



Original Article

Intelligent metallic loose part monitoring in three-dimensional structures using convolutional neural networks and the position-invariant loss function

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ABSTRACT

Loose parts in the primary systems of nuclear power plants pose a significant safety risk as they can collide with equipment inner surfaces, particularly in high-velocity areas like the steam generator. Such collisions, especially near welded joints in the steam generator tubes, can cause cracks and lead to coolant leakage. To mitigate these risks, nuclear plants use loose parts monitoring systems that trigger alarms based on rule-based algorithms. However, current methods rely on the analyst's proficiency, and timely analysis is challenging. To address these issues, this paper proposes a convolutional neural network (CNN) model to estimate the mass of any detected loose parts and their impact location. The CNN model uses a position-invariant loss function derived from Cartesian coordinates and the Wigner-Ville distribution. The proposed method is validated using impact signals acquired from a quarter-scale testbed specifically designed to simulate a nuclear power plant's primary system steam generator. This study experimentally confirms that the proposed method can estimate the impact location and mass of internal loose parts in three-dimensional structures more accurately and rapidly than existing methods.

1. Introduction

The monitoring of loose parts within the primary system of a nuclear power plant is crucial for enhancing operational safety and mitigating the risk of potential failures, which could lead to significant hazards. Loose parts, which can arise from component degradation, debris, or maintenance activities, present serious threats when they collide with the internal surfaces of the steam generator or reactor [1,2]. Any collisions involving these loose parts can cause structural damage and can lead to severe consequences, such as coolant leakage [3]. Therefore, an accurate and rapid loose parts monitoring system (LPMS) is essential to enable prompt detection of internal impacts, thereby enabling timely maintenance interventions and ensuring the overall safety of a nuclear plant.

Considerable efforts have been devoted to estimating the location and mass of loose parts in nuclear power settings. Conventional approaches are generally divided into two categories: impact location estimation and mass estimation methods. In terms of impact location estimation, one of the mostly widely used techniques is the triangulation method [4–6], which involves calculating the time difference in multiple signal arrivals to determine the distance from each sensor to the loose part. Another method, the circle-intersection method [7,8], can be viewed as a specific case of triangulation, where the location is defined by finding the intersection point of circles drawn around individual sensors, based on the circular wavefronts of signals. As an alternative strategy, frequency and stress characteristics can be analyzed to estimate the mass of the loose part. The frequency characteristic (FC) chart [9] involves examining the frequencies of the signals and correlating their

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features with the mass of the loose part. Additionally, the Hertzian method [10,11] can be employed to describe the contact stress and deformation at the point of impact. However, these methods have significant drawbacks: 1) degraded accuracy due to structural complexity and signal distortion, such as curvature within the primary system, 2) the need for skilled signal analysis, and 3) the time-intensive nature of the process.

Recent advancements have introduced deep-learning methods for estimating the impact location and mass of loose parts, offering promising solutions to the limitations of conventional methods. As a data-driven approach, deep learning generally leverages its capability for nonlinear approximation between the input and output [12,13]. Several deep-learning-based models have been proposed to predict the impact location and mass of loose parts. For instance, Moon et al. [14] developed a two-dimensional convolutional neural network (CNN) model to predict the impact location and mass of loose parts with the ultimate goal of implementing the method to detect loose parts in a nuclear power plant. A dataset of time-series impact signals was generated from the finite element analysis of flat plates. This study demonstrated that the proposed CNN model, which used time-series impact signals as input data, outperformed other machine learning methods, including Gaussian process (GP) and support vector machine (SVM), which used impact parameters such as acceleration amplitude, center frequency, and time difference of arrival (TDOA) for estimation. Additionally, Su et al. [15] proposed a mass estimation method for loose parts in a flat structure using a deep belief network (DBN) and virtual experimental data. The virtual experimental data were converted into auto power spectrum before using them as inputs to the DBN. The authors demonstrated that the proposed model achieved mass-estimation accuracy comparable to real impact cases. Dipierangelo et al. [16] estimated impact locations on aluminum plates using two different methods including polynomial regression and shallow neural networks. Both methods presented the estimation errors of 1.5 mm and 1.2 mm, respectively. This study demonstrated the effectiveness of machine learning methods for impact location estimation in the field of structural health monitoring (SHM). Huang et al. [17] also proposed a distance-assisted graph neural network (DAGNN) for impact location estimation. The DAGNN model employs graph-based supervised learning, considering sensors as nodes and representing the relationships between nodes as a matrix. Additionally, DAGNN incorporates physics-informed constraints, using the physical distances between the impact location and the sensors during the training process. The proposed method was validated on metal plates and demonstrated higher accuracy compared to conventional GNN, GCN, and ANN methods. Seno et al. [18] proposed an ANN model for estimating impact locations on structures under operating conditions, along with a robust time-of-arrival (TOA) feature extraction method suitable for such environments. The ANN model was validated through impact experiments on both flat and curved plates. The results confirmed that it outperformed conventional TOA extraction methods in estimating impact locations. Huang et al. [19] addressed the computational burden of traditional source localization methods by proposing an ANN model that uses a time delay between accelerometer signals as input. The proposed method was validated on plate structures, achieving estimation errors ranging from 0.01 m to 0.1 m. Li et al. [20] introduced a multi-output support vector regression (MSVR) model. The MSVR model uses multi-dimensional features of impact signals as input and outputs the coordinates of the impact location. The proposed approach was validated on plate and cylindrical structures, showing acceptable accuracy levels compared to existing methods.

While these studies have shown the potential of deep-learning methods for estimating the impact parameters of loose parts, the existing literature primarily focuses on simple structures, such as flat plates. It is challenging due to the complexity of three-dimensional structures in real applications, such as those with curvature, which are common in steam generators and reactors. Three-dimensional structures exhibit

different bending wave propagation characteristics and signal distortion, as compared to simple flat structures [21]. The difference underscores the need for advanced methods that can be generalized to more complex geometries of actual nuclear plant components. This is the motivation for this study.

To address the aforementioned limitations, here, we introduce a deep-learning approach capable of accurately predicting the impact location and mass of loose parts in a more generalized structure. Specifically, we propose a CNN-based method that is tailored to handle the complexity of three-dimensional structures that are typical in nuclear power plant components. Our method is integrated with the Wigner-Ville distribution (WVD) and incorporates a novel position-invariant loss function that enhances the model's accuracy in estimating impact parameters in three-dimensional structures with curved surfaces. During impact events, the accelerometer captures transient signals, which are analyzed using time-frequency signal analysis methods. Common techniques include short-time Fourier transform (STFT), wavelet transform (WT), and Wigner-Ville distribution (WVD). Among these, WVD offers superior time and frequency resolution, enabling the generation of high-resolution two-dimensional images that preserve critical signal features. In this study, WVD is employed as the time-frequency analysis method, ensuring precise extraction of impact location from the time domain signal and loose part mass from the frequency domain. Regression models with a conventional loss function can produce significant errors due to misrepresentation of angular coordinates at the lower part of the steam generator. For instance, suppose the actual impact longitude is 0° and the predicted longitude is 359° . An MSE calculation without appropriate coordinate representation yields a large error value of $\sqrt{(0-359)^2} = 359^\circ$. However, in the actual hemispherical geometry, this corresponds to an error of only 1° . The position-invariant loss function effectively addresses the issue and ensures accurate prediction of impact positions. Most of the previously studied ML/DL methods were validated on flat structures. They did not encounter the issues arising from the hemispherical shape in three-dimensional structures. Therefore, To the best of our knowledge, this study is the first attempt that tackles the problem through labeling. We validate our method through both quantitative and qualitative results using a quarter-scale steam generator testbed. Validation results demonstrate significant improvements in both impact location and mass-estimation performance, as compared to existing studies. Additionally, we emphasize the advantage of our approach in online monitoring applications, showcasing its high computational efficiency in rapid impact location and mass estimation. Our findings highlight the potential of the proposed method to advance the identification of impact parameters for loose parts, contributing to the development of accurate and rapid loose parts monitoring systems.

The remainder of this paper is organized as follows: Section 2 provides the research background and an overview of the fundamentals of steam generators within the primary system of a nuclear plant. In Section 3, we introduce our proposed deep-learning framework, detailing its architecture, training, and evaluation strategies. Section 4 describes the impact experiments conducted on a quarter-scale steam generator and explains the acquired dataset. Section 5 presents both quantitative and qualitative results from multiple perspectives, assessing the model's performance in location and mass estimation. Finally, Section 6 summarizes the conclusions of this research.

2. Steam generators in the primary system of a nuclear power plant

The primary system of a nuclear power plant consists of key components centered around the reactor, including coolant pumps, a pressurizer, and steam generators. When loose parts move along the internal flow within the primary system of a nuclear power plant and collide with the outer walls of the steam generator and reactor, bending waves are generated and propagate outward. Multiple accelerometers mounted on the outer walls of the structure can detect these bending waves as they

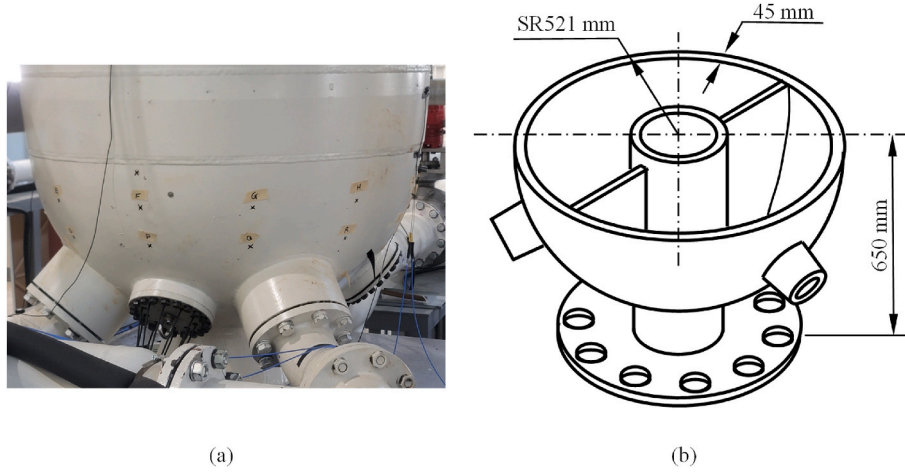


Fig. 1. The quarter-scale steam generator testbed: (a) Real image and (b) Schematic view.

propagate. The recorded accelerometer signals can be used to estimate the impact location and mass of the loose parts.

It is known that loose parts move along the coolant flow and impact the outer walls [22], which is why impacts primarily occur in the head section of the hot leg channel within the steam generator [23], where flow is present. The hot leg channel is located in the lower section of the steam generator. The tubes extending to the upper section of the steam generator are extremely thin. Loose parts capable of causing damage cannot pass through the tube and instead drift within the hot leg. Therefore, it is appropriate to estimate the impact location and mass of loose parts in the lower section of the quarter-scale steam generator. The lower section of the quarter-scale steam generator is made of 304 stainless steel, as shown in Fig. 1(a). The hemispherical shell is with the radius of 521 mm and the thickness of 45 mm, positioned 650 mm above the ground, as presented in Fig. 1(b).

The loose parts used for validation experiments are carbon steel balls, selected due to their uniform shape and material properties, which are suitable for controlled impact testing. A total of five different sizes were employed, with diameters of 25.4 mm, 30.2 mm, 31.8 mm, 34.9 mm, and 36.5 mm, respectively, and their corresponding masses were 69 g, 113 g, 132 g, 174 g, and 199 g. The use of carbon steel ensured consistent impact characteristics across all tests. It is worth noting that the degree of damage to the steam generator varies depending on the amount of impulse during the collision. It is essential to prioritize mass as a critical factor in the evaluation process. The shape of the loose part does not affect changes in impulse.

Although the impact in the quarter-scale does not perfectly match the impact in the real steam generator, the proposed method was validated through impact experiments performed on the quarter-scale model, which closely resembles the actual geometry, showing that it is likely to work in complex three-dimensional shell structures like those of a real steam generator. Regarding the similarity to actual loose parts irregularly shaped objects such as bolts and nuts may exist in real-world scenarios, so the spherical shape used in the experiments may not be identical. However, Hertz's theory of impact is not restricted to spheres and can generally be applied to objects that behave as rigid bodies during impact [10]. Numerous experiments have used spheres for testing [11,15,18,24]. Therefore, this study also used spherical objects for practical reasons when conducting the experiments.

3. Proposed methodology

3.1. Impact signal pre-processing

The impact signals generated by collisions of internal loose parts are typical examples of non-stationary signals. Non-stationary signal

analysis methods are categorized as either 1) instantaneous frequency analysis or 2) time-frequency analysis. Instantaneous frequency analysis provides the evolution of instantaneous frequency values of a signal over time. Unlike the fast Fourier transform (FFT) analysis, instantaneous frequency analysis displays time on the x-axis and frequency values on the y-axis. It is helpful to analyze the characteristics of transient phenomena in signals since this approach enables the reading of frequency values over time. However, instantaneous frequency analysis is limited in that it displays only one frequency value at a given time and is susceptible to noise.

Time-frequency methods are powerful techniques for analyzing non-stationary signals and can be used to extract useful features for condition monitoring. For example, the short-time Fourier transform (STFT), a type of time-frequency method, computes the Fourier spectrum as a function of time shift for a short time window that moves along the time axis. While STFT offers the benefit of easily tracking frequencies that change over time, it is limited by a trade-off between time and frequency resolution, which prevents simultaneous improvement of both resolutions. Another technique, the wavelet transform, decomposes a signal using a set of fixed-shape wavelets that move along the time axis, rather than using sinusoidal functions. The wavelet transform is defined as the convolution of the time-series signal with the wavelet family. The wavelet transform approach provides better time and frequency resolution than STFT and enables detailed frequency analysis of different signal types through the use of diverse wavelets. Nevertheless, the wavelet transform requires numerous parameters to be set by the user and has the drawback of reduced time resolution in the low-frequency range.

In addition, the Wigner distribution (WD), a time-frequency analysis technique, can be derived from the generalized relationship between the power spectrum and the autocorrelation function of non-stationary, time-variant processes. Thus, the Wigner distribution is defined by Equation (1) [25]:

$$W(t, f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right) x^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau \quad (1)$$

where t denotes time and f represents frequency. Meanwhile, the Wigner distribution of an analytic signal is called the Wigner-Ville Distribution (WVD), and the analytic signal is defined as shown in Equation (2) [26]:

$$x_A(t) = x(t) + j\hat{x}(t) \quad (2)$$

where $\hat{x}(t)$ represents the Hilbert transform of the input signal $x(t)$.

In any of these methods, the information regarding the impact location is indicated by the difference in arrival times of the impact wave at each accelerometer [27]. The mass of the loose part can be

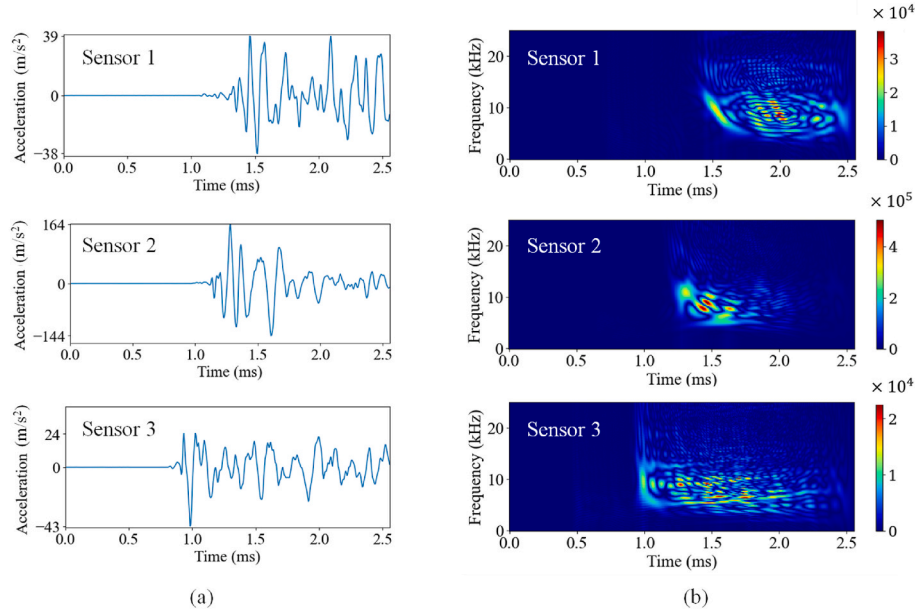


Fig. 2. Representative example of measured signals from accelerometers (a) One-dimensional time series and (b) Two-dimensional image processed by WVD.

determined from the frequency of the impact signal [28]. WVD offers superior resolution in both the time and frequency domains, as compared to the time-frequency methods of STFT and wavelet transform discussed earlier. Therefore, WVD provides more accurate information about the signal's inherent group delay and instantaneous frequency [27]. However, WVD can produce cross-terms due to the nonlinearity of its autocorrelation function, regardless of the actual signal. To remove cross-terms with unclear physical meaning, kernels in the time domain or frequency domain are sometimes used [29]. However, when using WVD for pattern recognition, removing cross-terms can lead to a reduction in diagnostic performance [26]. As a result, this study did not use a separate kernel to eliminate cross-terms when transforming acceleration signals into WVD images.

Fig. 2 illustrates an example of an image preprocessed using WVD from impact signals; the image was acquired from the quarter-scale steam generator tested. Fig. 2(a) illustrates the signals obtained from three accelerometers, showing time differences in their measurements, while Fig. 2(b) displays the WVD-transformed image with clearly visible frequency components. The impact experiment yields time-series impact signals, as shown in Fig. 2(a). When the WVD method is employed, the time domain signals are transformed into two-dimensional images, as illustrated in Fig. 2(b), where the x-axis represents time and the y-axis represents frequency. After employing the WVD, key features emerge from the transformed data. Specifically, the TDOA of impact signals across sensors, as depicted in Fig. 2(b), serve as critical features for determining the impact location. Additionally, the center frequency and signal intensity become significant features for estimating the mass of the loose part.

3.2. Proposed neural network architecture

This section presents a novel CNN-based neural network model that estimates the impact location and mass of loose parts based on time-frequency images of impact waves. The proposed model consists of convolutional layers for feature learning and fully connected layers for impact location and mass regression. The detailed structure of the proposed approach is shown in Fig. 3.

The feature learning layers are constructed with a series of 2D convolution blocks, each consisting of 2D convolution layers and ReLU activation functions to extract high-dimensional feature maps from

WVD images. The convolution layers are designed to gradually shrink the receptive field towards the following layers, extracting for the abstraction of time-frequency information. In object recognition, conventional CNN architectures employ a combination of convolutional layers and max-pooling layers to progressively reduce the dimensions of the feature maps [30]. However, this process can lead to data loss [31]. To prevent this limitation, the model proposed in this paper substitutes the stride convolution layer for the max-pooling layer, thereby mitigating data loss and enabling the use of learnable kernels.

Regression layers are used to accurately estimate the impact location and loose part mass using the extracted high-dimensional feature maps. These feature maps are passed through a series of fully connected layers to predict the impact location and mass at the output layer. The learning process uses backpropagation and the Adam optimization algorithm to minimize the loss function.

As mentioned in Section 3.1, in WVD images, the time axis is associated with impact location information, while the frequency axis and signal intensity are related to the impact mass. For a single impact, the WVD images of the impact signals measured from three sensor channels are stacked into a three-channel image, similar to an RGB image, forming a unique representation of the specific impact. The CNN model is trained to estimate the impact location and mass based on these three-channel WVD images. The rationale for selecting CNN lies in its specialization in image-based tasks [32], such as image classification, making it particularly effective for processing and analyzing image data like the WVD representations used in this study.

3.3. Position-invariant loss function

Several existing methods have been proposed to represent positions in three-dimensional space. Conventional methods include the Cartesian coordinate system, the cylindrical coordinate system, and the polar coordinate system. In addition, there are curvilinear coordinates, which are a type of coordinate system where the coordinate lines are curved in Euclidean space [33–36]. Another example is the Barycentric coordinate system, which references the vertices of a tetrahedron in three-dimensional space to specify positions [37–40].

Mildenhall et al. [41] proposed a method that combines the spatial coordinates of an object with the observation positions to represent space. This method was used to generate images from a new viewpoint

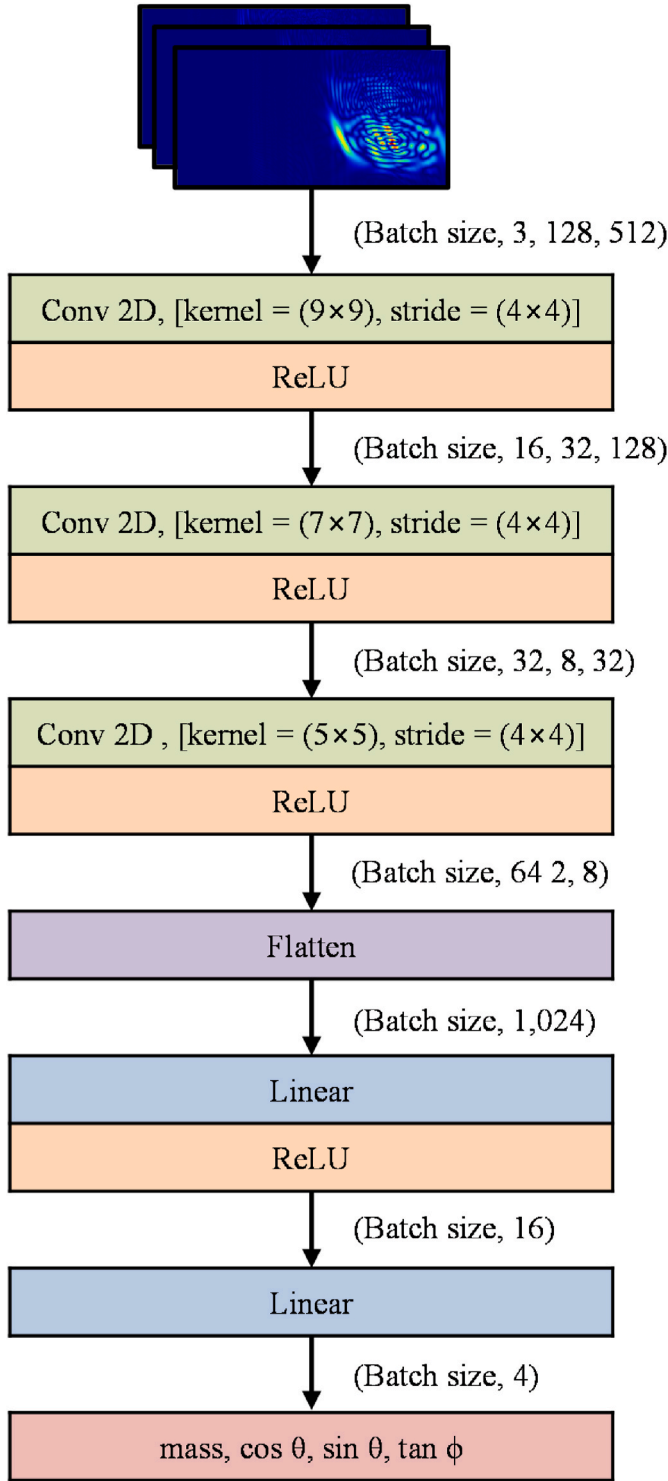


Fig. 3. The proposed neural network architecture.

by using images taken from various angles. Lee et al. [42] proposed a new positional representation method for estimating multiple noise source locations in a spherical microphone array system. This method involves flattening a spherical target map that represents the true noise source locations to express the positions of the noise sources.

Polar coordinates can be used to represent specific locations in the lower section of a steam generator, as demonstrated in Fig. 4. When the true impact location has a longitude of 0° and the neural network estimates the longitude as 360° , a non-zero loss function value is observed,

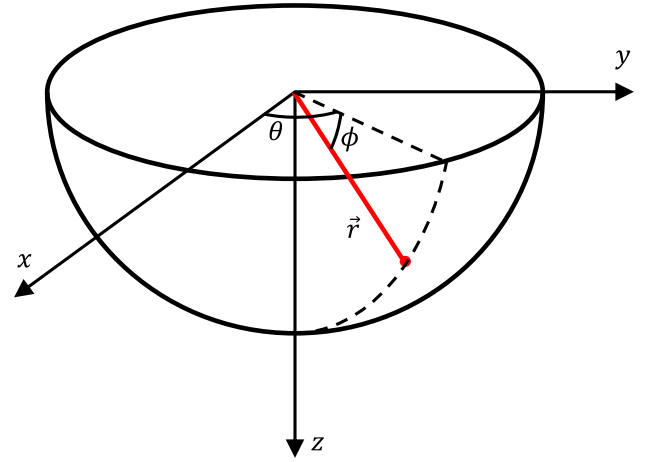


Fig. 4. Polar coordinate notation for the quarter-scale steam generator testbed.

despite the accurate longitude estimation. This is contradictory to the physical behavior. Hence, this paper proposes a new labeling method based on Cartesian coordinates.

The proposed labeling method begins with the process of converting the polar coordinate system to the Cartesian coordinate system. For the coordinate space shown in Fig. 4, polar coordinates can be transformed into Cartesian coordinates using Equation (3):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} |\vec{r}| \cos \theta \cos \phi \\ |\vec{r}| \sin \theta \cos \phi \\ |\vec{r}| \sin \theta \sin \phi \end{bmatrix} = \begin{bmatrix} |\vec{r}| \cos \theta \cos \phi \\ |\vec{r}| \sin \theta \cos \phi \\ |\vec{r}| \tan \phi \cos \phi \end{bmatrix} \quad (3)$$

where x , y , and z represent the Cartesian coordinates of the impact location, $\vec{r}$ is the vector from the center of the hemisphere to the impact location, θ is the longitude of the impact location, and ϕ is the latitude of the impact location. On the right side of Equation (3), the terms $\cos \phi$ and $|\vec{r}|$ are duplicated. Then, the right side can be simplified as shown in Equation (4) and used as the label for the impact location:

$$\frac{1}{|\vec{r}| \cos \phi} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ \tan \phi \end{bmatrix} \quad (4)$$

The proposed method addresses the issues that arise when using polar coordinates for labeling. For example, when the true impact location has a longitude of 0° and the network estimates the impact location as 360° , as shown in Fig. 5, the error is determined as follows:

$$\left(\begin{bmatrix} \cos 0^\circ \\ \sin 0^\circ \\ \tan \phi \end{bmatrix} - \begin{bmatrix} \cos 360^\circ \\ \sin 360^\circ \\ \tan \hat{\phi} \end{bmatrix} \right)^2 = 0 \quad (5)$$

where ϕ represents the latitude of the actual impact location, and $\hat{\phi}$ denotes the latitude of the impact location estimated by the network. Unlike methods that use polar coordinates directly as labels, the proposed method adjusts the labeling such that the mean squared error (MSE) is zero when the true longitude is 0° and the estimated longitude is 360° , as shown in Equation (5).

The neural network proposed in Section 3.2 is trained to minimize the mean squared error defined in Equation (6).

$$\text{Mean Squared Error} = \frac{1}{n} \sum_{i=1}^n (\hat{Y}_i - Y_i)^2 \quad (6)$$

where n denotes the number of training samples, Y represents the actual

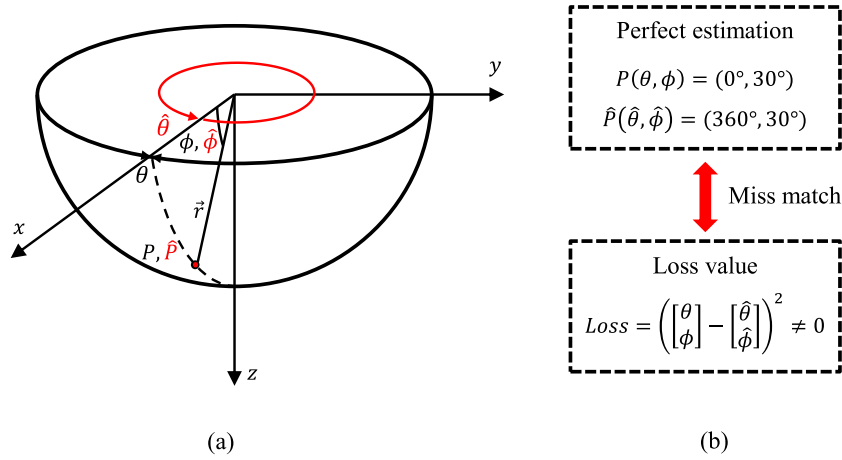


Fig. 5. Example of a problem that uses the coordinate value of the polar coordinate system as a label. (a) Example of a problem in a steam generator bottom ($|\vec{r}|$: constant, $P(\theta, \phi)$: True location, $\hat{P}(\hat{\theta}, \hat{\phi})$: Estimated location) and (b) Mismatch between estimated results and the loss function.

Table 1
Labeling type for Experiment 1.

Name	Labeling methods
Label 1	mass, θ, ϕ
Label 2	mass, $\cos \theta, \sin \theta, \tan \phi$
Label 3	mass, $\cos \theta, \sin \theta, \tan \phi, \theta, \phi$

values for mass and impact location coordinates, and $\hat{Y}$ refers to the predicted values estimated by the neural network.

3.4. Comparison strategies

The method proposed in this paper is a model that simultaneously estimates impact location and mass. Therefore, to validate the performance of this model, it is necessary to separately evaluate the performance of impact location estimation and mass estimation. The performance metric for impact location estimation uses the geodesic distance between the true impact location and the estimated impact location. The geodesic distance is the shortest path connecting two points on a curved surface. The impact location estimation metric is calculated as follows:

$$e_{location} = |\vec{r}| \cos^{-1} (\cos \phi \cos \theta \cos \hat{\phi} \cos \hat{\theta} + \cos \phi \sin \theta \cos \hat{\phi} \sin \hat{\theta} + \sin \phi \sin \hat{\phi}) \quad (7)$$

where $\vec{r}$ is the vector from the center of the hemisphere to the impact location, θ is the longitude of the true impact location, ϕ is the latitude of the true impact location, $\hat{\theta}$ is the longitude of the estimated impact location, and $\hat{\phi}$ is the latitude of the estimated impact location. The performance metric for mass estimation is defined as the percentage error, as follows:

$$e_{mass} = \frac{|m - \hat{m}|}{m} \times 100\% \quad (8)$$

where m denotes the true mass and $\hat{m}$ represents the estimated mass.

To evaluate the performance of the proposed method, three experiments are conducted. Experiment 1 assesses the performance of the proposed position-invariant loss function by comparing the impact location and mass-estimation results for three labeling methods, as shown in Table 1. The input data for this experiment is preprocessed using WVD. A CNN model with max pooling is used to estimate the impact location. Label 1 represents the method using the polar coordinate values of the impact location directly, which encounters the issue

Table 2
Model cases for Experiment 2.

Input data	Architecture
Wigner-Ville distribution	2D CNN
Continuous wavelet transform	2D CNN
Short time Fourier transform	2D CNN
Time series	1D CNN
Time series	ANN

Table 3
Model cases for Experiment 3.

Input data	Architecture	Labeling method
Wigner-Ville distribution	2D CNN	mass, $\cos \theta, \sin \theta, \tan \phi$
Time series	2D CNN	mass, θ, ϕ
Auto power spectrum	DBN	mass, θ, ϕ

shown in Fig. 5. Label 2 is the method proposed in this paper, as described in Section 3.3, which resolves the issue presented in Fig. 5. Label 3 combines Label 1 and Label 2. For mass estimation, all three methods employ min-max scaling.

The second experiment, Experiment 2, is designed to compare the estimation results based on different input data and neural network architectures. According to Table 2, this experiment evaluates the impact location and loose part mass-estimation results using the 2D CNN model proposed in Fig. 3, with input data preprocessed by various time-frequency methods, such as STFT, CWT, and WVD. Additionally, impact location and mass estimations are performed using 1D CNN and ANN with raw time series data, which do not involve time-frequency pre-processing. The neural networks employed in the experiments are configured with an equivalent number of trainable parameters for fair comparison.

In Experiment 3, the performance of the proposed method is compared with existing machine learning approaches for impact location and mass estimation. Moon et al. [14] introduced a method that estimates the impact location and mass using a 2D CNN with time-series impact data from plate simulations. Su et al. [15] proposed a deep belief network (DBN) for estimating the mass of loose parts that utilizes the auto-power spectrum (APS) of impact signals as its input. This experiment compares the results of impact location and mass estimation obtained using the approaches proposed by Moon and Su with those from the proposed method, as shown in Table 3. As in Experiment 2, the neural networks used in Experiment 3 are configured with similar numbers of trainable parameters, for fair comparison.

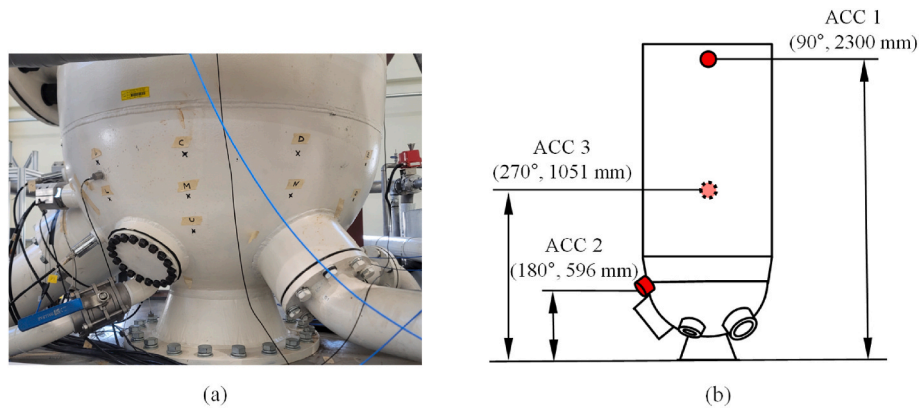


Fig. 6. Experimental setting (a) Picture of the experimental setting and (b) Acceleration sensor location.

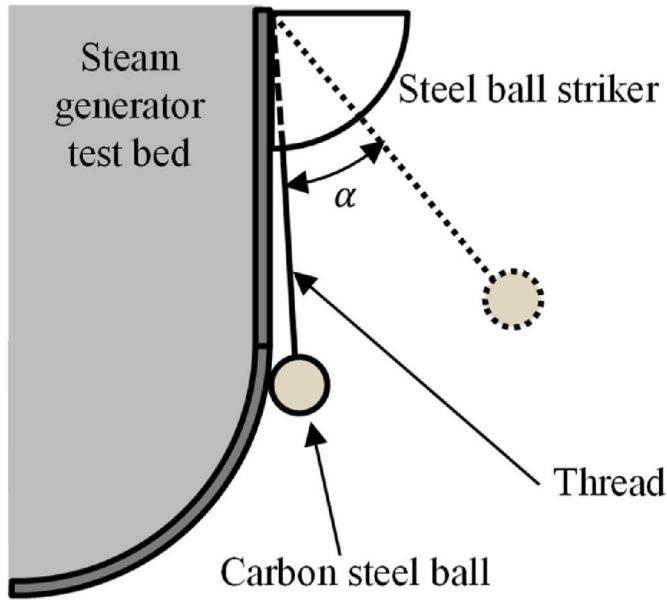


Fig. 7. Experiment setup for impact test with the steel ball striker (α : impact angle).

4. Experiments

In this study, a quarter-scale steam generator test bed was specially designed to accurately reflect the geometry of a primary system steam generator. Due to internal fluid flow, loose parts entering the reactor system accumulate in the bottom of the steam generator. Impact experiments were performed in this area, as Fig. 6(a) illustrates. Fig. 6(a) shows the bottom of the quarter-scale steam generator with marked impact points, while Fig. 6(b) shows the locations of the accelerometers, which were attached based on their actual positions in a full-scale steam generator, adjusted for the quarter-scale dimensions. As shown in Fig. 7, the experimental setup shows that the steel ball striker impacts the outer wall of the quarter-scale steam generator using gravity.

The key point in this study is that, for a given mass of the steel ball, it must strike the outer wall of the steam generator with the same impulse at all locations. As illustrated in Fig. 6(a), the hemispherical shape of the lower steam generator causes the impact velocity to vary depending on the position, when using the steel ball striker from Fig. 7 at a uniform angle. Consequently, even with the same carbon steel ball mass, the impulse differs across locations. To resolve this issue, we varied the impact angle α according to the latitude to ensure a uniform velocity of 1.2 m/s just before impact, thereby achieving consistent impulse across

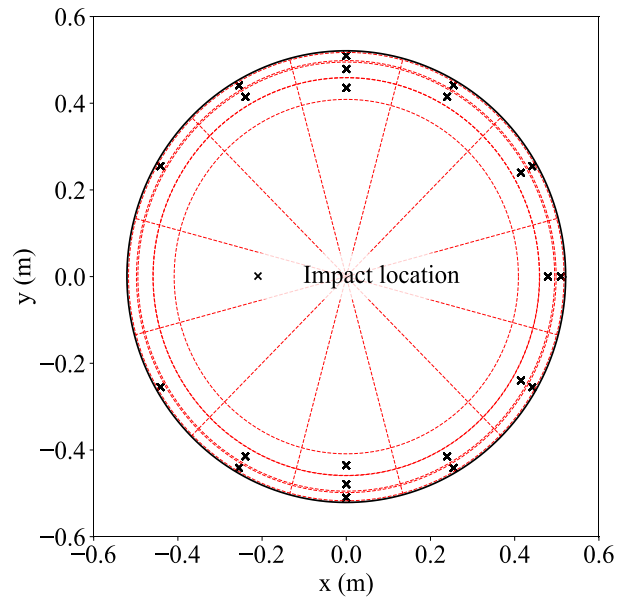


Fig. 8. Impact locations (bottom view).

all points in the experiment.

Fig. 8 shows the impact test locations from a bottom view. The impact tests were conducted at positions on the lower section of the quarter-scale steam generator where such tests are feasible. A total of 22 impact locations were selected, with latitudes of 12°, 23°, and 33°, and spaced at 30° intervals in longitude. The impact tests utilized five different masses of steel balls: 69 g, 113 g, 132 g, 174 g, and 199 g. Fig. 9 illustrates a subset of the experimental data obtained from the quarter-scale steam generator test bed. For each impact location and mass condition, the experiment was repeated 30 times, resulting in a total of 3300 impact test data points. Of these, 2640 (80 %) were used for training and validation, while the remaining 660 (20 %) were used for testing. The training and validation data were randomly selected for each mass and impact location to ensure a balanced and representative dataset.

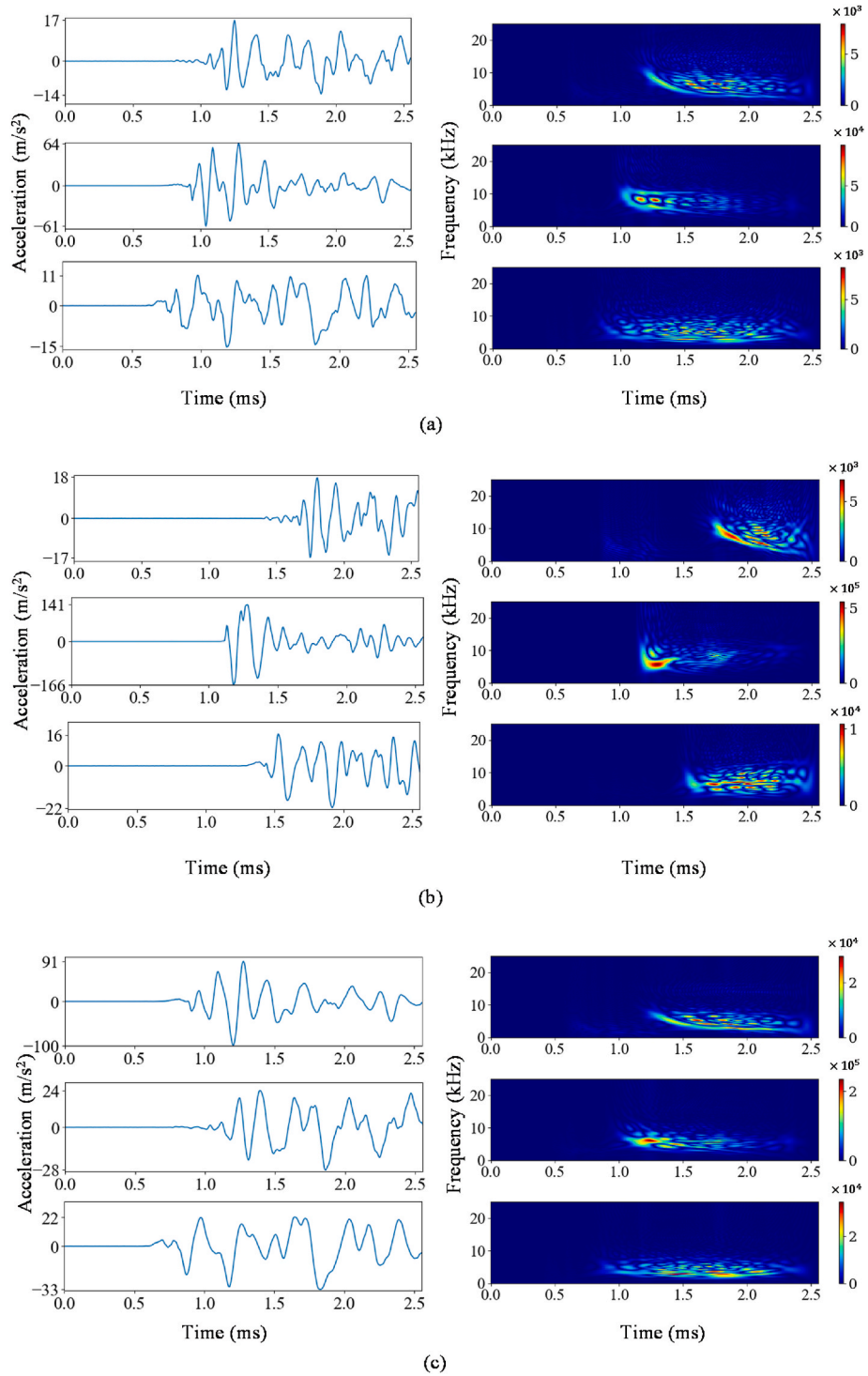


Fig. 9. Quarter-scale steam generator impact test data (a) Mass: 69g, longitude: 0°, latitude: 12°, (b) Mass: 69g, longitude: 150°, latitude: 12°, and (c) Mass: 199g, longitude: 0°, latitude: 12°.

5. Results and discussion

5.1. Quantitative performance evaluation

As discussed in Section 3.4, the performance of estimating the impact location and mass of loose parts is evaluated using $e_{location}$, defined by equation (7), and e_{mass} , defined by equation (8). Three experiments were conducted to experimentally validate the effectiveness of the proposed method. Experiment 1 evaluates the performance of the Position-

invariant Loss function by assessing the accuracy of the impact location and mass estimation with and without the function applied. Experiment 2 examines the impact location and mass-estimation performance based on the type of training data and model, comparing the proposed WVD-CNN combination with other approaches. Experiment 3 compares the performance of the proposed WVD-CNN-Position-invariant Loss method with previously established deep-learning-based LPMS methods, specifically Moon's method [14], and Su's method [15].

The first experiment involved evaluating a standard CNN model with

Table 4

Estimation result based on labeling method (Experiment 1).

Name	Location estimation error (mm)		Mass estimation error (%)	
	Average	Standard deviation	Average	Standard deviation
Label 1	65.2	53.8	4.84	4.91
Label 2	15.2	11.3	2.76	3.04
Label 3	22.1	16.1	4.00	3.79

Table 5

Estimation results based on input data type and architecture (Experiment 2).

Architectures	Location estimation error (mm)		Mass estimation error (%)	
	Average	Standard deviation	Average	Standard deviation
WVD – 2D CNN	4.86	4.24	1.63	1.72
CWT – 2D CNN	5.28	3.50	1.71	1.67
STFT – 2D CNN	5.47	5.10	1.41	1.44
Time series – 1D CNN	7.14	7.05	2.33	2.40
Time series – ANN	12.1	9.35	2.45	2.62

Table 6

Estimation result based on previous method (Experiment 3).

Architectures	Location estimation error (mm)		Mass estimation error (%)	
	Average	Standard deviation	Average	Standard deviation
WVD – 2D CNN	4.86	4.24	1.63	1.72
Time series – 2D CNN	9.09	15.0	1.94	1.96
APS– DBN	14.3	11.7	1.53	1.64

a max pooling layer based on the labeling method. Table 4 presents the mean and variance of e_{location} and e_{mass} for this experiment. The results reveal that the mean e_{location} value for Label 2, as proposed in this study, is 15.2 mm, representing the lowest average error among the three methods. Additionally, it was found that the mean e_{mass} value for Label 2 is the smallest, at 2.76 %.

Experiment 2 involves comparing methods for estimating the impact location and mass of loose parts, considering different input data preprocessing and neural network models. In this experiment, Label 2, which demonstrated the lowest average e_{location} and e_{mass} values in Experiment 1, was employed. Table 5 displays the results for Experiment 2. The proposed WVD-2DCNN model shows the smallest average e_{location}

value of 4.86 mm, compared to the other four models. Additionally, the mean e_{mass} value for this model is 1.63 %, making it the second-smallest error.

Experiment 3 involves comparing the performance of existing deep-learning-based methods that enhance LPMS. Table 6 displays the results of this experiment. The results indicate that the proposed WVD-2DCNN model records the smallest average e_{location} value among the three models. For e_{mass} , Su's APS-DBN method achieves the lowest average error of 1.53 %, while the proposed model ranks second in terms of the lowest e_{mass} value.

Experiment 1 demonstrates that the proposed method excels in both location and mass estimation, as compared to the alternative methods. Experiment 2 shows that the proposed WVD-2DCNN model achieves the lowest average impact location error, though it differs only marginally from the two other models that utilize time-frequency techniques, with a discrepancy of under 1 mm. Furthermore, the proposed model's average mass-estimation error is the second smallest, slightly larger than the STFT preprocessing method, with only a 0.22 % difference from STFT. However, the STFT method has the drawback of requiring users to determine parameters such as window type and size. Further, the CWT method also demands user selection of wavelet type and width. Conversely, the WVD method does not require such user-defined parameters, providing an advantage in automation. In Experiment 3, the proposed model recorded an average mass-estimation error 0.1 % higher than Su's method; however, it should be noted that this is a minimal difference. The proposed model also shows a significantly lower average location estimation error, half of that of Moon's method [14] and three times smaller than Su's method [15]. As described in Section 2, a quarter-scale steam generator was tested to simulate nuclear plant conditions, using carbon steel balls of various masses to replicate impact scenarios. The results showed that the proposed WVD-2DCNN model effectively estimated both the location and mass of loose parts, providing a robust and reliable method for real-world applications. The results show that the proposed WVD-2DCNN model effectively estimates both the location and mass of these loose parts, providing a robust and reliable method for real-world applications.

5.2. Qualitative performance evaluation

Fig. 10 presents the location estimation results for Experiment 1. Fig. 10(b) shows the results from the proposed method, which, when compared to the results from the other two methods depicted in Fig. 10 (a) and (c), demonstrate that the proposed method estimates the impact location more closely to the true impact position. Fig. 11 illustrates the mass-estimation results for Experiment 1. The results from the proposed method, shown in Fig. 11(b), reveal that the difference between the Q3 and Q2 values in the boxplot is smaller, compared to those of the other

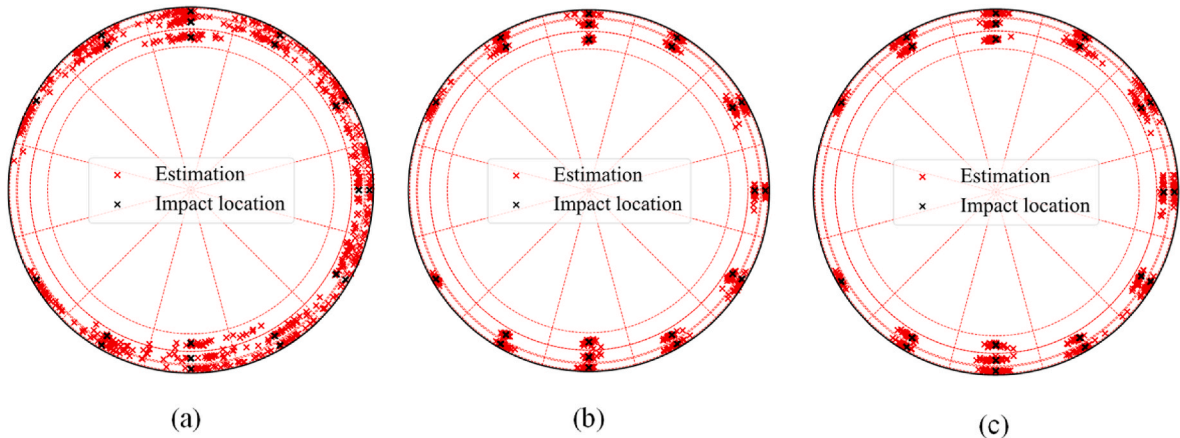


Fig. 10. Impact location estimation results of Experiment 1 (a) Label 1, (b) Label 2, and (c) Label 3.

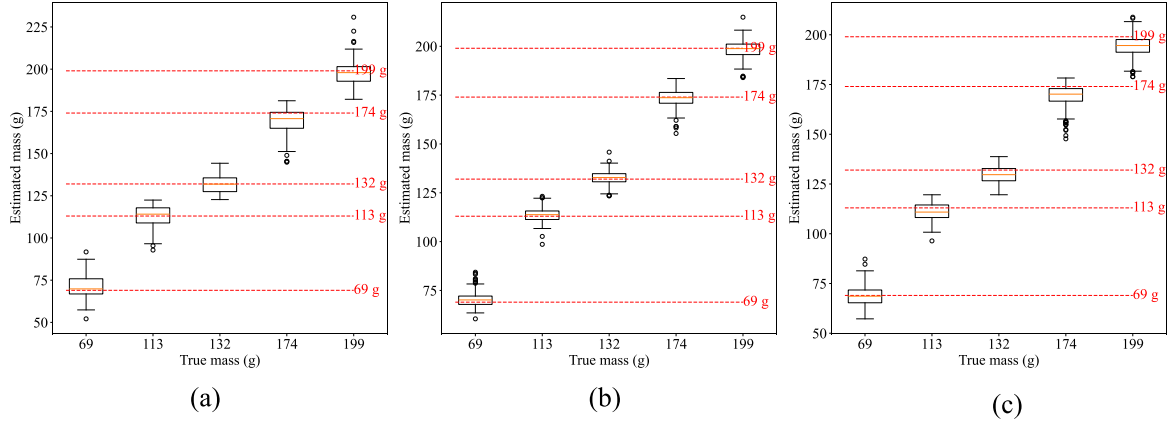


Fig. 11. Impact mass estimation results of Experiment 1 (a) Label 1, (b) Label 2, and (c) Label 3.

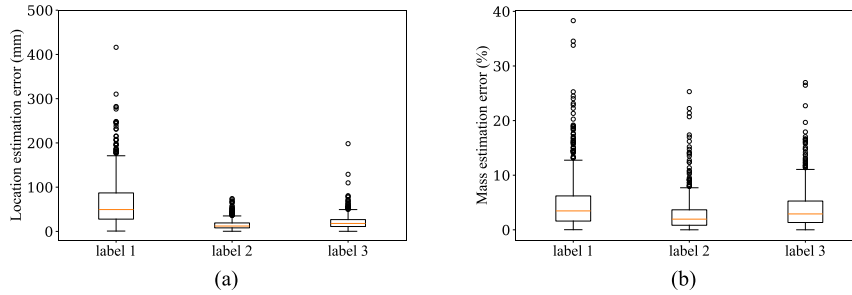


Fig. 12. Results comparison for Experiment 1 (a) Impact location estimation error and (b) Mass-estimation error.

two methods, which are presented in Fig. 11(a)–(c). Fig. 12 compares the results of Experiment 1 across various metrics. Fig. 12(a) depicts the location estimation errors for Experiment 1 in a boxplot, while Fig. 12(b) shows the mass-estimation errors. For both location and mass-estimation errors, the proposed method exhibits a lower median and the smallest differences between Q3 and Q2 values compared to the other methods, indicating superior accuracy and precision.

Fig. 13 presents the results of location estimation for Experiment 2. The models using time-series inputs, shown in Fig. 13(d)–(e), show minor differences compared to the proposed method, which is shown in Fig. 13(a). Conversely, models using the same time-frequency techniques, illustrated in Fig. 13(b)–(c), display no notable differences. Similarly, Fig. 14, which details mass-estimation results, shows that methods using time-series inputs differ from the proposed method, whereas mass estimation using time-frequency techniques does not exhibit significant deviations. Fig. 15 displays the location and mass-estimation errors for Experiment 2. As shown in Fig. 15(a), the proposed method demonstrates greater accuracy and precision compared to time-series input models; however, the proposed method shows similar performance to other models that use the same time-frequency approach. Fig. 15(b) confirms that the proposed method also achieves higher accuracy and precision in mass estimation, as compared to time-series models, with results that are comparable to time-frequency method models. However, as noted in Section 4.2, the WVD method proposed in this study offers the advantage of automation without requiring user-defined parameters, unlike the other time-frequency methods, which require manual parameter configuration.

Fig. 16 illustrates the location-estimation results from Experiment 3, comparing the proposed method with previous deep-learning techniques applied to LPMS. The results shown in Fig. 16(a) demonstrate

that impact locations estimated by the proposed method are closer to the actual impact locations than those obtained using either Moon's method (Fig. 16(b)) or Su's method (Fig. 16(c)). However, Fig. 17, which shows the mass-estimation results, reveals that there is no substantial difference between the proposed method and the methods proposed by Moon and Su. Fig. 18 provides a comparison of location and mass-estimation errors across all techniques used in Experiment 3. Fig. 18(a) confirms that the proposed method achieves higher accuracy and precision in location estimation, as compared to the other methods. On the other hand, Fig. 18(b) indicates that all three methods demonstrate comparable accuracy and precision in mass estimation.

5.3. Discussion about variation of the model training parameters

The width and depth of neural networks affect prediction accuracy [43]. Accordingly, this paper examines the impact of kernel size and layer count on location and mass-estimation results for the proposed WVD-CNN-Position-invariant loss function combination. Tables 7 and 8 provide the location estimation error results for different kernel sizes and convolution layer counts in the proposed CNN model. The kernel size indicated refers to the largest size used in the model's initial convolutional layer, with each successive layer employing kernels that are reduced by a size of 2. For example, if the specified kernel size is 7 and the model has 3 layers, the kernels used would be 7×7 , 5×5 , and 3×3 , in that order.

Location estimation errors generally decrease as the model becomes deeper, indicating that increasing the number of layers reduces errors. However, trends related to kernel size for location estimation errors are less apparent. The same trend applies to mass-estimation errors, where a higher number of layers leads to reduced errors, but the effect of kernel

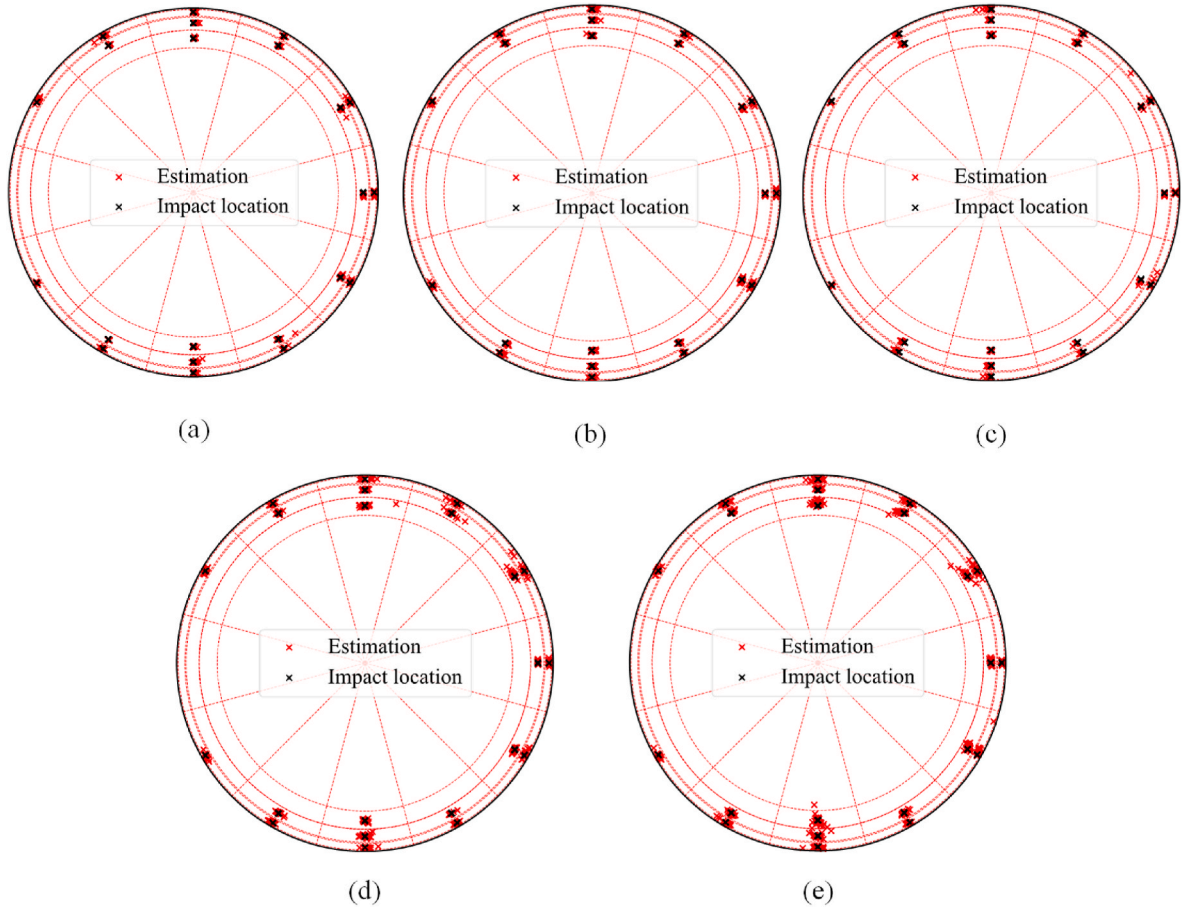


Fig. 13. Impact location estimation results of Experiment 2 (a) WVD-2D CNN, (b) CWT-2D CNN, (c) STFT-2D CNN, (d) Timeseries-1D CNN, and (e) Time series-ANN.

size is not easily discernible. The range between the minimum and maximum location estimation errors is 2.63 mm, while the variation in mass-estimation errors is 0.85 percentage points. This suggests that the proposed model exhibits robustness against changes in learning parameters. In conclusion, the model with a maximum kernel size of 9×9 and 3 layers provides results closest to the median for both impact location and mass-estimation errors.

5.4. Discussion about the possibility of online monitoring

The LPMS used in steam generators and reactors forms part of the primary system in a nuclear power plant, specifically, the LPMS is part of the zones where hazardous radiation is present. Thus, there is a need for rapid monitoring of the plant's operational status. Upon detection of impact signals caused by foreign objects, it is important to promptly evaluate the extent of any potential damage. This study assesses the online monitoring capability of the proposed method by calculating the time required for impact location and mass estimation within a 0.1 s interval.

Fig. 19 depicts how the proposed model estimates the impact location of foreign objects from continuous impact signals. Fig. 19(a) shows the scenario of consecutive 0.1 s impacts from a 132 g object, and Fig. 19(b) displays the model's continuous estimation of the impact location. Given that the time required for impact location and mass estimation within each 0.1 s interval is 0.0017 s, the proposed method demonstrates the potential for online monitoring within the primary system of a nuclear power plant.

6. Conclusions

This paper introduces a data-driven deep-learning approach to estimate the impact location and mass of foreign objects on irregularly shaped surface structures. The methodology involves transforming impact acceleration signals into Wigner-Ville distribution (WVD) images, which are fed into a convolutional neural network (CNN) model to estimate both the impact location and the mass of foreign objects. A novel position-invariant loss function based on Cartesian coordinates is proposed to ensure accurate positioning in three-dimensional structures. To validate the proposed approach, experiments were carried out using a quarter-scale test bed of a steam generator based on the primary system of an actual nuclear power plant, where data were collected and the proposed neural network model was trained and tested.

This study conducted three experiments: validating the position-invariant loss function's effectiveness, evaluating image transformation techniques with neural networks, and comparing the method to existing approaches for estimating the mass or location of foreign objects. The results from these three experiments indicated that the proposed method achieved an average impact location error of 4.86 mm and an average loose part mass-estimation error of 1.63 %, demonstrating its capability to effectively estimate impact locations and loose part masses in three-dimensional primary system structures.

The major contributions of this study are as follows: 1) The proposed approach offers a more accurate and rapid method, as compared to the traditional triangulation method and the circle-intersection method, helping signal analysts make informed decisions within the given time frame. 2) Unlike previous deep-learning approaches that were validated on simplified models, this study uses experimental setups identical to the actual structures of primary system equipment to train and validate the

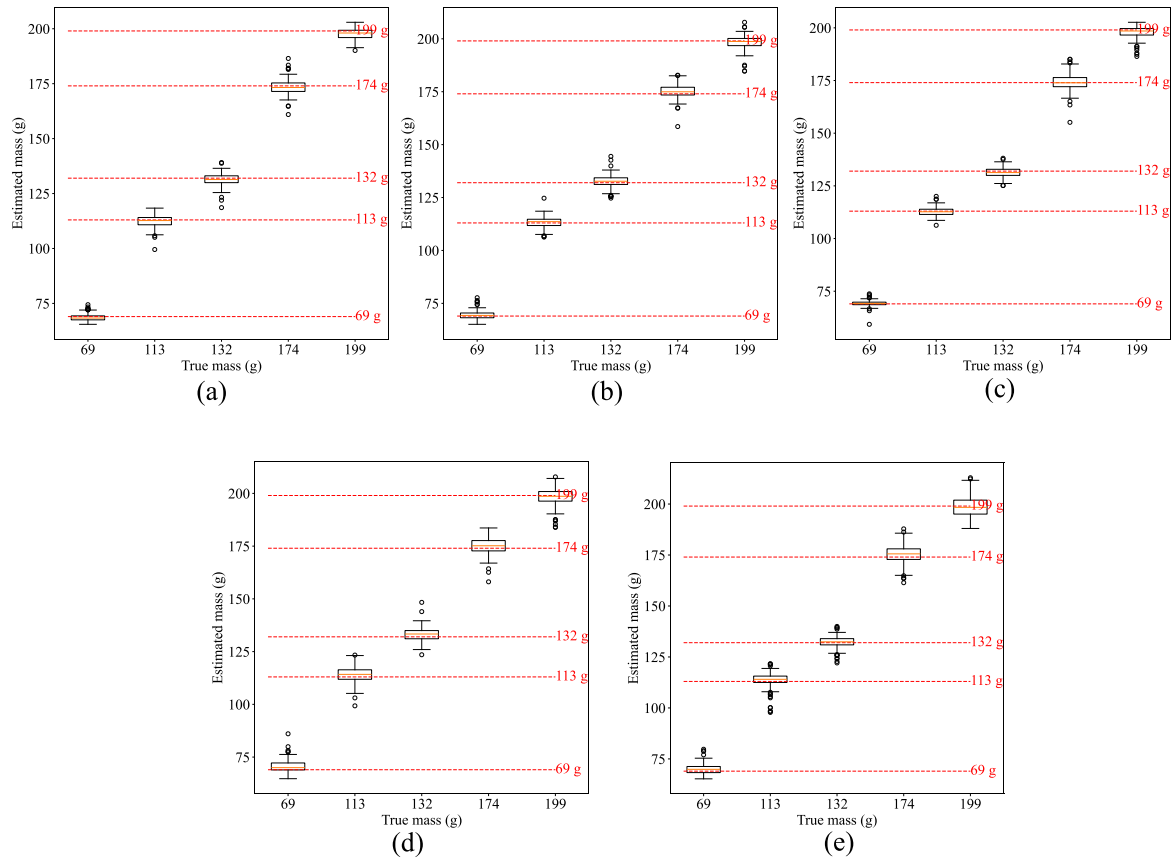


Fig. 14. Impact mass estimation results of Experiment 2 (a) WVD-2D CNN, (b) CWT-2D CNN, (c) STFT-2D CNN, (d) Timeseries-1D CNN, and (e) Time series-ANN.

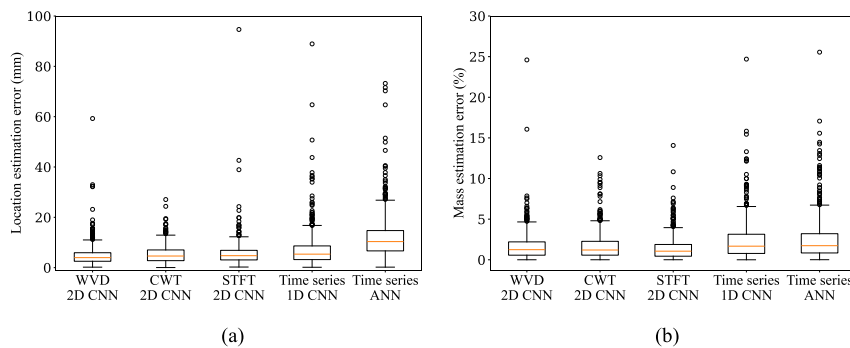


Fig. 15. Results comparison for Experiment 2 (a) Impact location estimation error and (b) Mass-estimation error.

proposed method. 3) To address issues arising from three-dimensional shell structures, the proposed approach introduces a novel position-invariant loss function. Consequently, this research establishes a foundation for rapidly and accurately assessing the extent of damage from foreign object impact events in the primary systems of nuclear power plants.

The proposed approach in this study is data-driven and relies on the quality and quantity of training data. While this study still presents challenges in implementing the method directly to real nuclear power plant primary systems, the challenges can be addressed through future research. To reflect real-world scenarios, future research should focus on the development of domain adaptation techniques, allowing the model

to generalize across primary systems with different scales. Moreover, integrating physics-based AI techniques can help mitigate data scarcity by incorporating physical principles, and enhance the interpretability of the model, making it more transparent and reliable for real-time decision-making in systems.

CRediT authorship contribution statement

Jungsik Choi: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Data curation, Conceptualization. **Jeongmin Oh:** Investigation. **Taeyoung Ko:** Methodology, Investigation, Data curation. **Bunyoung Chung:** Writing – review & editing,

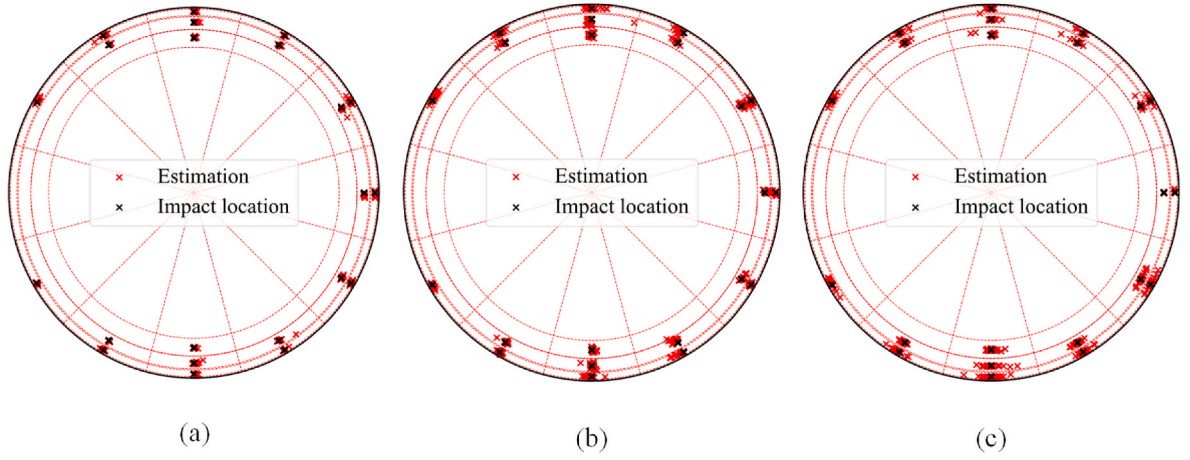


Fig. 16. Impact location estimation results of Experiment 3 (a) WVD-2D CNN, (b) Timeseries-2D CNN, and (c) APS-DBN.

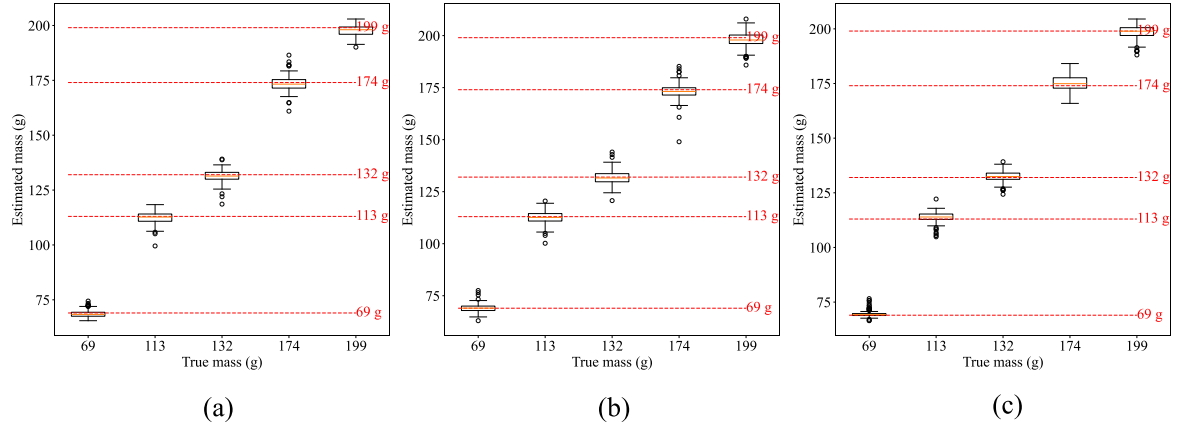


Fig. 17. Impact mass estimation results of Experiment 3 (a) WVD-2D CNN, (b) Timeseries-2D CNN, and (c) APS-DBN.

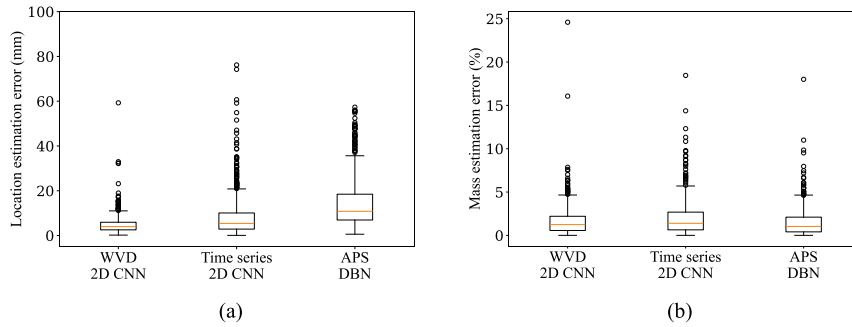


Fig. 18. Comparison results for Experiment 3 (a) Impact location estimation error and (b) Mass-estimation error.

Table 7

Impact location estimation average error to hyperparameter fluctuations in the WVD-CNN model (mm, standard deviation in parentheses).

Number of layers \ Kernel size	11	9	7
2	5.72 (5.89)	4.41 (6.64)	4.52 (5.69)
3	5.06 (6.73)	4.86 (4.24)	6.73 (11.7)
4	4.10 (7.37)	4.85 (5.14)	4.53 (8.37)

Table 8

Mass estimation average error to hyperparameter fluctuations in the WVD-CNN model (% , standard deviation in parentheses).

Number of layers \ Kernel size	11	9	7
2	1.78 (2.06)	1.84 (2.01)	1.85 (1.91)
3	2.20 (2.18)	1.63 (1.72)	2.22 (2.29)
4	1.47 (7.37)	1.51 (1.91)	1.37 (1.64)

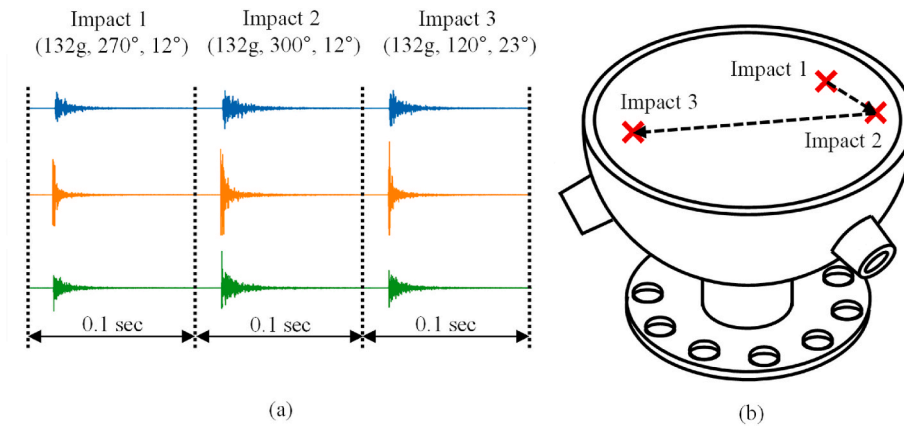


Fig. 19. Online monitoring (a) Sequential impacts and (b) Sequential estimation.

Methodology, Investigation, Data curation. **Young-Chul Choi**: Writing – review & editing, Methodology, Investigation. **Sooyoung Lee**: Writing – review & editing, Writing – original draft, Supervision, Conceptualization. **Hyunseok Oh**: Writing – review & editing, Validation, Supervision, Resources, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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