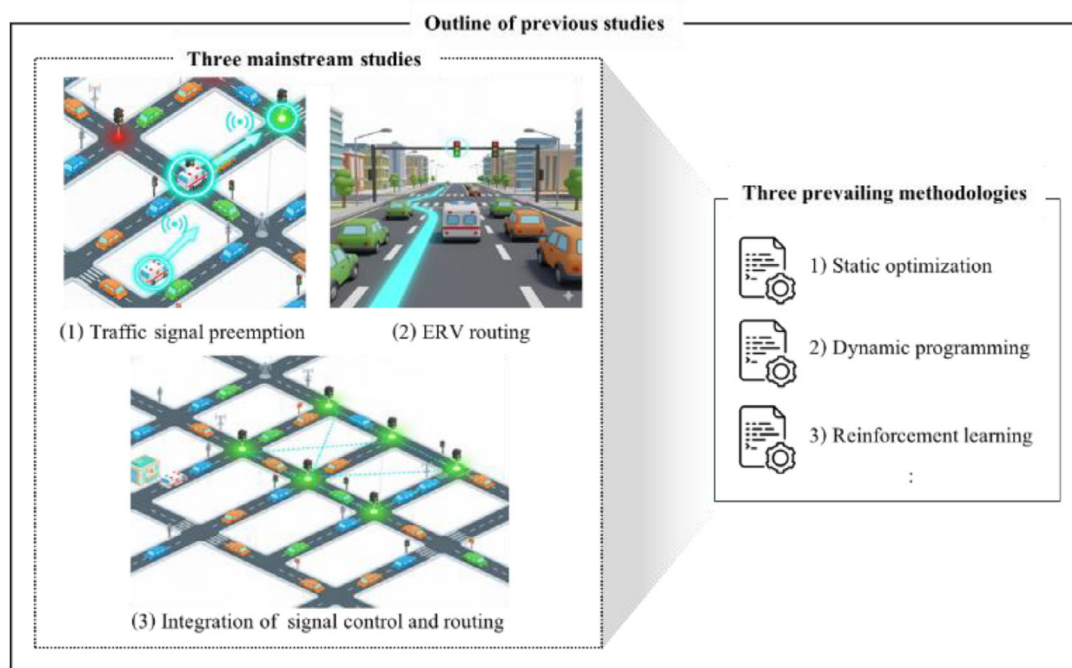


FIGURE 3 Breakdowns of previous studies regarding ERV prioritization.

approaches to identify time-efficient routes, as shown by Sarkar et al. (2022). However, such static routing proves inadequate in dynamically varying traffic conditions in a road segment.

To address this challenge, diverse advanced strategies have been proposed. Zeng et al. (2021) developed a lane pre-clearing scheme to coordinate non-ERVs yielding in advance, while Liu et al. (2024) introduced a bi-level trajectory optimization framework that accounts for fine-grained vehicle scheduling. Liu et al. (2022) proposed an IoV-based cooperative control model, facilitating platoon-based yielding behavior through V2V communication.

A particularly influential contribution comes from Hannoun et al. (2019), who developed an ILP model to optimize ERV routing in fully connected vehicle environments. This model laid the groundwork for spatial reallocation strategies and has become a foundational reference in the field.

Building on this, their subsequent work (Hannoun et al., 2020) expanded the ILP framework from isolated segments to a broader network of intersections, addressing meso- and microscopic levels of traffic control. This extension highlighted the model's scalability and its applicability to more complex urban road networks.

To address the limitations of full connectivity assumptions, Hannoun et al. (2021) introduced a sequential optimization framework tailored to partially connected vehicle systems. By incorporating real-time updates and dynamic adjustments, this approach enhanced adaptability in more realistic, mixed-traffic scenarios.

Complementing these technical advancements, Hannoun & Menéndez (2022) explored an architectural solution by proposing modular vehicle configurations for emergency medical services. Their design emphasized flexibility and rapid deployment, offering a practical pathway to improve operational efficiency across diverse urban contexts.

These works collectively underscore the evolution of ERV routing research—from static and structured models toward more adaptive, communication-aware, and system-integrated approaches—while highlighting the ongoing importance of robust optimization under varying levels of connectivity.

2.4 | Integration of signal control and routing

Recognizing the limitations of isolated signal control and segment-level routing, several studies have explored integrated approaches that jointly manage traffic signal operations and ERV routing under dynamic conditions.

Elalouf (2012) proposed an optimization model that accounts for uncertainty in urban traffic, enabling more effective ERV routing by considering real-time delays and stochastic travel times. This approach moves beyond static routing by adapting paths in response to evolving congestion conditions.

Building on this, Mu et al. (2018) introduced a route-based signal preemption framework that proactively



adjusts traffic signals along an ERV's expected path. By integrating route information into signal control logic, their method reduces intersection delays and improves coordination across links.

Jung et al. (2021), on the other hand, proposed a reinforcement learning-based system that dynamically prioritizes ERV movement through real-time signal adjustments. Leveraging V2X communication and simulation-trained policies, their approach adapts signal phases to spontaneous disruptions in traffic.

Additionally, focusing on evacuation rather than ERV dispatch, Liu et al. (2024) presented a bi-level optimization to determine optimal departure times and routes at the upper level, while refining traffic signal operations at the lower level. This integrated framework combines macro- and micro-scopic representations to improve overall evacuation performance, significantly reducing total clearance time and individual travel delays.

Despite these advancements, many integrated systems still rely on simplified assumptions about driver behavior and structured traffic movement. Informal yielding practices—such as lane straddling, shoulder driving or temporary lane changes—are rarely incorporated into these models, which limits their practical effectiveness in highly congested or unpredictable urban environments.

3 | MODELING FRAMEWORK

Despite various efforts to improve the ERV mobility—such as intersection preemption, routing optimization, and coordinated control—most existing models assume that all vehicles strictly adhere to their designated lanes. This assumption originates from under-saturated traffic conditions and underlies a wide range of approaches, including MILP-based optimization (Firooze et al., 2018; Kaufman et al., 1998), dynamic control systems (Polimeni & Vitetta, 2011; Katsuma & Yoshida, 2018), advanced dynamic control systems (Alruwaili et al., 2025; Duan et al., 2022), and learning-based methods (Alzubaidi et al., 2023; Su et al., 2023; Ding et al., 2024).

In reality, however, drivers often exhibit more active yielding behaviors—such as moving onto shoulders or straddling lanes—to make way for ERVs. While these actions are voluntary, they play a crucial role in enabling ERV passage even under jammed traffic conditions. Although Hannoun et al. (2019, 2020 and 2021), proposed a rigorous ILP model that offers a strong foundation for structured ERV trajectory optimization, their formulation assumes so strict discipline that such flexible yielding behaviors are not allowed. To bridge this gap, the present study extends the ILP so that lane straddling can be allowed for ERV routing to better reflect how drivers respond to ERV in the real world.

More concretely, the present study is different from Hannoun et al. (2019, 2020 and 2021) in the following three aspects. First, vehicle positions are treated in a totally different manner to allow vehicle straddling and shoulder utilization. That is, conditions for residing vehicles in informal positions are more restrictive than that for normal lanes. Second, the constraints of the Hannoun model are simplified to make it easier to understand. Last, the objective function is diversified in the present study to accommodate the loss of non-ERVs, which brings out the most crucial difference.

3.1 | Layout of initial and final stages

The proposed model, based on a static ILP, requires the initial state of all vehicles as input and presents their final locations and an ERV's optimal route as output. The conceptual representation of a road segment and vehicle layout is shown in Figure 4. To allow lane straddling, cells cover space both within and between lanes. The proposed framework formalizes vehicle reallocation using a discretized, cell-based representation of a road segment. Each cell accommodates one vehicle and has distinct longitudinal and lateral coordinates. Based on the cells, a road segment is discretized to capture road geometry and vehicle occupations.

At the initial stage, existing non-ERVs in the road segment are captured just before an ERV enters the scene. In this snapshot, each non-ERV occupies a specific (x, y) cell, where x denotes the vehicle's position along the longitudinal direction and y corresponds to its lateral placement. The speed of each non-ERV at the initial stage should also be given to determine the range of its final position to yield to ERV.

The lateral axis (y -axis) spans from $y = 0$ to $y = Y_{\text{Max}}$, where the boundaries are the shoulder and center line. Odd numbers are assigned to normal lanes, while even numbers are assigned to shoulder, center line, and lane delineation lines. This numbering scheme allows the model to consider active yielding behaviors such as lane straddling or shoulder running—commonly observed in the real-world.

Non-ERVs at the initial stage are sequentially numbered according to the convention of Hannoun et al. (2019), first along the longitudinal direction and then laterally. The final number, thus, indicates the total number of non-ERVs, reflecting the level of congestion. All vehicles are assumed to be homogeneous in size and mechanical performance, and then this assumption is relaxed to allow two different types of non-ERVs such as cars and heavy vehicles. Heavy vehicles include buses and trucks.

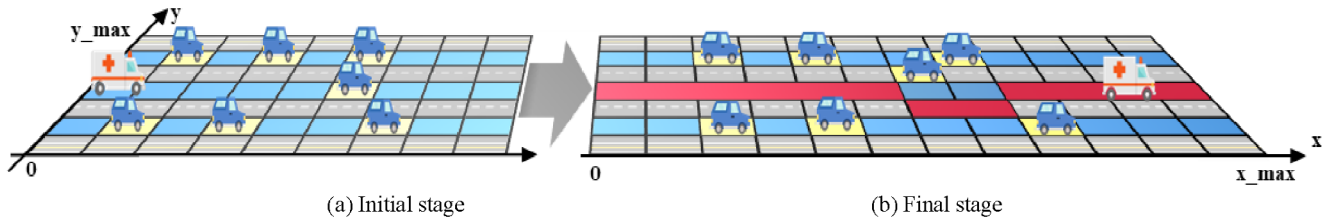


FIGURE 4 Layouts of initial and final states of non-ERVs. Each cell in road segment stands for an available space that can accommodate a vehicle at a time. Each cell is 3.5 m long. This cell length approximates the average vehicle length with clearance. Cells in normal lane are shaded in light cyan, and those for straddling between normal lanes are shaded in grey. Cells on road shoulder and center line are also marked in grey with white longitudinal lines, and for convenience, both are regarded as identical. Operationally, cells occupied by non-ERVs are highlighted in light yellow, and an ERV's route is painted in red.

To restrict non-ERVs' final positions, the minimum stopping distance (MSD) is adopted, based on which the range of each vehicle's final position is constrained. The MSD for each vehicle is computed by its initial speed. Based on the MSD, the minimum final position of each non-ERV is computed as follows:

$$x'_j = x_j^0 + \text{MSD}_j$$

where:

- x_j^0 is the initial longitudinal position of vehicle j , expressed as a cell count from the entry of the segment
- MSD_j is the minimum stopping distance of vehicle j measured as cell count
- x'_j is the longest boundary of final position of vehicle j .

The longest boundary of each non-ERV's stopping position is then used to define its feasible stopping range that spans from x'_j to $x'_j + c_j$, wherein c_j is a buffer that is randomly varies up to two or four cells. The range of stopping position should be predefined and fed into formulation as parameters. The impact of varying the range on deriving solutions is examined later in Section 5.4.2.

3.2 | Decision variables and parameters

The baseline model is first set such that a single type of non-ERVs yield to a single ERV. This baseline model is extended to accommodate multiple types of non-ERVs with multiple ERVs.

3.2.1 | Decision variables

Binary decision variables for indicating the presence of an ERV and non-ERVs at the final stage are represented by w

and v , respectively. Both sets of decision variables indicate the optimal ERV path and the final position of non-ERVs after yielding.

The speed variables (s_x , $s_x^{\text{env}1}$, and $s_x^{\text{env}2}$) emerging in the objective function are related with the binary decision variables via constraints. The speed variables are confined to positive integers to reduce the formulation into an ILP. It should be noted that the speed variables do not equal physical speeds but reflect the surrogate index for ERV speed to facilitate formulation (Hannoun et al., 2019). Each of three surrogate variables represents the primary speed affecting only by lane change, the first auxiliary speed reduced only by side friction, and the second auxiliary speed reduced only by straddling, respectively. The baseline is the primary speed, and the remaining two auxiliary speeds are reduced from this baseline speed, for the purpose of separating the impact of side friction and straddling.

The remaining variables (z , z^- , z^+ , and a) are binary or ternary and mobilized to linearize non-linear constraints. Table 1 lists the variable types and descriptions.

3.2.2 | Parameters

Table 2 lists parameter notations and descriptions. Special care should be taken to define the initial location of non-ERVs. (x_j^0, y_j^0) denotes coordinates of the j th non-ERV at the initial stage. The longitudinal coordinate increases in the direction of vehicle progression, and the lateral coordinate increases from the rightmost shoulder to the center line.

The utility and effect of each parameter will be described while formulating the problem in what follows. The solution varies considerably according to how parameters are set up. The sensitivity analysis will be conducted later in Section 5.4 to investigate the impact of varying parameters on solution properties.

**TABLE 1** Description of decision variables.

Decision variable	Type	Description
$w_{x,y} \in \{0, 1\}$	Binary	1 if a cell (x, y) is included in ERV path, and 0 otherwise.
$v_{x,y,j} \in \{0, 1\}$	Binary	1 if a cell (x, y) is occupied by the j th non-ERV, and 0 otherwise.
$s_x \in \text{integer}$	Integer	Primary ERV speed at a longitudinal position x
$s_x^{\text{env1}} \in \text{integer}$	Integer	Auxiliary ERV speed at position x affected solely by the side friction of adjacent non-ERVs
$s_x^{\text{env2}} \in \text{integer}$	Integer	Auxiliary ERV speed at position x related only to running in informal positions rather than a normal lane
$r_{x,y,j} \in \text{integer}$	Integer	Auxiliary integer variable to linearize the objective function term representing the loss of non-ERVs
$z_{x,y}^- \in \{0, 1\}$	Binary	Auxiliary binary variables to linearize the constraint for lane-change to left without interference
$z_{x,y}^+ \in \{0, 1\}$	Binary	Auxiliary binary variables to linearize the constraint for lane-change to right without interference
$a_{x,y}^{\text{env1}} \in \{0, 1, 2\}$	Ternary	Auxiliary ternary variables to indicate the side friction due to non-ERVs for speed adjustment.

3.3 | Objective function

The present study sets up two different objective functions according to whether to consider the disutility of non-ERVs. Equation (1) denotes the first objective function (Obj. A) that considers only an ERV's utility, representing the weighted sum of three different ERV speeds along its path.

$$\text{Max Obj. A} = \sum_x s_x + \sum_x \alpha_1 s_x^{\text{env1}} + \sum_x \alpha_2 s_x^{\text{env2}} \quad (1)$$

$$\begin{aligned} \text{Max Obj. B} = & \sum_x s_x + \sum_x \alpha_1 s_x^{\text{env1}} + \sum_x \alpha_2 s_x^{\text{env2}} \\ & - \sum_j \sum_x \sum_y \alpha_3 \cdot |y \cdot v_{x,y,j} - y_j^0| \end{aligned} \quad (2)$$

The first term reflects the sum of an ERV's primary speeds along its longitudinal path which is affected solely by lane changes, the second term is the auxiliary ERV speed to consider the impact of the side friction due to adjacent non-ERVs, and the last term is included to dwindle an ERV's speed when it runs straddling between normal

TABLE 2 Description of parameters.

Parameter	Type	Description
α_1	Real	Weight in objective function for the auxiliary ERV speed affected by the side friction of adjacent non-ERVs
α_2	Real	Weight in objective function for the auxiliary ERV speed related only to running along a delineation line rather than a lane
α_3	Real	Weight of objective term that accounts for changes in lateral position of non-ERVs
c_j	Real	The buffer of stopping distance of the j th non-ERV
x_j^0, y_j^0	Integer	Position of the j th non-ERV at the initial stage
MSD_j	Real	Minimum stopping distance of vehicle j measured as cell count
$s_{\min}, s_{\text{free}}$	Real	Minimum and free-flow speeds of ERV
γ	Real	Speed reduction factor for lane-change of ERV
β_1	Real	Temporary speed reduction factor for lane-change of ERV
β_2	Real	Temporary speed reduction factor for the side friction of an adjacent non-ERV
M	Real	Sufficiently large value for linearization
Y	Integer	Number of lateral positions (lanes, lane delineation lines, and shoulders)
N	Integer	Number of non-ERVs
L	Integer	Road segment length measured in cells

lanes. The reason for this separation lies in the advantage that real-world driving behavior is modeled more accurately, preventing overly aggressive slowdowns of the ERV speed due to side friction or lane straddling.

Matching the surrogate speed variables with physical ERV speed is possible in principle by removing the separation and computing a single surrogate speed. However, in practice, it is hard to obtain data to fit the surrogate speed into physical speed. Nonetheless, the rationale for adopting the surrogate variables in the objective function lies in the fact that the physical speed is enhanced as the surrogate speed increases.

Maximizing the first objective function pursues the advantage of an ERV, which might lead to drastic changes in non-ERVs' positions. To overcome this, the second



objective function (Obj. B) accommodates the loss of non-ERVs. More concretely, non-ERVs' positional change in the lateral direction is minimized simultaneously while maximizing the ERV speeds. Equation (2) denotes the extended objective function, and the fourth term in the objective function denotes the negative loss of non-ERVs which will be maximized together with the three ERV speeds.

$$\max \sum_x s_x + \sum_x \alpha_1 \cdot s_x^{\text{env1}} + \sum_x \alpha_2 \cdot s_x^{\text{env2}} - \sum_j \alpha_3 \cdot d_j \quad (3)$$

$$r_{x,y,j} \geq (y - y_j^0) \cdot v_{x,y,j} \quad \forall x, y, j \quad (4)$$

$$r_{x,y,j} \geq -(y - y_j^0) \cdot v_{x,y,j} \quad \forall x, y, j \quad (5)$$

$$\text{where, } d_j = \sum_x \sum_y r_{x,y,j} \quad (6)$$

The absolute value in the additional term makes the second objective function nonlinear. This function is linearized and transformed into Equation (3) with constraints of Inequalities (4) and (5). New auxiliary integer variables ($r_{x,y,j}$) are added for the linearization. All linearization schemes used in the present formulation are clearly introduced in Table A.2 in the Appendix with original non-linear formulas. The formulation demonstrates the balance between maximizing the ERV utility and minimizing the loss of non-ERVs.

3.4 | Constraints

3.4.1 | Vehicle positioning constraints

Inequalities (7) denote the constraint that only one vehicle can exist per cell. Equation (8) forces that non-ERVs must stop within a feasible stopping range that is predefined based on their initial speeds.

$$w_{x,y} + \sum_j v_{x,y,j} \leq 1 \quad \forall x, y \quad (7)$$

$$\sum_{x=x_j'}^{x_j'+c_j} \sum_y v_{x,y,j} = 1 \quad \forall j \quad (8)$$

Inequality (9) ensures that non-ERVs maintain the initial positional order without passing. That is, if non-ERV A was initially downstream of non-ERV B, non-ERV A must not end up upstream of non-ERV B. Inequality (10) ensures that non-ERVs avoid weaving conflicts. Two non-ERVs cannot switch their lateral positions in between initial and

final stages. These two constraints use the big-M method to linearize the original conditional restrictions.

$$\sum_{x' < x} \sum_y v_{x',y,j'} \leq M \left(1 - \sum_y v_{x,y,j} \right) \quad \forall j' > j, \quad x_j^0 < x_{j'}^0 \quad (9)$$

$$\sum_{y' < y} v_{x,y',j'} \leq M (1 - v_{x,y,j}) \quad \forall j' > j, \quad y_j^0 < y_{j'}^0 \quad (10)$$

Inequalities (11) and (12) guarantee that no two non-ERVs can be laterally adjacent in the same longitudinal position. That is, when a non-ERV occupies a lane, another non-ERV cannot straddle the lane, and vice versa.

$$\sum_j v_{x,y,j} + \sum_{j'} v_{x,y-1,j'} \leq 1 \quad \forall (x, y), \forall j' \neq j \quad (11)$$

$$\sum_j v_{x,y,j} + \sum_{j'} v_{x,y+1,j'} \leq 1 \quad \forall (x, y), \forall j' \neq j \quad (12)$$

3.4.2 | Vehicle motion constraints

Equation (13) denotes the constraint to ensure an ERV's continuous forward movement, and inequality (14) ensures that non-ERVs cannot laterally occupy, at the same time, cells more than the number of normal lanes.

$$\sum_y w_{x,y} = 1 \quad \forall x \quad (13)$$

$$\sum_{y'} \sum_j v_{x,y,j} \leq (Y - 1) / 2 \quad \forall x \quad (14)$$

Inequalities (15) and (16) are lane-change constraints for ERV to guarantee the following conditions. An ERV can move to an adjacent normal lane, if it previously straddled between normal lanes (= odd-numbered position), and it can either straddle an adjacent lane or move to an adjacent normal lane, if it was previously in a normal lane (= even-numbered position). These constraints also hold when an ERV runs along the road edges such as shoulder or center line by ensuring $w_{x,y}$ is set at 0 for all y 's beyond both lateral boundaries.

$$w_{x+1,y} \leq w_{x,y-1} + w_{x,y} + w_{x,y+1} \quad \forall x, \quad \forall y \in \text{odd} \quad (15)$$

$$w_{x+1,y} \leq w_{x,y-2} + w_{x,y-1} + w_{x,y} + w_{x,y+1} + w_{x,y+2} \quad \forall x, \quad \forall y \in \text{even} \quad (16)$$

Inequalities (17) and (18) ensure that non-ERVs cannot be laterally adjacent to an ERV in the proposed lane-numbering convention. These constraints are necessary



because normal lanes are even-numbered, and shoulder, center line, and straddling positions are odd-numbered. In other words, this numbering scheme allows the present formulation to treat normal lanes and informal positions in an incorporated manner.

$$\sum_j v_{x,y-1,j} \leq M(1 - w_{x,y}) \quad \forall x, y \quad (17)$$

$$\sum_j v_{x,y+1,j} \leq M(1 - w_{x,y}) \quad \forall x, y \quad (18)$$

For an ERV to change a lane without interference with non-ERVs, the following conditions should be met. If an ERV switches to the right lane or straddling position, the next cell in the progressing direction of the ERV should be vacated, and vice versa to the left. For these constraints, a new variable taking the product of two binary variables (w and v) is required, which then converts the problem nonlinear. Two auxiliary binary variables ($z_{x,y}^+$, $z_{x,y}^-$) are mobilized to linearize these conditional constraints with the product variable. Inequalities (19)–(22) linearized constraints for the ERV movement to the right. Inequalities (23)–(26) correspond to the same restrictions for an ERV's move to the left.

$$z_{x,y}^+ \leq w_{x,y} \quad \forall x, y \quad (19)$$

$$z_{x,y}^+ \leq w_{x+1,y-1} \quad \forall x, y \quad (20)$$

$$z_{x,y}^+ \geq w_{x,y} + w_{x+1,y-1} - 1 \quad \forall x, y \quad (21)$$

$$\sum_j v_{x+1,y,j} \leq M(1 - z_{x,y}^+) \quad \forall x, y \quad (22)$$

$$z_{x,y}^- \leq w_{x,y} \quad \forall x, y \quad (23)$$

$$z_{x,y}^- \leq w_{x+1,y+1} \quad \forall x, y \quad (24)$$

$$z_{x,y}^- \geq w_{x,y} + w_{x+1,y+1} - 1 \quad \forall x, y \quad (25)$$

$$\sum_j v_{x+1,y,j} \leq M(1 - z_{x,y}^-) \quad \forall x, y \quad (26)$$

3.4.3 | Speed constraints

Inequalities (27) and (28) deal with the ERV speed adjustment due to side friction from adjacent non-ERVs. An auxiliary ternary variable ($a_{x,y}^{\text{env1}}$) is adopted to reflect that non-ERVs adjacent laterally from an ERV's position impedes the ERV's progression. For the proposed layout, 1-cell separation in lateral axis between vehicles is not

allowed for safety. 2-cell separation thus indicates that vehicles are laterally adjacent. Inequality (28) reflects that the auxiliary speed stays equivalent to the ERV speed plus 1 when there is no adjacent non-ERV, decreases by β_1 when only one vehicle resides adjacently, or decreases by $2\beta_1$ when there are two non-ERVs on both sides. This inequality is, however, nonlinear because of the product of two variables. Mobilizing another auxiliary ternary variable is necessary for linearization. For brevity, the linearized formulas are listed in Table A.2 in the Appendix.

$$a_{x,y}^{\text{env1}} \geq w_{x,y-2} + w_{x,y+2} \quad \forall x, y \quad (27)$$

$$s_x^{\text{env1}} \leq s_x + 1 - \beta_1 \sum_y \sum_j a_{x,y}^{\text{env1}} \cdot v_{x,y,j} \quad \forall x \quad (28)$$

Inequality (29) denotes the constraint to adjust the second auxiliary speed of ERV, so that it decreases by β_2 when it straddles normal lanes or resides in shoulders (= even-numbered positions).

$$s_x^{\text{env2}} \leq s_x + 1 - \beta_2 \sum_{y \in \text{even}} w_{x,y} \quad \forall x \quad (29)$$

The primary ERV speed is penalized by the lane changing maneuver. Inequality (30) considers that an ERV's primary speed should decrease by γ every time it conducts a lane change. The lane change encompasses both movements from lane to lane and from lane to adjacent informal position. The auxiliary variables ($z_{x,y}^-$ and $z_{x,y}^+$) are necessary to express the lane-changing constraint.

$$s_{x+1} \leq s_x + 1 - \gamma \sum_y (z_{x,y}^- + z_{x,y}^+) \quad \forall x \quad (30)$$

Final constraints (31)–(33) restrict the range of speed variables.

$$s_{\min} \leq s_x \leq s_{\text{free}} \quad \forall x \quad (31)$$

$$s_{\min} \leq s_x^{\text{env1}} \leq s_{\text{free}}^{\text{env1}} \quad \forall x \quad (32)$$

$$s_{\min} \leq s_x^{\text{env2}} \leq s_{\text{free}}^{\text{env2}} \quad \forall x \quad (33)$$

3.5 | Model extensions

The baseline formulation above is established assuming a single type of non-ERVs with a single ERV. The assumption is relaxed in this section by adjusting corresponding variables and constraints that include them.

**TABLE 3** Additional variables and parameters for extension of non-ERV types.

Variable	Type	Description
$h_{x,y,k}$	Decision variable/binary	1 if a cell (x, y) is occupied by the k th heavy vehicle's tail, and 0 otherwise.
$r_{x,y,k}^{\text{hev}}$	Auxiliary variable/integer	Auxiliary integer variable to linearize the objective function term representing the loss of heavy vehicles
$r_{x,y,j}$	Auxiliary variable/integer	Auxiliary integer variable to linearize the objective function term representing the loss of cars
$\text{occ}_{x,y}^{\text{tail}}$	Definitional variable/binary	1 if a cell (x, y) is occupied by a heavy vehicle's tail, and 0 otherwise.
$\text{occ}_{x,y}^{\text{head}}$	Definitional variable/binary	1 if a cell (x, y) is occupied by a heavy vehicle's head, and 0 otherwise.
$\text{occ}_{x,y}^{\text{hev}}$	Definitional variable/binary	1 if a cell (x, y) is occupied by a heavy vehicle, and 0 otherwise.
$X_k^{\text{tail}}, X_k^{\text{head}}$	Definitional variable/integer	Longitudinal position of the k th heavy vehicle
X_j^{car}	Definitional variable/integer	Longitudinal position of the j th car
x_j^0, y_j^0	Parameter/integer	position of the j th car at the initial stage
$x_k^{\text{h0}}, y_k^{\text{h0}}$	Parameter/integer	position of the k th heavy vehicle at the initial stage
K	Parameter/integer	Number of heavy vehicles
α_3^{hev}	Parameter/real	Weight of objective term that accounts for changes in lateral position of heavy vehicles
β_1	Parameter/real	Temporary speed reduction factor for the side friction of an adjacent car
β_{11}	Parameter/real	Temporary speed reduction factor for the side friction of an adjacent heavy vehicle
A_k	Set	Set of cars that reside ahead of the k th heavy vehicle at the initial stage
B_k	Set	Set of cars that reside behind the k th heavy vehicle at the initial stage

3.5.1 | Different types of non-ERVs

Two different types of non-ERVs are considered in the formulation. Non-ERVs are divided into cars and heavy vehicles such as buses and trucks. For the extension, new decision variables and parameters are added as shown in Table 3.

Only changed parts form the baseline formulation are described in this section. The second objective function (Obj. B) should have an additional term to indicate the lateral displacement of heavy vehicles as a loss. The objective function for two types of non-ERVs is expressed as Equation (34).

$$\max \sum_x s_x + \sum_x \alpha_1 s_x^{\text{env1}} + \sum_x \alpha_2 s_x^{\text{env2}} - \sum_j \alpha_3 d_j - \sum_k \alpha_3^{\text{hev}} d_k^{\text{hev}} \quad (34)$$

A new decision variable ($h_{x,y,k}$) is necessary to denote the position of heavy vehicles, whereas the existing position variable is confined only to cars. A heavy vehicle is assumed to occupy two cells in a row. $h_{x,y,k}$ is defined as the tail position of a heavy vehicle. For the facilitation of

extended formulation, three definitional variables need to be set up. $\text{occ}_{x,y}^{\text{tail}}$, $\text{occ}_{x,y}^{\text{head}}$, and $\text{occ}_{x,y}^{\text{hev}}$ denotes the existence of tail, head, and either, respectively. Equations (35)–(37) correspond to each of the definitional variables.

$$\text{occ}_{x,y}^{\text{tail}} = \sum_k h_{x,y,k} \quad \forall x \in \{0, \dots, L-1\}, \forall y \quad (35)$$

$$\text{occ}_{x,y}^{\text{head}} = \sum_k h_{x-1,y,k} \quad \forall x \in \{1, \dots, L\}, \forall y \quad (36)$$

$$\text{occ}_{x,y}^{\text{hev}} = \text{occ}_{x,y}^{\text{tail}} + \text{occ}_{x,y}^{\text{head}} \quad \forall x, \forall y \quad (37)$$

The existing constraint (7) changes to Equation (38) to secure that a cell accommodates only one vehicle.

$$\sum_j v_{x,y,j} + \text{occ}_{x,y}^{\text{hev}} + w_{x,y} \leq 1 \quad \forall x, \forall y \quad (38)$$

The existing constraints (11) and (12) change to inequalities (39)–(42) to purge adjacent positions in the lateral direction of heavy vehicles.

$$\sum_j v_{x,y-1,j} + \text{occ}_{x,y-1}^{\text{hev}} \leq M \left(1 - \text{occ}_{x,y,k}^{\text{tail}} \right) \quad (39)$$



$$\sum_j v_{x+1,y-1,j} + \text{occ}_{x+1,y-1}^{\text{hev}} \leq M (1 - \text{occ}_{x,y}^{\text{tail}}) \quad (40)$$

$$\sum_j v_{x,y+1,j} + \text{occ}_{x,y+1}^{\text{hev}} \leq M (1 - \text{occ}_{x,y}^{\text{tail}}) \quad (41)$$

$$\sum_j v_{x+1,y+1,j} + \text{occ}_{x+1,y+1}^{\text{hev}} \leq M (1 - \text{occ}_{x,y}^{\text{tail}}) \quad (42)$$

$$\forall x \in \{0, \dots, X-1\}, \forall y$$

Inequalities (43) and (44) are the updated versions of existing constraints (17) and (18), when heavy vehicles are included, to vacant positions adjacent in the lateral direction of ERV trajectory.

$$\text{occ}_{x,y-1}^{\text{hev}} \leq M (1 - w_{x,y}) \quad \forall x \in \{0, \dots, X-1\}, \forall y \quad (43)$$

$$\text{occ}_{x,y+1}^{\text{hev}} \leq M (1 - w_{x,y}) \quad \forall x \in \{0, \dots, X-1\}, \forall y \quad (44)$$

Because cars and heavy vehicles are independently numbered at the initial stage, additional constraints are necessary to hamper passing between non-ERVs. Overtaking between heavy vehicles is prevented based on a constraint denoted by Inequality (45).

$$\sum_{x'=0}^{x-1} \sum_y h_{x',y,k'} \leq M (1 - \sum_y h_{x,y,k}) \quad (45)$$

$$\forall k < k', \forall x \in \{1, \dots, L-1\}$$

To restrict the passing between cars, three definitional variables are necessary, which indicate the longitudinal position of cars and heavy vehicles. Equations (46)–(48) denote the definitional variables, respectively. Inequalities (49) and (50) restrict the range of the definitional variables for heavy vehicles.

$$X_k^{\text{tail}} = \sum_x x \left(\sum_y h_{x,y,k} \right) \quad \forall k \quad (46)$$

$$X_k^{\text{head}} = X_k^{\text{tail}} + 1 \quad \forall k \quad (47)$$

$$X_j^{\text{car}} = \sum_x x \left(\sum_y v_{x,y,j} \right) \quad \forall j \quad (48)$$

$$0 \leq X_k^{\text{tail}} \leq L-1 \quad (49)$$

$$1 \leq X_k^{\text{head}} \leq L \quad (50)$$

Two sets for cars should be identified before formulating constraints to limit overtaking between a car and a heavy vehicle.

$$A_k = \left\{ i \in N : \left| y_j^0 - y_k^{h0} \right| \leq 1, x_j^0 \geq x_k^{h0} + 1 \right\} \quad (51)$$

$$B_k = \left\{ i \in N : \left| y_j^0 - y_k^{h0} \right| \leq 1, x_j^0 \leq x_k^{h0} \right\} \quad (52)$$

Using the two sets, constraints to prohibit passing between cars and heavy vehicles are established as shown in Inequalities (53) and (54).

$$X_k^{\text{head}} \leq X_j^{\text{car}} \quad \forall k, \forall j \in A_k \quad (53)$$

$$X_j^{\text{car}} \leq X_k^{\text{tail}} \quad \forall k, \forall j \in B_k \quad (54)$$

Among existing constraints (19)–(26) for an ERV to change a lane without interference with non-ERVs, only Inequalities (22) and (26) need to be changed to Inequalities (55) and (56), respectively, to consider heavy vehicles.

$$\sum_j v_{x+1,y,j} + \text{occ}_{x+1,y}^{\text{hev}} \leq M (1 - z_{x,y}^+) \quad \forall x, \forall y \quad (55)$$

$$\sum_j v_{x+1,y,j} + \text{occ}_{x+1,y}^{\text{hev}} \leq M (1 - z_{x,y}^-) \quad \forall x, \forall y \quad (56)$$

The existing constraint (28) changes to Equation (57), so that the ERV speed reduction due to side frictions can be differentiated between cars and heavy vehicles. Inequality (57) denotes the speed reduction constraint separated by vehicle type.

$$s_x^{\text{env}1} \leq s_x + 1 \quad (57)$$

$$-\beta_{11} \sum_y (a_{x,y} \text{occ}_{x,y}^{\text{hev}}) - \beta_1 \sum_y (a_{x,y} \sum_j v_{x,y,j})$$

Last, additional constraints are necessary to include the loss of heavy vehicles in the objective function. The same linearization scheme is adopted as when adopting a unified vehicle type. Constraints (58)–(60) are added to do so.

$$r_{x,y,k}^{\text{hev}} \geq (y - y_k^{h0}) h_{x,y,k} \quad \forall x \in \{0, \dots, L-1\}, y, k \quad (58)$$

$$r_{x,y,k}^{\text{hev}} \geq -(y - y_k^{h0}) h_{x,y,k} \quad (59)$$

$$\forall x \in \{0, \dots, L-1\}, y, k$$

$$d_k^{\text{hev}} \geq \sum_x \sum_y^{L-2} r_{x,y,k}^{\text{hev}} \quad \forall k \quad (60)$$

3.5.2 | Simultaneous accommodation multiple ERVs

Multiple ERVs move together or may be encountered at the same time on a road segment. Considering such a case, the baseline formulation is extended to simultaneously accommodate multiple ERVs. Compared to the extension of non-ERV types, accommodating multiple ERVs is much simpler because only one dimension of ERV decision variable needs to be added. This section introduces what should be changed or added for the treatment based

**TABLE 4** Additional variables and parameters for extension of ERVs.

Variable	Type	Description
$w_{x,y,m}$	Decision variable/binary	1 if a cell (x, y) is occupied by the m th ERV, and 0 otherwise.
$z_{x,y,m}^-$	Auxiliary variable/binary	Auxiliary binary variables to linearize the constraint for the m th ERV's lane-change to right without interference
$z_{x,y,m}^+$	Auxiliary variable/binary	Auxiliary binary variables to linearize the constraint for the m th ERV's lane-change to left without interference
$a_{x,y,m}^{\text{env1}}$	Auxiliary variable/ternary	Auxiliary ternary variables to linearize the constraint for the speed adjustment of the m th ERV based on surroundings
$s_{x,m}$	Decision variable/integer	Speed of the m th ERV at a longitudinal position x
$s_{x,m}^{\text{env1}}$	Decision variable/integer	Auxiliary speed of the m th ERV at position x affected solely by the side friction of adjacent non-ERVs
$s_{x,m}^{\text{env2}}$	Decision variable/integer	Auxiliary speed of the m th ERV at position x related only to running along a delineation line rather than a lane
M	Parameter/integer	Number of ERVs

on the baseline model. Table 4 lists additional variables and parameters necessary for the extension. Equation (61) denotes the changed objective function that includes the speed of multiple ERVs.

$$\max \sum_m \sum_x s_{x,m} + \alpha_1 \sum_m \sum_x s_{x,m}^{\text{env1}} + \alpha_2 \sum_m \sum_x s_{x,m}^{\text{env2}} - \alpha_3 \sum_j d_j \quad (61)$$

A new constraint is necessary to ban overtaking between ERVs, whereas remaining constraints could be changed by adding an additional dimension, considering the changes in existing conditions that should be met. Inequality (62) denotes the additional constraint. This extension assumes that each ERV has its own trajectory that is not shared with other ERVs. This is a handicap that will be overcome with a dynamic programming.

$$\sum_{y'=y}^Y w_{x,y',0} \leq M(1 - w_{x,y,1}) \quad \forall x, \forall y \quad (62)$$

Equations (63)–(77) denote changed constraints by adding one dimension to variables, each of which is updated version of its corresponding constraint in the baseline model.

$$w_{x+1,y,m} \leq w_{x,y-2,m} + w_{x,y-1,m} + w_{x,y,m} + w_{x,y+1,m} + w_{x,y+2,m} \quad (63)$$

$$w_{x+1,y,m} \leq w_{x,y-1,m} + w_{x,y,m} + w_{x,y+1,m} \quad (64)$$

$$\sum_j v_{x,y-1,j} \leq M(1 - w_{x,y,m}) \quad (65)$$

$$\sum_j v_{x,y+1,j} \leq M(1 - w_{x,y,m}) \quad (66)$$

$$z_{x,y,m}^+ \leq w_{x,y,m} \quad (67)$$

$$z_{x,y,m}^+ \leq w_{x+1,y-1,m} \quad (68)$$

$$z_{x,y,m}^+ \leq w_{x,y,m} + w_{x+1,y-1,m} - 1 \quad (69)$$

$$\sum_j v_{x+1,y,j} \leq M(1 - z_{x,y,m}^+) \quad (70)$$

$$z_{x,y,m}^- \leq w_{x,y,m} \quad (71)$$

$$z_{x,y,m}^- \leq w_{x+1,y+1,m} \quad (72)$$

$$z_{x,y,m}^- \leq w_{x,y,m} + w_{x+1,y+1,m} - 1 \quad (73)$$

$$\sum_j v_{x+1,y,j} \leq M(1 - z_{x,y,m}^-) \quad (74)$$

$$a_{x,y,m}^{\text{env1}} \geq w_{x,y-2,m} + w_{x,y+2,m} \quad (75)$$

$$s_{x,m}^{\text{env2}} \leq s_{x,m} + 1 - \beta_2 \cdot \sum_{y \in \text{even}} w_{x,y,m} \quad (76)$$

$$s_{x+1,m} \leq s_{x,m} + 1 - \gamma \cdot \sum_y (z_{x,y,m}^- + z_{x,y,m}^+) \quad (77)$$

$\forall m, \forall x \in \{0, \dots, X-1\}$

4 | EXPERIMENTAL SET-UPS

The testbed is discretized into cells, where each cell represents 3.5 m in the longitudinal (x) direction. The lateral (y) dimension is also discretized, and the maximum y coordinate (Y) is defined as the number of all available lanes and



TABLE 5 Parameters of baseline scenario.

Baseline scenario									
Parameters						Traffic condition	Physical environment		
α_1	α_2	α_3	β_1	β_2	γ	Number of non-ERVs	Number of lanes	Length of initial stage	Length of final stage
1	1	1	1	5	0.05	30	3	20	25

informal positions. In this representation, odd-numbered positions ($y = 1, 3, 5, \dots$) correspond to actual driving lanes, while even-numbered positions ($y = 0, 2, 4, \dots$) represent the informal positions such as shoulder, center line, and lane delineation lines.

All experiments are conducted over a fixed longitudinal range ($x = 0 \sim 25$) with a single ERV. The initial speed values are set to $s_0 = 10$, $s_0^{\text{env1}} = 1$, $s_0^{\text{env2}} = 1$ in all subsequent experiments.

4.1 | Base line scenario definition

Table 5 presents the baseline settings used in this study. The testbed is a 25-cell long 3-lane road segment. 30 non-ERVs are randomly assigned to 20×5 front grids at the initial stage. The road length is extended to 25-cell long at the final stage, which is to accommodate the final position of non-ERVs after yielding to an ERV. The rightmost non-ERV should be positioned within this length, and the ERV speed accumulated along this length is regarded as an objective function value for evaluating diverse operational conditions.

External parameters α_1 , α_2 , and α_3 appear in the objective function. For the baseline scenario, α_1 and α_2 are equally set to 1 to ensure balanced contributions from each speed term. In contrast, α_3 differently weighs the disutility of non-ERVs. β_1 , β_2 , and γ are internal parameters embedded in the model constraints.

Judging whether traffic is congested depends on the failure of the existing algorithm. The existing algorithm assigns vehicles to empty positions only along normal lanes. If the initial stage has vehicles more than the capacity of this normal assignment, the existing algorithm fails in finding available space at the final stage. The threshold is 33 in vehicle counts, at which the existing algorithm starts to fail. Considering this threshold, 30 non-ERVs are determined as baseline traffic. More congested conditions are tested later in the sensitivity analysis up to 50 non-ERVs.

Preliminary tests with $\gamma = 1$ revealed that the lane-change exerted a strong influence on the ERV's speed, possibly resulting in the minimum ERV speed. γ was then adjusted to 0.05 by sensitivity analysis, where the maximized objective function is the smallest (see Sec-

tion 5.4.1). A larger penalty parameter $\beta_2 (= 5)$ was selected to discourage excessive straddling between lanes.

Centered on the baseline setting, the following subsections introduce diverse scenarios to demonstrate the performance of the proposed optimization model. The analysis results from the baseline and other scenarios are discussed in chapter 5.

4.2 | Impact of different objective functions

Obj. A and Obj. B are discriminated by whether to consider the disutility of non-ERVs. Obj. A ensures more flexible maneuver for non-ERVs when they make way for an ERV, facilitating moves in a lateral direction. In contrast, Obj. B restricts non-ERVs' lane-changing behavior for yielding, thereby enhancing the model's capability to mimic actual traffic dynamics.

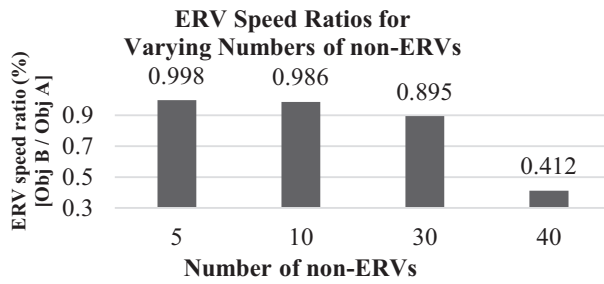
The impact of these two objective functions is compared by examining the deterioration in ERV speed when considering non-ERVs' disutility. Specifically for the comparison, the ERV speed—measured by $\sum_x s_x + s_x^{\text{env1}} + s_x^{\text{env2}}$ without weights—was evaluated for both objective functions. It should be noted that even for Obj. B the result is evaluated by this measure excluding the non-ERVs' disutility term.

Comparing both different objective functions also considers varying conditions: the number of vehicles and the number of lanes. This comparison allows us to assess how incorporating realistic behavior of non-ERVs affects the performance of ERV.

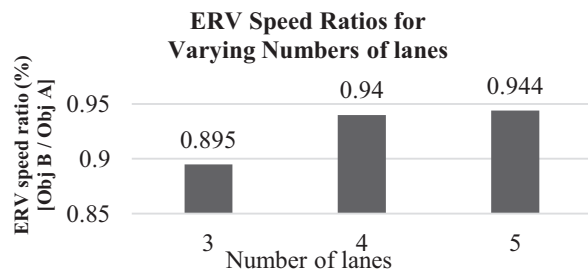
4.3 | Various layouts of road segment

The number of lanes is altered from three to five to examine how the road width influences an ERV's trajectory choice and particularly whether to straddle between lanes. For each case, the final stage shows how an ERV behaves as spatial flexibility varies, when all other being equal.

The proposed model derives the optimal trajectory of an ERV including the entry and exit positions of the road segment of concern. For real-time applications, the entry position of a road segment should match the exit position



(a) Impact of traffic congestion level on the ERV speed ratio



(b) Emergency vehicle speed change rate of various number of lanes

FIGURE 5 Impact of varying environment on the ERV speed ratio.

of the upstream road segment. Fixing an ERV's entry position needs to be tested. Even for the same road geometry and the same initial conditions for non-ERVs, an ERV's different entry points lead to different objective function values.

4.4 | Impact of varying traffic conditions

The spatial distribution of non-ERVs at the initial stage influences an ERV's optimal route and speed. Based on the baseline setting, different traffic conditions are tested. As the initial spatial distribution of non-ERVs differs, an ERV's optimal speed and trajectory vary even for the same number of non-ERVs. To cover severely congested conditions, up to 40 non-ERVs are tested.

This analysis highlights the influence of localized congestion on ERV routing. According to where non-ERVs are clustered in a road segment, the ERV routing may vary. Consequently, the insight gained from this experiment assists to better guide the ERV routing in real-world road networks.

4.5 | Sensitivity analysis by parameters

Sensitivity analysis is necessary to examine how parameters influence an ERV's optimal trajectory and non-ERVs' final yielding positions. As mentioned earlier, parameters

are categorized into two types: internal parameters (β_1, β_2, γ), and external parameters ($\alpha_1, \alpha_2, \alpha_3$). Each parameter varies separately, and its impact on an ERV's routing and non-ERV's positions can be examined.

Aside from model parameters, the impact of varying MSD on solutions is also examined because the proposed ILP model accepts MSD as input. To determine the range of non-ERVs' final positions, a random buffer is added to MSD. Altering both MSD and buffer, how solutions change is investigated.

All experiments are performed under the baseline setting, with only parameters of concern altered. The result from this analysis is expected to provide an insight into how the proposed model adapts into different road and traffic scenarios through parameter tuning.

4.6 | Extensions to accommodate multiple non-ERV types with multiple ERVs

The impact of extensions is first examined by altering the proportion of heavy vehicles. Three different proportions (10%, 20%, and 30%) of heavy vehicles are tested considering the real mix of heavy vehicles in the field. The new parameter (β_{11}) that indicates the side friction of heavy vehicles is chosen larger than the existing one (β_1) of cars.

Second, the impact of multiple ERVs on solutions is tested when two different ERVs simultaneously appear in a road segment, assuming that an ERV's trajectory is not overlapped with the other ERV's trajectory. Last, the case where two ERVs' trajectories are optimized when 10% of non-ERVs are heavy vehicles.

5 | SIMULATION EXPERIMENTS

5.1 | Results from different objective functions

To evaluate the influence of including the disutility of non-ERVs, both objective functions are tested by varying the number of non-ERVs and the number of lanes, respectively.

5.1.1 | Both objectives with varying numbers of vehicles

The number of non-ERVs is set to 5, 10, 30, and 40, with all other road conditions being equal to the base-

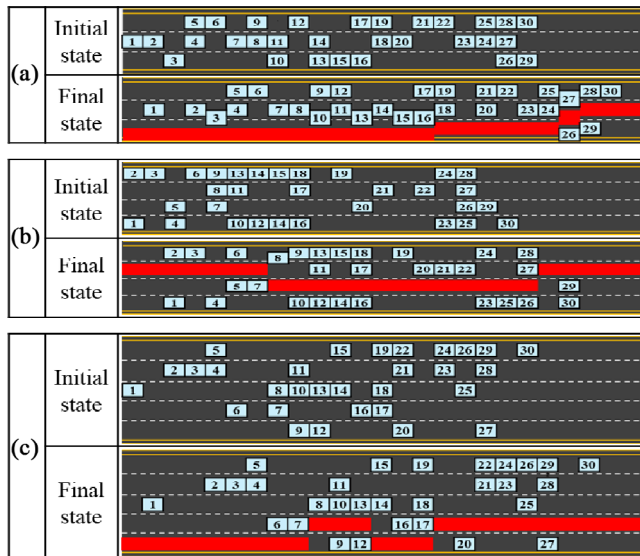


FIGURE 6 Optimal ERV paths from different road widths.

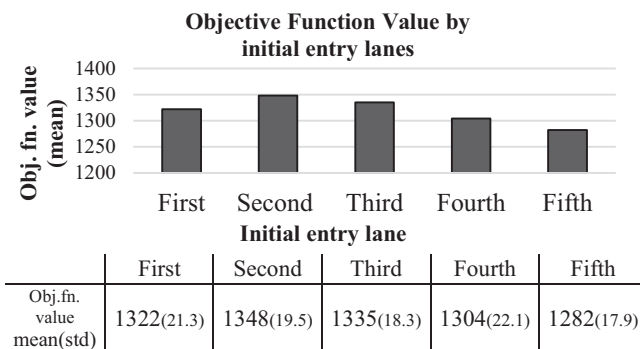


FIGURE 7 ERV speed by changing initial entry lanes in a 5-lane road.

line setting. As mentioned earlier, 40 non-ERVs exceeds the threshold at which the existing algorithm starts to fail. This setting is to investigate the impact of congestion levels on the ERV routing with different objectives. The performance measure is the ratio of ERV speed under Obj. B with respect to that under Obj. A. A larger ratio implies that the ERV speed would be less affected by considering the disutility of non-ERVs. Figure 5a shows that as the congestion level increases, the ERV speed ratio decreases from 99.8% to 41.2%. Nonetheless, the experiment reveals that the speed reduction is negligible before the number of non-ERVs exceeds 30. That is, despite including the damage reduction of non-ERVs in the objective function, an ERV can maintain a higher speed unless the traffic is severely congested. It turns out that the proposed model can thus provide a threshold of traffic condition that is not affected by including non-ERVs' disutility.

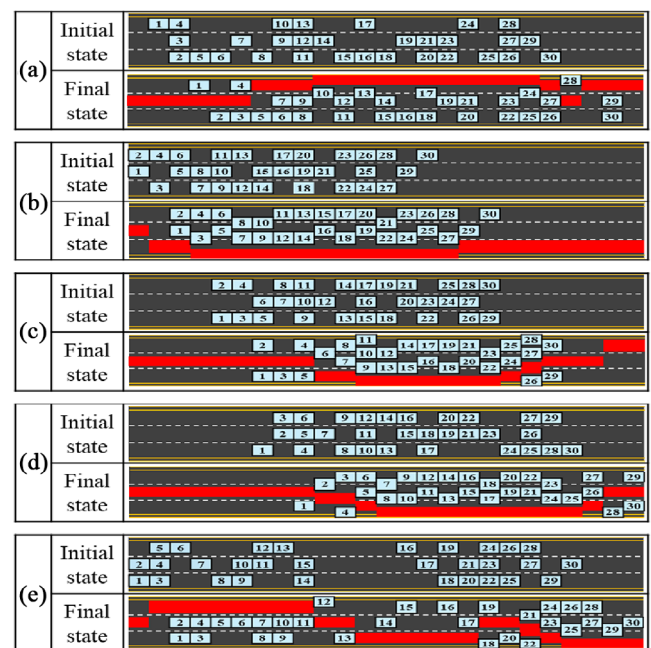


FIGURE 8 The optimal ERV path according to varying positional distributions of non-ERVs.

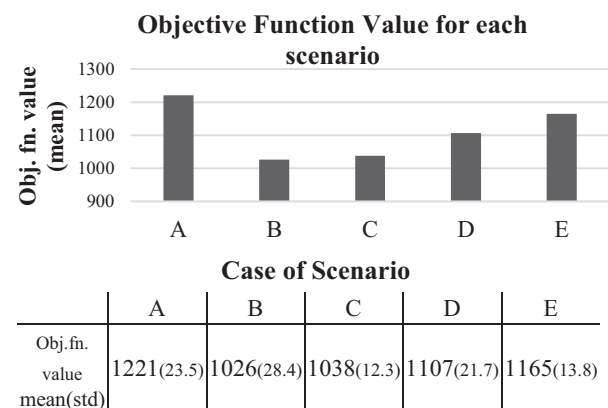


FIGURE 9 Objective function values of ERV under different traffic density distributions.

5.1.2 | Both objectives with varying numbers of lanes

Figure 5b shows the impact of both objective functions by altering the number of lanes from three to five, with the number of non-ERVs fixed at 30. The ERV speed under Obj. B was 89.5% of that under Obj. A for the 3-lane configuration and 94.4% for the 5-lane scenario. The difference in speed ratios is not that large, so it turns out that even a 3-lane road can take care of non-ERVs within a tolerant reduction (−10.5%) in ERV speed. Once the number of lanes exceeded 3, there is negligible difference between the ERV speed from both objective functions. In conclu-



sion, if a highway has sufficient capacity with multiple lanes, considering the disadvantage of non-ERVs in the objective function does not significantly reduce the ERV speed.

5.2 | Various layouts of road segment

5.2.1 | ERV routing under different lane configuration

The present study investigates the optimal trajectory of an ERV for different numbers of lanes. This analysis focuses on how the road width affects an ERV's straddling between normal lanes.

As shown in Figure 6, scenario A represents a near-congested traffic condition of 30 non-ERVs on a 3-lane road segment, where the limited road width restricts an ERV's smooth passage. Under this setting, an ERV mainly utilizes road shoulder to compose its optimal path and straddles between lanes just for lane change. This result demonstrates that the proposed model can secure an ERV passage under restricted road width. Compared to the 3-lane scenario, scenarios B and C with four and five lanes, respectively, demonstrates more flexibility in lateral movement. ERV trajectories are more likely to follow normal lanes, and lane changes are performed more smoothly. In contrast to the 3-lane scenario, the optimal path does not include informal positions. This confirms that the model's validity to show rational behaviors of ERV according to road traffic conditions.

5.2.2 | Impact of an ERV's initial entry position

The impact of an ERV's entry position on its optimal speed is examined, being other conditions equal to the baseline setting. Depending on entry lane, an ERV may face different levels of interference with non-ERVs, even under the same traffic volume. Experiments are conducted for a 5-lane road segment, and the results from five different entry positions are examined. For each case, the proposed model derives an ERV's optimal trajectory and speed with the ERV entry position fixed.

As shown in Figure 7, an ERV achieves different objective function values for varying entry positions. This result suggests that the proposed model works well with an additional constraint that fixes the entry position of an ERV. Thus, the recursive implementation of the proposed model makes it possible to match the entry position with the exit

position of the previous upstream road segment when road segments are connected in a row.

5.3 | Impact of spatial distribution of non-ERVs

The influence of spatial distribution of non-ERVs at the initial stage is examined on ERV routing. Five scenarios (A to E), each with the same number of non-ERVs but different distributions of vehicle positions, are tested. The test results are shown in Figure 8.

Each of the five scenarios represents a distinct spatial distribution of non-ERVs along the road segment. In scenario A, vehicles are evenly dispersed along the segment. Scenario B features non-ERVs near the road entry, while scenario C places non-ERVs only in the middle of road. In scenario D, a vehicle cluster resides near the road exit. Finally, Scenario E shows a bimodal distribution, with vehicles clustered at both ends of the road, leaving the central region vacated.

Figure 8 presents the optimal ERV trajectory for five scenarios, and Figure 9 compares the average objective function value over 10 model implementations for each scenario. The standard deviation is indicated in parenthesis. The ERV speed is used interchangeably with the objective function value in what follows. Scenario A, wherein traffic is most evenly distributed, achieves the best ERV speed. Scenarios B, C, and D, wherein vehicle platoons cluster, results in relatively lower ERV speeds. The location of vehicle platoons significantly affects the ERV speed.

The closer to the road entry a vehicle platoon is, the more exacerbated the ERV progression is. The ERV speed in scenario E, wherein two vehicle clusters reside, is higher than those for three other scenarios of a single vehicle cluster but is smaller than that of scenario A with evenly distributed non-ERVs. In summary, as non-ERVs get more evenly dispersed along the road segment at the initial stage, the ERV speed turns out to be higher.

Another important finding is that across all five scenarios an ERV's dependence on road shoulder is larger than that on straddling between lanes, which implies that lane straddling is adequate only for temporary lane changing. This is because lane straddling over a longer distance is constrained in the formulation. However, if lane straddling is not allowed, a feasible ERV trajectory cannot be derived. The main advantage of the proposed model is that it allows an ERV to pass through road sections that are partially blocked by traffic.

Effects of Varying Internal Parameter Values on ERV speed

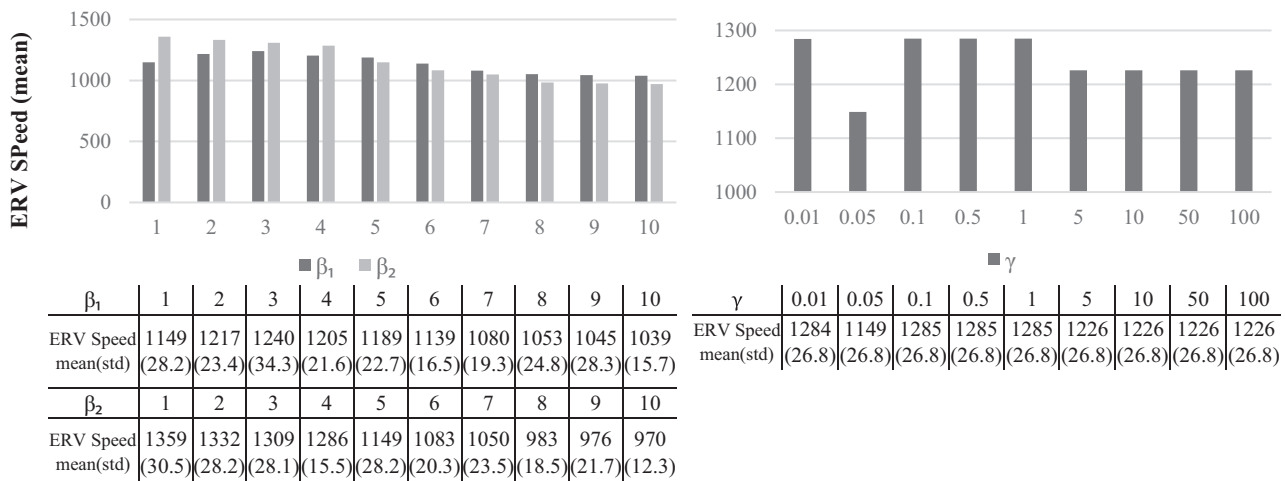


FIGURE 10 Impact of internal parameters on objective function value.

5.4 | Sensitivity analysis of parameters

5.4.1 | Sensitivity analysis of internal and external parameters

Figure 10 provides insights into how internal parameters affect the ERV routing under various traffic distributions. All other conditions align with the baseline setting for this analysis. The ERV speed in graphs denotes the objective function value, averaged over results from ten different random vehicle position patterns at the initial stage. The standard deviation is in the parenthesis for reference.

β_1 penalizes the side friction from adjacent non-ERVs. When β_1 is set to a moderate level ($= 3$), the ERV speed ($=$ the objective function value) is maximized. As β_1 increases over 3, the ERV speed dwindles and stabilized. β_2 penalizes the ERV speed for the use of informal positions. The ERV speed decreases as β_2 increases from 1 to 7, and beyond 7, the speed stabilizes.

The parameter γ penalizes an ERV's lane changes. The lowest ERV speed is found when γ is 0.05, and this value is chosen as the baseline. Other values for γ less than 1 are at a similar level. When increasing γ to larger values than 1, there is a marginal decrease in ERV speed.

Parameters (β_1 , β_2 , γ) for the speed reduction could be calibrated using data. How an ERV slows down when they change lanes can be measured in the field. The speed reduction due to side friction and straddling is also observable. If observing data is possible, parameters can be estimated with a simple linear regression model. When estimating parameters, speed scaling is required, rather than using physical speeds directly.

How external parameters α_1 , α_2 , and α_3 influence the ERV speed is also investigated. All other conditions align with the baseline setting for this sensitivity analysis. Because external parameters changes the objective function value, the impact of altering each external parameter is measured by the speed sum ($\sum_x s_x + s_x^{\text{env1}} + s_x^{\text{env2}}$) without weighting, which is called the ERV speed in Figure 11. α_1 and α_2 has an increasing effect on the ERV speed. When s_x^{env1} is heavily weighted with higher α_1 , the model tends to reduce the side friction and increase the ERV speed.

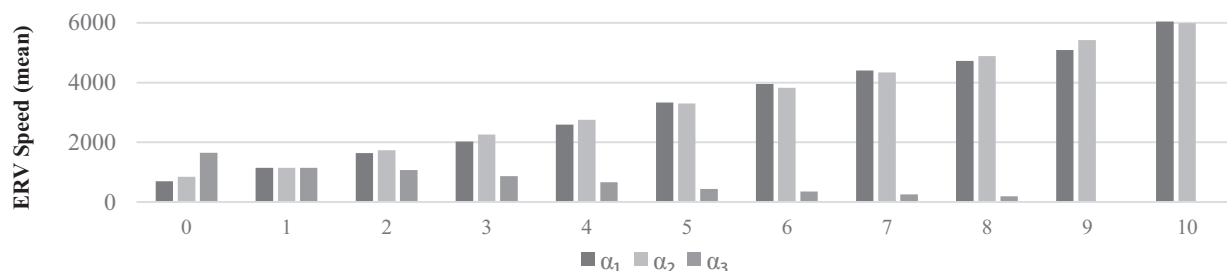
Increasing α_2 that weighs s_x^{env2} also leads to a higher ERV speed. When straddling is more penalized with a higher α_2 , an ERV tends to stay in a normal lane, which implies the advantage of running on normal lanes. As the speed reduction due to straddling lessens, the ERV speed tends to increase.

In contrast to α_1 and α_2 that penalize the ERV movement, α_3 penalizes the lateral movements of non-ERVs. As α_3 increases, the ERV speed declines. This drop indicates that respecting non-ERVs' utilities leads to an ERV's slower progression. If α_3 exceeds a threshold ($= 8$), the ERV speed drops below zero which is unrealistic, and the speed is excluded in the table.

Notably, Figure 12 shows that when α_3 is large ($= 7$), the ERV path becomes more winding, making it difficult to get better emergency operations. For the larger α_3 , ERV tends to follow a route that depends more on straddling or lane changing. This implies that as non-ERVs more stick to their initial lateral positions, it becomes harder to for an ERV to find a straight trajectory.

Overall, maximizing the ERV speed requires three conditions: minimizing side friction with surrounding vehi-

Effects of Varying External Parameter Values on ERV speed



α_1	0	1	2	3	4	5	6	7	8	9	10
ERV Speed	690	1149	1641	2024	2588	3337	3955	4400	4723	5093	6048
Mean(std)	(26.8)	(28.2)	(22.5)	(15.3)	(18.7)	(18.5)	(21.4)	(17.3)	(25.6)	(26.2)	(40.4)
α_2	0	1	2	3	4	5	6	7	8	9	10
ERV Speed	841	1149	1731	2265	2752	3301	3823	4337	4884	5426	5993
Mean(std)	(30.1)	(28.2)	(25.3)	(30.1)	(18.6)	(22.6)	(12.8)	(30.5)	(35.1)	(41.8)	(40.7)
α_3	0	1	2	3	4	5	6	7	8	9	10
ERV Speed	1654	1149	1075	865	658	439	348	254	189	-	-
Mean(std)	(26.4)	(28.2)	(19.3)	(21.2)	(22.5)	(17.5)	(19.1)	(24.1)	(23.7)	-	-

* $\alpha_3 > 8$: ERV speed drops below 0

FIGURE 11 Impact of external parameters on objective function value.



FIGURE 12 ERV trajectory comparison under different α_3 settings.

cles, promoting uninterrupted movement along normal lanes, and ensuring sufficient lateral space for non-ERVs. Moreover, the ERV management strategies must prioritize the usage of normal lanes, while allowing lane straddling only to pass through local traffic concentration.

5.4.2 Sensitivity analysis for MSD

The impact of varying MSDs on the difference in vehicle position between initial and final stages is investigated. Altering MSDs reflects various applications of initial speeds, deceleration rates, vehicle types, and partial road conditions. More concretely, the impact of varying ranges of stopping positions is examined. The final position of each non-ERV must be within a random buffer after its MSD. Figure 13 shows that the final yielding position tends to be farther from the initial position as the sum of MSD and buffer gets larger.

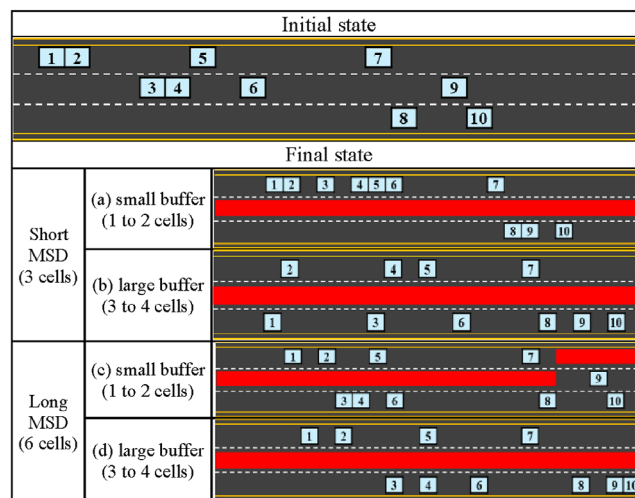


FIGURE 13 The impact of varying MSDs and buffers.

5.5 | Test results from extended models

5.5.1 | Accommodating heavy vehicles

Figure 14 shows the initial and final stages of non-ERVs when a single ERV is interfered with two different non-ERV types. The results show that an ERV's trajectory gets straighter as the proportion of heavy vehicles increases. The ERV speed, however, deteriorates as the proportion increases. This is because the large proportion of heavy vehicles increases side friction. This is due to the larger length of heavy vehicles limiting their movement for yielding.

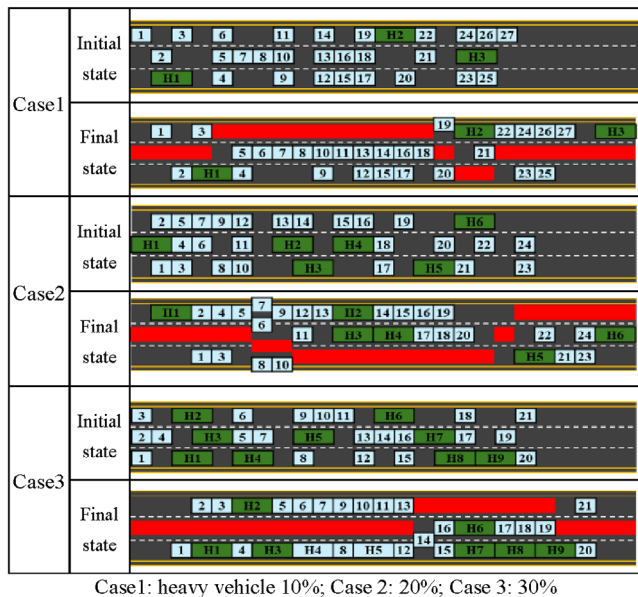


FIGURE 14 The impact of heavy vehicles on the ERV trajectory according to different mix rates.

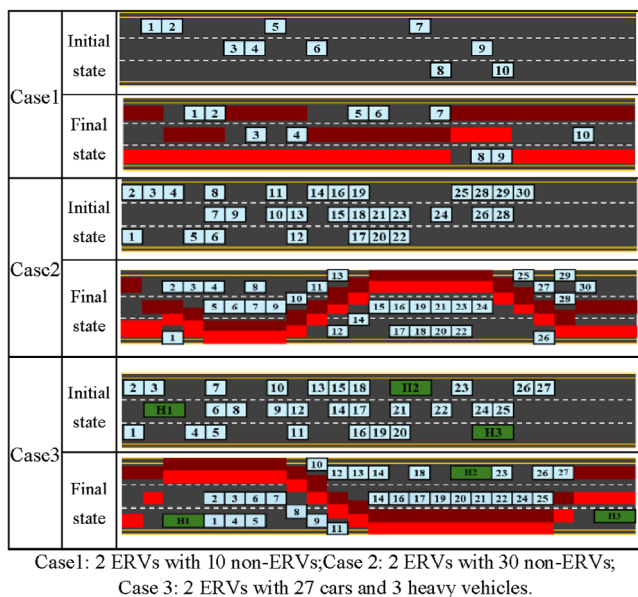


FIGURE 15 The impact of two ERVs appearing simultaneously.

5.5.2 | Accommodating multiple ERVs at the same time

Figure 15 shows the impact of two ERVs on solutions when they simultaneously appear in a road segment. According to cases 1 and 2 in the figure, two ERV tend to be adjacent to each other along the longitudinal trajectory as the number of non-ERVs increases. The proposed model thus shows that treating two ERVs together is more efficient for

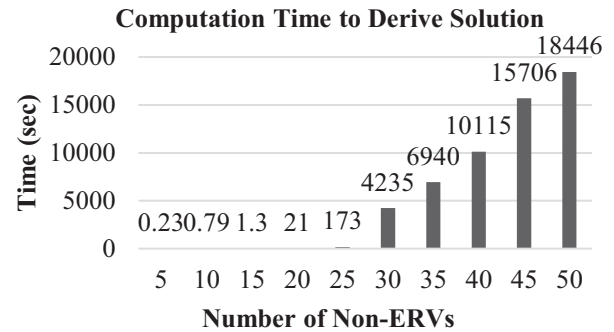


FIGURE 16 Computation times to solve the proposed ILP by varying non-ERV volumes.

congested conditions. When heavy vehicles are included (= case 3), there is no drastic change from case 2 without heavy vehicles, keeping two ERVs moving in parallel.

5.6 | Computation time to implementation

5.6.1 | Computation time

Figure 16 shows computing times according to varying numbers of non-ERVs. Computation time increases exponentially with the number of non-ERVs. While the model run completes in a second when 10 non-ERVs are involved, it requires over 18,000 s for a severely congested traffic condition with 50 vehicles. This trend poses a scalability issue; however, such a complication is not confined only to the present study but remains unresolved for every large-scale ILP that depends on a partial enumeration algorithm such as branch-and-cut or branch-and-price. An online application of the proposed model thus would require considerable computing resources, as the number of non-ERVs increases. All tests in the present study are executed using the CPLEX solver on a computing platform equipped with a 3.7–1.4 GHz CPU and 256 GB of RAM.

Two practical measures are taken in this study to resolve the long computing time issue. First, we focus on the technical scheme to reduce the computing time. The computing time presented in Figure 16 is based on a personal computer. The computation is moved into a server with multiple CPUs. Unfortunately, CPLEX doesn't support GPU-based computing. Second, a meta-heuristic algorithm is devised to deal with large-scale problems. In Section 5.7 a genetic algorithm (GA) is devised to solve the proposed problem. As a result, adopting GA and multi-thread computing turns out to reduce computing times for deriving solutions. The test results are shown in Section 5.7.5.

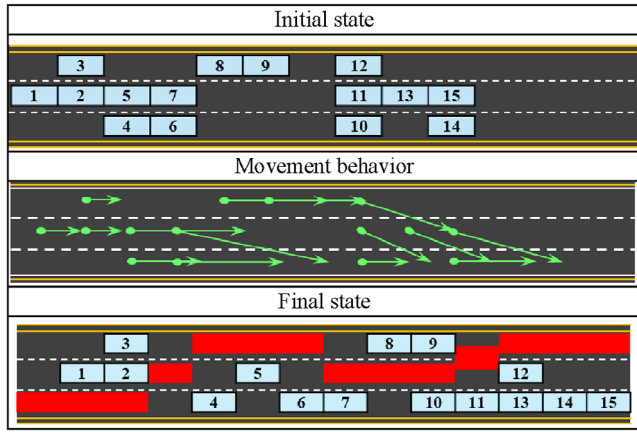


FIGURE 17 Non-ERVs' transition to their yielding positions.

5.6.2 | Implementation

In real-time applications, the model solving for the next road segment is done while ERVs are passing through the previous road segment. Moving non-ERVs to optimal yielding positions in the next road segment is also done during this period after deriving a solution. The computing time should be sufficiently short to secure the transition of non-ERVs. GA computing time is sufficient to fulfill these two tasks (see Section 5.7.5).

Since moving non-ERVs to their yielding positions takes time, such a preemptive implementation of the proposed model is inevitable. Figure 17 shows an example of moving non-ERVs without conflicts. Owing to constraints (Inequalities 9 and 10) to avert overtaking between non-ERVs, every solution ensures non-ERVs' fast movement.

5.7 | Design and application of genetic algorithm

To address the high computational cost observed when solving the exact ILP formulation, a heuristic framework based on GA is developed. The GA aims to efficiently approximate the optimal routing and coordination of an ERV and non-ERVs under the same spatial and operational constraints described in Section 3.

5.7.1 | Chromosome representation

Each individual (chromosome) encodes the lateral trajectory of an ERV along the longitudinal axis $x = 0, \dots, L$.

The gene sequence $C = [y_0, y_1, \dots, y_L]$ specifies the ERV's lateral position at each longitudinal position, restricted by the continuity constraint $|y_x - y_{x-1}| \leq 2$.

A small auxiliary vector R encodes local displacement rules for non-ERVs, which guide their repositioning during evaluation.

5.7.2 | Decoding and evaluation

Given a chromosome (C, R) , the final configuration of non-ERVs is generated using a greedy placement with repair procedure:

- For each non-ERV j , a feasible stopping range $[x_j^{\min}, x_j^{\max}]$ is computed based on the MSD rule.
- Vehicles are sequentially assigned to admissible (x, y) cells following their initial order, applying the rule R to prioritize available shoulder or lane positions.
- If a constraint violation occurs (e.g. overlap, adjacent occupancy, order inversion), a repair step relocates the vehicle longitudinally $(x + 1, x + 2)$ within its feasible range.

This decoding process enforces all operational constraints such as continuity, single occupancy, order preservation, minimum distance, and lateral safety without solving an optimization sub-problem, thereby ensuring linear computational complexity in the number of vehicles.

5.7.3 | Fitness function

The fitness function denoted as Equation (78) approximates the original objective function by balancing an ERV's moving efficiency and non-ERVs' inconvenience:

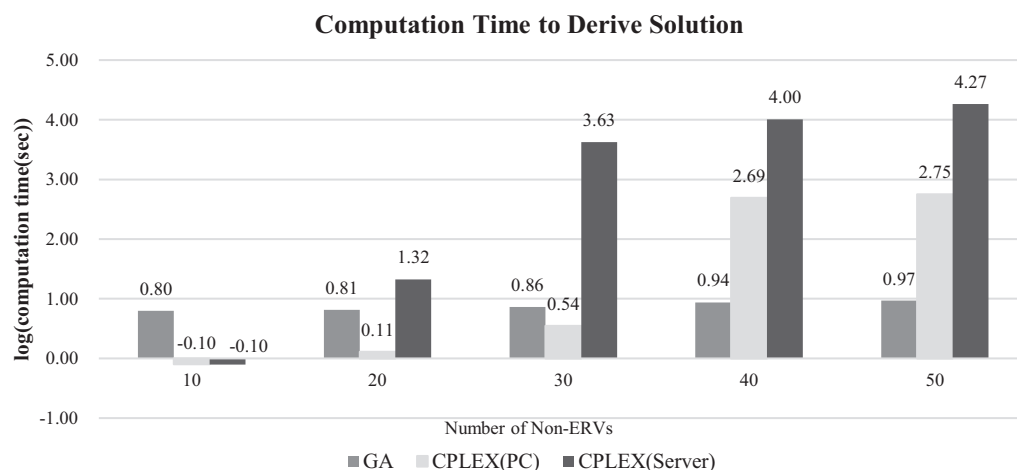
$$F = \sum_{x=1}^L [s_x - \gamma 1(y_x \neq y_{x-1}) - \beta_1 C_x - \beta_2 E_x] - \alpha_3 \sum_j |y_j^{\text{final}} - y_j^0| - M \cdot V \quad (78)$$

where C_x and E_x denote side friction and shoulder/straddle penalties, respectively, and V is the total number of violated constraints.

This formulation maintains interpretability with the coefficients $\alpha_i, \beta_i, \gamma$ defined in Section 3.

5.7.4 | Genetic operations

The present GA employs a population of 100 individuals with tournament selection, two-point or block crossover, and mutation through random lateral shifts or smoothing operations. Elitism preserves the top $k (= 3)$ individuals



Number of Non-ERV	Computation time(sec)			log(Computation time)			Objective function value		(A) / (B) (%)	(C)* (%)	(D)** (%)
	GA	CPLEX (PC)	CPLEX (Server)	GA	CPLEX (PC)	CPLEX (Server)	GA (A)	CPLEX (B)			
10	6.3	0.8	0.8	0.80	-0.10	-0.10	998	1,108	90.7	94.7	90
20	6.5	21	1.3	0.81	0.11	1.32	998	1,095	91.1	94.7	90
30	7.3	4.2	3.5	0.86	0.54	3.63	993	1,060	93.9	87.5	80
40	8.7	1,0115	491	0.94	2.69	4.00	576	590	97.6	58.3	60
50	9.3	18,446	564	0.97	2.75	4.27	304	335	90.7	58.3	60

* (C) : GA success rate of deriving the exact solution out of feasible solutions

** (D) : GA rate of deriving a feasible solution

FIGURE 18 Computing time comparison across different solution algorithms.

per generation. A local 1-opt search is periodically applied to elite chromosomes to fine-tune lateral positions and shorten straddle segments.

5.7.5 | Experimental results from GA

GA converges within 100th generations and achieves feasible near-optimal solutions with two to three orders of magnitude shorter computation time compared to the exact solver (CPLEX) for more than 20 non-ERVs. The population size is set to 100, and the mutation rate is set to 3% in the experiment.

Performance metrics such as average ERV speed, number of lane changes, total non-ERV displacement, and computation time were evaluated under the same test settings as Section 4.

The results demonstrate that the GA can preserve the essential behavioral patterns of the optimal solution while substantially improving computational efficiency, making it suitable for real-time or large-scale emergency routing applications. It should be, however, noted that GA does not guarantee the exact solution every time implemented. Even it might fail in finding a feasible solution.

Figure 18 shows the comparison of computing times of

three different schemes. Computing times presented are the average of 20 experiments, each of which has a different spatial distribution of non-ERVs at the initial stage. The multi-thread computing in server reduces the computing time, but the computing time is still inadequate for real-time applications. GA achieves practical computing times at the expense of losing optimality or feasibility. The percentage of obtaining a feasible solution decreases as the number of non-ERVs, and the success rate of deriving the exact solution also decreases with the number of non-ERVs. The objective function value averaged over feasible solutions, however, is very close to that of the exact solution obtained by CPLEX.

A more promising result is that the computing time of GA is consistent across increasing numbers of non-ERVs. This advantage makes it possible to adapt the proposed model to real-time applications.

6 | CONCLUSION

This study proposed an ILP-based modeling framework for determining optimal trajectories for an ERV in congested traffic conditions. The model allows an ERV to run along shoulders and lane boundaries—to generate an optimal



path. By assigning final positions to non-ERVs, the model ensures optimal yielding behaviors.

Experimental results confirmed that the proposed method can identify a viable ERV trajectory even under severely congested conditions. A sensitivity analysis shows that model parameters significantly affect ERV speed and path generation.

Unlike previous models that assume strict lane discipline and is applicable only to uncongested traffic conditions, the proposed model uniquely captures informal yielding behaviors—such as lane straddling and shoulder utilization—that frequently occur in real-world emergency scenarios. This advantage allows for realistic and robust ERV routing even when road sections are partially blocked by vehicles, setting a new direction for emergency management strategies in CAV environments.

The present model is set up under the strong assumption of a perfect CAV environment. Unfortunately, accommodating uncertainties due to partial observability, communication latency, and delay in response are not considered. Future work will extend this framework to relax the CAV environment assumption for field test. In addition, the proposed model will be extended to cover multi-segment links and intersections, incorporate time-varying elements, and explore more rigorous solution algorithms to resolve the scalability problem and to make possible the real-time applicability.

CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

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APPENDICES

Tables A1 and A2.

TABLE A1 Literature survey summary.

Topic	Reference	Method	Contribution
Traffic signal preemption	Samant & Adeli (2001)	Wavelet	Wavelet–NN incident detection; preemption trigger.
	Karim & Adeli (2003)	Wavelet	Wavelet energy incident detection; faster alarms.
	Wang et al. (2013)	Conceptual modeling	Classifies preemption (route/intersection/segment) and models differences
	Nellore & Hancke (2016)	Computer vision + WSN	Vision-based ERV detection to trigger signals without V2X
	Li et al. (2018)	Heuristic	Compensates V2X latency to improve preemption accuracy
	Jung et al. (2021)	RL	Real-time V2X-assisted signal priority
	Cao et al. (2022)	RL	Learns adaptive signal priority for approaching ERVs

(Continues)



TABLE A1 (Continued)

Topic	Reference	Method	Contribution
Segment routing & spatial reallocation (lane changing, dynamic routing)	Kaufman et al. (1998)	MILP	Time-dependent routing via time-expanded networks
	Hooshdar & Adeli (2004)	NN	NN-based VMS for work-zones; lane guidance. (work zones)
	Jiang & Adeli (2004)	DSS	IntelliZone DSS: capacity/queue estimates
	Jiang & Adeli (2004, ODOT)	DSS	IntelliZone DSS for work-zone operations
	Polimeni & Vitetta (2011)	Dijkstra	Modified Dijkstra for evacuation time-varying links
	Katsuma & Yoshida (2018)	VANET	Route reconstruction via vehicular ad-hoc networks
	Firooze et al. (2018)	MILP	Location-relocation with vehicle unavailability
	Hajibabai & Saha (2018)	MINLP	Joint siting of patrol stations and patrol routes
	Hannoun et al. (2019)	ILP	Fully connected ILP for ERV pathing in a segment
	Hannoun et al. (2020)	ILP	Extends segment ILP to network with intersections
	Hannoun et al. (2021)	Sequential ILP	Partially connected traffic via rolling optimization
	Zeng et al. (2021)	Rule-based	Coordinated lane pre-clearing (advance yielding)
	Sarkar et al. (2022)	Heuristic	Survey of static shortest-path/heuristics
	Liu et al. (2022)	V2V/IoV	IoV/V2V platoon-based yielding control
	Duan et al. (2022)	Metaheuristic	Joint ERV-evacuation trajectory planning (Salp swarm)
	Hannoun & Menéndez (2022)	Design/ops	Modular EMS vehicle architecture for flexible deployment
	Alzubaidi et al. (2023)	Deep RL	RL for faster ERV lane changes
	Ding et al. (2024)	Two-stage RL	V2X-assisted mobility-aware lane change
	Liu et al. (2024)	Bi-level	Trajectory optimization with microscopic scheduling
Integrated signal + routing	Adeli & Karim (2005)	Monograph	Wavelets + soft computing in ITS (survey).
	Elalouf (2012)	Robust/stochastic optimization	Routing under uncertainty (stochastic delays)
	Mu et al. (2018)	Integrated control	Route-aware preemption along ERV path
	Su et al. (2023)	MARL	Joint priority of route & signals with RL
	Liu et al. (2024, evacuation)	Bi-level integration	Upper-level departures/routes + lower-level signals
	Alruwaili et al. (2025)	DL (LSTM+ResNet)	Spatio-temporal traffic forecasting to aid routing & signals



TABLE A2 Original constraints before linearization.

Description	Original nonlinear constraints	Linearized constraints
The loss of non-ERVs in the objective function	$\sum_j \sum_x \sum_y \alpha_3 \cdot y \cdot v_{x,y,j} - y_j^0 $	$\sum_j \alpha_3 \cdot d_j$ (3) $r_{x,y,j} \geq (y - y_j^0) \cdot v_{x,y,j} \quad \forall x, y, j$ (4) $r_{x,y,j} \geq -(y - y_j^0) \cdot v_{x,y,j} \quad \forall x, y, j$ (5) Where $d_j = \sum_x \sum_y r_{x,y,j}$.
Non-ERVs maintain the initial positional order without passing	$(X_j^{\text{fin}} - X_{j'}^{\text{fin}})(x_{j0} - x_{j'0}) \geq 0, \quad \forall j' > j$ $X_j^{\text{fin}} = \sum_x x \sum_y v_{x,y,j}$ $(Y_j^{\text{fin}} - Y_{j'}^{\text{fin}})(y_{j0} - y_{j'0}) \geq 0, \quad \forall j' > j$ $Y_j^{\text{fin}} = \sum_y y \sum_x v_{x,y,j}$	$\sum_{x' < x} \sum_y v_{x',y,j'} \leq M(1 - \sum_y v_{x,y,j})$ $\forall j' > j, x_j^0 < x_{j'}^0$ (8) $\sum_{y' < y} v_{x,y',j'} \leq M(1 - v_{x,y,j})$ $\forall j' > j, y_j^0 < y_{j'}^0$ (9)
Non-ERVs cannot be laterally adjacent to an ERV in the proposed lane-numbering convention	$w_{x,y} \cdot (\sum_j v_{x,y-1,j}) = 0 \quad \forall x, y$ $w_{x,y} \cdot (\sum_j v_{x,y+1,j}) = 0 \quad \forall x, y$	$\sum_j v_{x,y-1,j} \leq M(1 - w_{x,y}) \quad \forall x, y$ (16) $\sum_j v_{x,y+1,j} \leq M(1 - w_{x,y}) \quad \forall x, y$ (17)
If an ERV switches to the right lane or straddling position, the next cell in the progressing direction of the ERV should be vacated, and vice versa to the left.	$w_{x,y} \cdot w_{x+1,y-1} \cdot (\sum_j v_{x+1,y,j}) = 0, \quad \forall x, y$ $w_{x,y} \cdot w_{x+1,y+1} \cdot (\sum_j v_{x+1,y,j}) = 0, \quad \forall x, y$	$z_{x,y}^+ \leq w_{x,y}, \quad \forall x, y$ (18) $z_{x,y}^+ \leq w_{x+1,y-1}, \quad \forall x, y$ (19) $z_{x,y}^+ \geq w_{x,y} + w_{x+1,y-1} - 1, \quad \forall x, y$ (20) $\sum_j v_{x+1,y,j} \leq M(1 - z_{x,y}^+), \quad \forall x, y$ (21) $z_{x,y}^- \leq w_{x,y}, \quad \forall x, y$ (22) $z_{x,y}^- \leq w_{x+1,y+1}, \quad \forall x, y$ (23) $z_{x,y}^- \geq w_{x,y} + w_{x+1,y+1} - 1, \quad \forall x, y$ (24) $\sum_j v_{x+1,y,j} \leq M(1 - z_{x,y}^-), \quad \forall x, y$ (25)
The ERV speed adjustment due to side friction from adjacent non-ERVs	$a_{x,y}^{\text{env1}} \geq w_{x,y-2} + w_{x,y+2} \quad \forall x, y$ (26) $s_x^{\text{env1}} \leq s_x + 1 - \beta_1 \sum_y \sum_j a_{x,y}^{\text{env1}} \cdot v_{x,y,j} \quad \forall x$ (27)	$p_{x,y,j}^{\text{env1}}$ = auxiliary ternary variable $p_{x,y,j}^{\text{env1}} \leq a_{x,y}^{\text{env1}},$ $p_{x,y,j}^{\text{env1}} \leq 2 v_{x,y,j},$ $p_{x,y,j}^{\text{env1}} \geq a_{x,y}^{\text{env1}} - 2(1 - v_{x,y,j})$ $p_{x,y,j}^{\text{env1}} \geq 0$ $s_x^{\text{env1}} \leq s_x + 1 - \beta_1 \sum_y \sum_j p_{x,y,j}^{\text{env1}}, \quad \forall x$