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RESEARCH ARTICLE

Resilient Adaptive Multi-Robot Formation Tracking for Achieving Guaranteed Dynamic Obstacle Avoidance Within Limited Inter-Agent Communication Ranges

BONG SEOK PARK¹ AND SUNG JIN YOO², (Member, IEEE)

¹Division of Electrical, Electronic, and Control Engineering, Kongju National University, Cheonan 31080, South Korea

²School of Electrical and Electronics Engineering, Chung-Ang University, Dongjak-gu, Seoul 06974, Republic of Korea

Corresponding author: Sung Jin Yoo (sjyoo@cau.ac.kr)

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ABSTRACT This paper presents a resilient adaptive formation control design strategy to achieve guaranteed dynamic obstacle avoidance within limited inter-agent communication ranges in nonholonomic multi-robot systems. Unlike existing literature, the main contribution lies in the development of a Lyapunov-based formation tracker utilizing guidance signals to avoid dynamic obstacles within limited inter-agent communication ranges in the presence of unknown deception attacks and unknown velocities of dynamic obstacles. Error variables, constructed using the guidance signals, are employed to design a resilient adaptive formation tracker within a neural network-based command-filtered backstepping framework. Through Lyapunov stability analysis, the guidance signals are designed to ensure that connected robots remain within limited inter-agent communication ranges during obstacle avoidance by adjusting formation errors that arise during obstacle avoidance. Additionally, adaptive laws are devised to compensate for unknown obstacle velocities and deception attacks affecting the leader's velocity. The analysis guarantees that all error signals within closed-loop systems remain bounded and can be arbitrarily reduced in the Lyapunov sense. Simulation results confirm the effectiveness of the proposed theoretical framework.

INDEX TERMS Guaranteed connectivity, dynamic obstacle avoidance, limited inter-agent communication ranges, nonholonomic multi-robot systems, formation tracking.

I. INTRODUCTION

Over the past decades, the formation control for mobile robots has received a lot of attention due to its economic and military efficiency. Various formation controllers using virtual structure [1], leader-follower [2], and graph theory-based [3] methods have been proposed. A common requirement for implementing these methods is communication between robots. However, as is known to all, network communication is vulnerable to external attacks, thus increasing the need

for resilient control. In addition, collisions with obstacles and restrictions on the connection between robots are critical issues in mission performance using multiple mobile robots. Therefore, it is necessary to consider the issue of ensuring the safety of mobile robot formation in environments such as external attacks, collision with obstacles, collision with robots, and constraints on connectivity between robots.

Resilient control categorizes external attacks into denial-of-service [4], deception attack [5], and replay attack [6]. Particularly, the deception attack manipulates robots according to the attacker's intent by injecting false data into communication information, making detection challenging

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for defenders due to its secretive nature [7]. Consequently, extensive research has focused on resilient control for deception attacks. Adaptive techniques were utilized in previous works to estimate time-varying attacks [8], [9]. A resilient estimation technique was proposed to safeguard the states of the network control system [10]. Additionally, a security correction control method was introduced to estimate heterogeneous attack effects [11]. An adaptive resilient control method using neural networks was presented for uncertain nonlinear cyber-physical systems [12], while time-varying deception attacks were considered in designing an adaptive controller for second-order nonlinear systems [13]. A distinct contribution is found in the adaptive resilient tracking approach presented for multiple nonholonomic mobile robots facing deception attacks [14]. Furthermore, a finite-time adaptive attack estimation observer was proposed for resilient control in vehicle platoon systems [15]. Nevertheless, there remains a shortage of resilience control results addressing all challenges, including collision avoidance, obstacle avoidance, and connectivity maintenance, in the presence of unknown deception attacks.

The real-time operation of collision avoidance, obstacle avoidance, and connectivity maintenance algorithms is essential for mobile robots with limited sensing ranges. The potential field method has been widely used because real-time processing is possible, ensuring formation tracking, collision avoidance, obstacle avoidance, and connectivity maintenance. In [16], potential functions were used to avoid collisions with static obstacles and connected robots. In [17], inter-agent collisions and connectivity maintenance were considered. In [18], the potential function was used for collision avoidance between connected robots, and the connectivity controller was designed to maximize the second smallest eigenvalue of the graph Laplacian matrix. In [19], a bounded barrier function was introduced to deal with sensor limitations and collision avoidance. To address the local minimum problem [20], a decentralized control method using the dipolar navigation function was proposed in [21], and the rotating matrix-based method was presented in [22]. On the other hand, studies that do not use the potential field method have also been proposed. An error transformation method was proposed for ensuring collision avoidance and connectivity maintenance in [23]. In [24], a leader-follower tracking problem with the avoidance of static obstacles was addressed for uncertain multiple nonholonomic mobile robots without external attacks. An adjustable formation shape was introduced for obstacle avoidance [25]. A distributed model predictive control method was presented to consider static and dynamic obstacles as the constraints of the optimization control problem in [26]. While these studies have addressed various challenges, they did not completely resolve all safety-related issues, primarily due to the following difficulties:

D1) The potential field method involves designing control inputs by linearly combining individual potential

functions for collision avoidance, obstacle avoidance, formation tracking, and connectivity. Consequently, in the presence of a conflict between these objectives, the control input may become zero, resulting in a loss of control. For instance, when the distance between robots increases to avoid collision, the objectives of collision avoidance and formation tracking conflict with each other. If the control inputs required for each objective have the same magnitude but opposite directions, it leads to a local minimum problem, causing the control input to be zero. Therefore, the potential field method faces the challenge of addressing this local minimum problem.

- D2) External attacks compromising the velocity information of the leader can undermine the objective of maintaining the connection beyond the communication and detection range of followers.
- D3) In the context of avoiding dynamic obstacles with unknown velocities, accurate information regarding their directions and velocities is essential. However, obtaining this information is challenging since robots cannot communicate with dynamic obstacles. Consequently, the task of avoiding dynamic obstacles proves to be more demanding than avoiding static ones.

Therefore, this paper aims to develop an adaptive connectivity-preserving collision avoidance mechanism against dynamic obstacles for resilient formation control of nonholonomic mobile robots with deception attacks and limited inter-robot communication ranges. It is assumed that there are unexpected static and dynamic obstacles, a limited communication and detection range, and velocity information of the leader compromised by unknown deceptive attacks. Under these circumstances, guidance signals are newly devised to avoid collisions with dynamic obstacles in the resilient nonholonomic multi-robot formation tracking framework. In the virtual control laws and the dynamics of guidance signals, adaptive tuning mechanisms are designed to compensate for the effects of unknown attack signals and velocities of unknown obstacles. The stability of the control system, including the above proposals, is analyzed in the Lyapunov stability sense. As a result, it is proven that all error signals for assuring the safety of leader-follower formation are uniformly ultimately bounded and can be made arbitrarily small.

Compared with previous studies, the contributions of this paper are as follows:

- C1) Unlike conventional methods [16], [17], [18], [19], [21], [22] that rely on the linear combination of potential functions, the design of a controller becomes possible for a unified control input that satisfies all control objectives. This is achieved by employing newly devised guidance signals based on transformed nonlinear formation error and collision avoidance error functions.

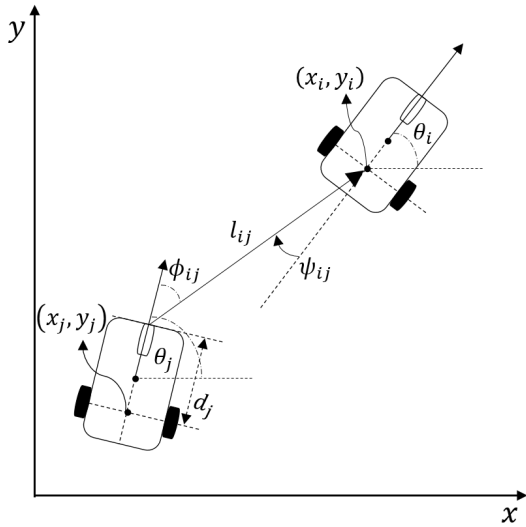


FIGURE 1. Leader-follower configuration [14].

C2) Compared to [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [21], [22], [23], [24], [25], and [26], this paper addresses an adaptive resilient control problem for range-constrained nonholonomic multi-robot formation systems. The adaptive compensation mechanism, designed to counteract deception attacks, is incorporated into the connectivity-maintaining formation tracking framework with adaptive collision avoidance.

The rest of this paper is organized as follows. In Section II, the kinematics and kinetics of nonholonomic mobile robots and the control objectives are presented. In Section III, a novel formation tracker design methodology with adaptive collision avoidance mechanism is established. In Section IV, simulations are performed to demonstrate the proposed theoretical results. Finally, conclusions are drawn in Section V.

II. PROBLEM FORMULATION

A. SYSTEM DESCRIPTION

Consider N nonholonomic mobile robots. The kinematics and kinetics of the n th robot are expressed below [27]:

$$\dot{\mathbf{q}}_n = \begin{bmatrix} \cos \theta_n & 0 \\ \sin \theta_n & 0 \\ 0 & 1 \end{bmatrix} \mathbf{v}_n \quad (1)$$

$$\mathbf{M}_n \dot{\mathbf{v}}_n + \mathbf{C}_n \mathbf{v}_n + \mathbf{D}_n \mathbf{v}_n + \boldsymbol{\tau}_{d,n} = \boldsymbol{\tau}_n \quad (2)$$

where $n = 0, \dots, N$, $\mathbf{q}_n = [x_n, y_n, \theta_n]^T$, (x_n, y_n) and θ_n denote the position and heading direction, respectively, $\mathbf{v}_n = [v_n, \omega_n]^T$ with the linear velocity v_n and the angular velocity ω_n , $\boldsymbol{\tau}_n = [\tau_{1,n}, \tau_{2,n}]^T$ denotes the control torque, and $\boldsymbol{\tau}_{d,n}$ is the bounded external disturbance. $\mathbf{M}_n$, $\mathbf{C}_n$, and $\mathbf{D}_n$ are given in [27].

Define the subscript i and j to denote the i th and j th mobile robots, respectively. Fig. 1 shows the configuration between the leader i and the follower j for the leader-follower

approach. It is assumed that the follower j uses onboard sensors to measure the relative distance and angle to the leader where the sensors have a limited measurement range. To derive the leader-follower model, the relative distance l_{ij} and angle ψ_{ij} between the leader i and the follower j are represented as follows:

$$l_{ij} = \sqrt{l_{ij,x}^2 + l_{ij,y}^2} \quad (3)$$

$$\psi_{ij} = \theta_i - \arctan\left(\frac{l_{ij,y}}{l_{ij,x}}\right)$$

where $l_{ij,x} = x_i - x_j - d_j \cos \theta_j$ and $l_{ij,y} = y_i - y_j - d_j \sin \theta_j$; d_j denotes the distance from the center position of the follower to the sensors. From Fig. 1, $l_{ij,x} = l_{ij} \cos(\theta_i - \psi_{ij})$ and $l_{ij,y} = l_{ij} \sin(\theta_i - \psi_{ij})$. Then, using (3), the model between the leader and the follower is derived as

$$\dot{l}_{ij} = v_i \cos \psi_{ij} - v_j \cos \phi_{ij} + d_j \omega_j \sin \phi_{ij}$$

$$\dot{\psi}_{ij} = \omega_i - \frac{1}{l_{ij}} v_i \sin \psi_{ij} + \frac{1}{l_{ij}} v_j \sin \phi_{ij} + \frac{1}{l_{ij}} d_j \omega_j \cos \phi_{ij} \quad (4)$$

where $\phi_{ij} = \psi_{ij} - \theta_i + \theta_j$.

It is assumed that the deception attack signals are added to the leader velocity information (v_i, ω_i) transmitted to the follower. Thus, the corrupted leader velocity information is only available for the control design of the follower robot.

Assumption 1: The follower j receives the corrupted velocity data ($\tilde{v}_i(t), \tilde{\omega}_i(t)$) from the leader i , where $\tilde{v}_i(t) = v_i(t) + a_{v,i}(t)$ and $\tilde{\omega}_i(t) = \omega_i(t) + a_{\omega,i}(t)$; $a_{v,i}(t)$ and $a_{\omega,i}(t)$ are time-varying deception attacks. The attack signals $a_{v,i}(t)$ and $a_{\omega,i}(t)$ are unknown, but bounded such that $|a_{v,i}(t)| \leq \bar{a}_{v,i}$ and $|a_{\omega,i}(t)| \leq \bar{a}_{\omega,i}$ where $\bar{a}_{v,i}$ and $\bar{a}_{\omega,i}$ are unknown constants.

Assumption 1 is reasonable in the resilient control area because the boundedness of attack signals is necessary to hide the attack from the defender [28].

B. CONTROL OBJECTIVE

The control objective is to design the resilient control torque τ_j with adaptive collision avoidance for the nonholonomic mobile robot given in (1) and (2) so that

$$(O-i) \quad R_{a,j} < l_{ij}(t) < R_{c,j}, \forall t \geq 0$$

$$(O-ii) \quad l_{oj}(t) > R_{a,j}, \forall t \geq 0$$

$$(O-iii) \quad \text{if there are no obstacles, then } \lim_{t \rightarrow \infty} |l_{ij}(t) - l_{d,ij}| \leq \mu_0 \text{ and } \lim_{t \rightarrow \infty} |\psi_{ij}(t) - \psi_{d,ij}| \leq \mu_1$$

$$(O-iv) \quad \text{if there are obstacles, then formation tracking errors } l_{ij} - l_{d,ij} \text{ and } \psi_{ij} - \psi_{d,ij} \text{ are bounded}$$

where $R_{c,j} > 0$ is the maximum connection range which is determined by the limited measurement range of sensors, $R_{a,j} > 0$ is the minimum avoidance range, $l_{d,ij} > 0$ and $\psi_{d,ij}$ denote the desired distance and angle between the leader i and the follower j , respectively, μ_0 and μ_1 are positive constants which can be reduced arbitrarily, and $l_{oj} = \sqrt{l_{oj,x}^2 + l_{oj,y}^2}$ is the relative distance from the nearest obstacle o to the follower j with $o \neq i$, $l_{oj,x} = x_o - x_j - d_j \cos \theta_j$, and $l_{oj,y} = y_o - y_j - d_j \sin \theta_j$. Here, (x_o, y_o) is the position of the obstacle o , and the obstacle o is allowed to be static and

dynamic. The follower j connected to its leader regards other followers as dynamic obstacles.

Assumption 2: The desired distance $l_{d,ij}$ and angle $\psi_{d,ij}$ are constants. In addition, the desired distance $l_{d,ij}$ is given by $R_{a,j} < l_{d,ij} < R_{c,j}$.

Assumption 2 is reasonable because designers can choose the desired values to meet the constraints of the connection and avoidance ranges.

Assumption 3: The kinematics of the nearest obstacle o to the follower j is used as the equation (1) with the subscript o . The linear velocity v_o is bounded such that $|v_o| \leq \bar{v}_o$ where $\bar{v}_o$ is an unknown positive constant.

Remark 1: The nearest obstacle, denoted as o , could be either a neighboring robot or an unknown object within the maximum connection range $R_{c,j}$. This implies the simultaneous consideration of collision avoidance between mobile robots and obstacle avoidance between the robot and objects.

Remark 2: It is noticed that

- (i) Objective (O-i) ensures connectivity maintenance and collision avoidance between the leader i and the follower j , regardless of the detection of the static or dynamic obstacles. Additionally, Objective (O-ii) prevents collisions between obstacles and follower j while maintaining connectivity between the leader i and the follower j .
- (ii) Objective (O-iii) indicates, in the absence of detected obstacles, the formation errors between the leader i and the follower j converge to arbitrarily small bounds.
- (iii) Objectives (O-iii) and (O-iv) prioritize obstacle avoidance over formation tracking, as collisions can result in more critical system damage than the temporary degradation of formation tracking performance. Furthermore, the formation errors remain bounded even during obstacle avoidance, as stated in Objective (O-iv).

III. MAIN RESULTS

In this paper, each follower tracks one leader to maintain the desired distance and angle, ensuring connectivity and avoiding collisions even in the presence of static and dynamic obstacles such as neighboring robots and unknown objects. To achieve this, a leader-follower-based formation control scheme is designed using guidance signals, nonlinear error functions, and command-filtered backstepping (CFB) technique.

Define the errors as

$$e_{1,j} = s_{ij} - l_{r,ij} \quad (5)$$

$$e_{2,j} = \psi_{ij} - \psi_{r,ij} \quad (6)$$

$$e_{3,j} = \max\{0, s_{oj}\} \quad (7)$$

$$e_{4,j} = [e_{4,1,j}, e_{4,2,j}]^T = \mathbf{v}_j - \bar{\boldsymbol{\alpha}}_j \quad (8)$$

where

$$s_{ij} = \ln \left(\frac{\delta_{ij} + p_{ij}}{\delta_{ij}(1 - p_{ij})} \right), \quad s_{oj} = -\ln \left(\frac{l_{oj} - R_{a,j}}{R_{s,j} - R_{a,j}} \right)$$

$$\bar{\boldsymbol{\alpha}}_j = [\bar{\alpha}_{1,j}, \bar{\alpha}_{2,j}]^T$$

with

$$\delta_{ij} = \frac{l_{d,ij} - R_{a,j}}{R_{c,j} - l_{d,ij}}, \quad p_{ij} = \frac{l_{ij} - l_{d,ij}}{R_{c,j} - l_{d,ij}}.$$

In these expressions, $l_{r,ij}$ and $\psi_{r,ij}$ denote the guidance signals to be defined later, and $R_{s,j}$ is the obstacle detection range satisfying $R_{a,j} < R_{s,j} < R_{c,j}$. The command-filtered signals $\bar{\alpha}_{n,j}$ for $n = 1, 2$ are generated by the filters

$$\dot{\bar{\alpha}}_{n,j} = -\varpi_{n,j}(\bar{\alpha}_{n,j} - \alpha_{n,j}) \quad (9)$$

where $\alpha_{n,j}$ are virtual controls, $\varpi_{n,j}$ are positive constants, and $\bar{\alpha}_{n,j}(0) = \alpha_{n,j}(0)$.

Remark 3: The design concepts of the error signals (5)–(7) are as follows:

- (i) The guidance distance $l_{r,ij}$ and angle $\psi_{r,ij}$ are designed to modify the desired signals $l_{d,ij}$ and $\psi_{d,ij}$ based on the presence or absence of obstacles.
- (ii) If the unified nonlinear function $s_{ij}(t)$ in (5) is bounded for $t \geq 0$, it ensures that $R_{a,j} < l_{ij}(t) < R_{c,j}$ for $t \geq 0$, even while avoiding obstacles. Thus, the controller is designed to ensure that s_{ij} is bounded.
- (iii) The error $e_{3,j}$ is an error function consisting of the obstacle avoidance function s_{oj} , which diverges to infinity as the follower j approaches the obstacle. If $e_{3,j}$ is bounded, it ensures that $l_{oj}(t) > R_{a,j}$. In s_{oj} , the obstacle detection range $R_{s,j}$ is introduced to set the distance at which it starts avoiding collisions with obstacles.
- (iv) The command-filtered signal $\bar{\alpha}_j$ is used to eliminate the requirement for differentiation of the virtual controls.

To compensate for the effect of the filtering error $\bar{\alpha}_{n,j} - \alpha_{n,j}$, we define the compensated tracking errors as

$$z_{1,j} = e_{1,j} - \xi_{1,j} \quad (10)$$

$$z_{2,j} = e_{2,j} - \xi_{2,j} \quad (11)$$

$$z_{3,j} = e_{3,j} - \xi_{3,j} \quad (12)$$

$$z_{4,j} = e_{4,j}. \quad (13)$$

Here, the compensating signals $\xi_{n,j}$ using neural networks are presented as

$$\dot{\xi}_{n,j} = -k_{n,j}\xi_{n,j} + \hat{\mathbf{W}}_{n,j}^\top \boldsymbol{\Phi}_{n,j}, \quad n = 1, 2, 3, \quad (14)$$

where $\xi_{n,j}(0) = 0$, $k_{n,j} > 0$ denote constants, and $\hat{\mathbf{W}}_{n,j}^\top \boldsymbol{\Phi}_{n,j}$ are the outputs of radial basis function networks (RBFNs) to be defined later.

Remark 4: In the CFB method [29], the stability of the feedback control system requires assuming the boundedness condition of the filtering errors $\bar{\alpha}_{n,j} - \alpha_{n,j}$ on a compact set. This assumption becomes feasible when the time derivatives of the virtual controls are bounded on the compact set, as demonstrated in the proof of Lemma 2 using Lemma 4 of [29]. The Dynamic Surface Design (DSC) method [30] also employs the semi-global stability concept to address this issue. However, in tackling this problem, our paper opts for

using RBFNs to design compensating signals in (14), rather than directly utilizing the errors $\tilde{\alpha}_{n,j} - \alpha_{n,j}$. Consequently, the proposed design, employing the modified CFB, holds the advantage that bounding conditions for the time derivatives of virtual controls are not necessary to establish the stability of the formation control system, distinguishing it from the CFB and DSC methods.

From (2), (4), and (8), the time derivatives of (5)–(7) are derived as follows:

$$\dot{e}_{1,j} = \frac{1}{\rho_{1,j}} \{ -(\tilde{\alpha}_{1,j} - \alpha_{1,j} + \alpha_{1,j} + e_{4,1,j}) \cos \phi_{ij} + d_j(\tilde{\alpha}_{2,j} - \alpha_{2,j} + \alpha_{2,j} + e_{4,2,j}) \sin \phi_{ij} + v_i \cos \psi_{ij} - \dot{l}_{r,ij} \} \quad (15)$$

$$\dot{e}_{2,j} = \frac{1}{l_{ij}} (\tilde{\alpha}_{1,j} - \alpha_{1,j} + \alpha_{1,j} + e_{4,1,j}) \sin \phi_{ij} + \frac{d_j}{l_{ij}} (\tilde{\alpha}_{2,j} - \alpha_{2,j} + \alpha_{2,j} + e_{4,2,j}) \cos \phi_{ij} + \omega_i - \frac{v_i}{l_{ij}} \sin \psi_{ij} - \dot{\psi}_{r,ij} \quad (16)$$

$$\dot{e}_{3,j} = \begin{cases} 0, & \text{if } l_{oj}(t) \geq R_{s,j} \\ g_{1,j}, & \text{otherwise} \end{cases} \quad (17)$$

$$\mathbf{M}_j \dot{e}_{4,j} = \boldsymbol{\tau}_j - \mathbf{M}_j \dot{\tilde{\alpha}}_j - (\mathbf{C}_j \mathbf{v}_j + \mathbf{D}_j \mathbf{v}_j + \boldsymbol{\tau}_{d,j}) \quad (18)$$

where

$$\rho_{1,j} = \frac{(R_{c,j} - l_{d,ij})(\delta_{ij} + p_{ij})(1 - p_{ij})}{1 + \delta_{ij}}$$

$$g_{1,j} = \frac{1}{\rho_{2,j}} \{ (\tilde{\alpha}_{1,j} - \alpha_{1,j} + \alpha_{1,j} + e_{4,1,j}) \cos \phi_{oj} - d_j(\tilde{\alpha}_{2,j} - \alpha_{2,j} + \alpha_{2,j} + e_{4,2,j}) \sin \phi_{oj} - v_o \cos \psi_{oj} \}$$

with $\rho_{2,j} = l_{oj} - R_{a,j}$, $\psi_{oj} = \theta_o - \arctan(l_{oj,y}/l_{oj,x})$, and θ_o is the heading direction of the obstacle o . Similar to the angle ϕ_{ij} in Fig. 1, the relative angle ϕ_{oj} between the follower j and the nearest obstacle o can be measured by onboard sensors of the follower j .

Define the matrix $\mathbf{B}_j$ as

$$\mathbf{B}_j = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Multiplying $\mathbf{B}_j$ to both sides of (18) yields

$$\overline{\mathbf{M}}_j \dot{e}_{4,j} = -\mathbf{B}_j(\mathbf{C}_j \mathbf{v}_j + \mathbf{D}_j \mathbf{v}_j + \boldsymbol{\tau}_{d,j}) - \overline{\mathbf{M}}_j \dot{\tilde{\alpha}}_j + \mathbf{B}_j \boldsymbol{\tau}_j \quad (19)$$

where $\overline{\mathbf{M}}_j = \mathbf{B}_j \mathbf{M}_j$. From the definitions of matrices $\mathbf{M}_j$ and $\mathbf{B}_j$, it can be easily shown that $\overline{\mathbf{M}}_j$ is positive-definite and the inverse matrix of $\mathbf{B}_j$ exists.

Remark 5: The main difficulty in achieving the control objectives (O-i)–(O-iv) is deriving the virtual control laws $\alpha_{1,j}$ and $\alpha_{2,j}$ to stabilize the error dynamics defined in (15)–(18). The challenges in designing the virtual control laws and the corresponding solutions are outlined as follows:

(i) The virtual control terms $\alpha_{1,j}$ and $\alpha_{2,j}$ are embedded in the dynamics (17) of the error function for obstacle

avoidance, as well as the relative position and angle error dynamics (15) and (16). Consequently, the coupling terms among the dynamics (15)–(17) are taken into account for the design of the virtual controllers $\alpha_{1,j}$ and $\alpha_{2,j}$. To address this challenge, we introduce the guidance signals $l_{r,ij}$ and $\psi_{r,ij}$ to provide design flexibility in the virtual control design of $\alpha_{1,j}$ and $\alpha_{2,j}$.

(ii) The corrupted velocity information $(\bar{v}_i, \bar{\omega}_i)$ of the leader is solely available for the design of the virtual control laws $\alpha_{1,j}$ and $\alpha_{2,j}$. Additionally, the obstacle velocity v_o is unknown to the follower j . Consequently, the adaptive compensation terms in $\alpha_{1,j}$ and $\alpha_{2,j}$ must be designed to counteract the effects of the unknown attack signals $a_{v,i}$ and $a_{\omega,i}$, as well as the unknown obstacle velocity v_o . We introduce the adaptive virtual controllers $\alpha_{1,j}$ and $\alpha_{2,j}$ along with the adaptive guidance signals $l_{r,ij}$ and $\psi_{r,ij}$, taking into consideration the coupling terms among the dynamics (15)–(17).

To overcome these design difficulties, the formation control scheme with adaptive collision avoidance is designed as

$$\alpha_{1,j} = -\zeta_{1,j} \cos \phi_{ij} + \zeta_{2,j} l_{ij} \sin \phi_{ij} \quad (20)$$

$$\alpha_{2,j} = \frac{1}{d_j} \zeta_{1,j} \sin \phi_{ij} + \frac{l_{ij}}{d_j} \zeta_{2,j} \cos \phi_{ij} \quad (21)$$

$$\boldsymbol{\tau}_j = \mathbf{B}_j^{-1} (-k_{4,j} \mathbf{e}_{4,j} - \widehat{\mathbf{W}}_{4,j}^\top \boldsymbol{\Phi}_{4,j} - \Upsilon_{1,j} \mathbf{z}_{1,j} - \Upsilon_{2,j} \mathbf{z}_{2,j} - \Upsilon_{3,j} \mathbf{z}_{3,j}) \quad (22)$$

where $\alpha_{1,j}$ and $\alpha_{2,j}$ are virtual control laws, $\boldsymbol{\tau}_j$ is the actual control law,

$$\begin{aligned} \zeta_{1,j} &= -k_{1,j} \rho_{1,j} e_{1,j} - \bar{v}_i \cos \psi_{ij} - \lambda_{1,j} \rho_{1,j} l_{r,ij} \\ &\quad + k_{3,j} \rho_{2,j} e_{3,j} \cos(\phi_{ij} - \phi_{oj}) - \hat{a}_{1,j} \tanh \left(\frac{\hat{a}_{1,j} \mathbf{z}_{1,j}}{\rho_{1,j} \epsilon_j} \right) \\ &\quad + \hat{a}_{3,j} \cos(\phi_{ij} - \phi_{oj}) \tanh \left(\frac{\hat{a}_{3,j} \mathbf{z}_{3,j}}{\rho_{2,j} \epsilon_j} \right) \\ \zeta_{2,j} &= -k_{2,j} e_{2,j} - \frac{k_{3,j} \rho_{2,j} e_{3,j}}{l_{ij}} \sin(\phi_{ij} - \phi_{oj}) - \bar{\omega}_i \\ &\quad + \frac{\bar{v}_i}{l_{ij}} \sin \psi_{ij} - \lambda_{2,j} (\psi_{r,ij} - \psi_{d,ij}) \\ &\quad - \frac{\hat{a}_{1,j}}{l_{ij}} \tanh \left(\frac{\hat{a}_{1,j} \mathbf{z}_{2,j}}{l_{ij} \epsilon_j} \right) - \hat{a}_{2,j} \tanh \left(\frac{\hat{a}_{2,j} \mathbf{z}_{2,j}}{\epsilon_j} \right) \\ &\quad - \frac{\hat{a}_{3,j}}{l_{ij}} \sin(\phi_{ij} - \phi_{oj}) \tanh \left(\frac{\hat{a}_{3,j} \mathbf{z}_{3,j}}{\rho_{2,j} \epsilon_j} \right) \end{aligned}$$

$$\Upsilon_{1,j} = [-\cos \phi_{ij} / \rho_{1,j}, d_j \sin \phi_{ij} / \rho_{1,j}]^\top$$

$$\Upsilon_{2,j} = [\sin \phi_{ij} / l_{ij}, d_j \cos \phi_{ij} / l_{ij}]^\top$$

$$\Upsilon_{3,j} = [\cos \phi_{oj} / \rho_{2,j}, -d_j \sin \phi_{oj} / \rho_{2,j}]^\top$$

with design constants $k_{1,j} > 0$, $k_{2,j} > 0$, $k_{3,j} > 0$, $k_{4,j} > 0$, $\lambda_{1,j} > 0$, $\lambda_{2,j} > 0$, and $\epsilon_j > 0$. In these expressions,

$\widehat{\mathbf{W}}_{4,j}^\top \Phi_{4,j}$ is the output of the RBFN to be defined later, $\widehat{\mathbf{W}}_{4,j} = [\widehat{W}_{4,j,1}, \widehat{W}_{4,j,2}]^\top$, and $\hat{a}_{1,j}$, $\hat{a}_{2,j}$, and $\hat{a}_{3,j}$ are the estimates of $\bar{a}_{v,i}$, $\bar{a}_{\omega,i}$, and $\bar{v}_o$, respectively. When there are no detected obstacles, $\hat{a}_{3,j}$ is set to $\hat{a}_{3,j} = 0$ because $\hat{a}_{3,j}$ is an estimate of the bound $\bar{v}_o$ of the obstacle's velocity.

The guidance signals $l_{r,ij}$ and $\psi_{r,ij}$ are updated by

$$\begin{aligned} \dot{l}_{r,ij} = & -\lambda_{1,j} l_{r,ij} + \frac{k_{3,j} \rho_{2,j} e_{3,j}}{\rho_{1,j}} \cos(\phi_{ij} - \phi_{oj}) \\ & + \frac{k_{1,j} \rho_{1,j} z_{3,j}}{\rho_{2,j}} \cos(\phi_{ij} - \phi_{oj}) \\ & + \frac{\hat{a}_{3,j}}{\rho_{1,j}} \cos(\phi_{ij} - \phi_{oj}) \tanh\left(\frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j} \epsilon_j}\right) \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{\psi}_{r,ij} = & -\lambda_{2,j} (\psi_{r,ij} - \psi_{d,ij}) - \frac{k_{3,j} \rho_{2,j} e_{3,j}}{l_{ij}} \sin(\phi_{ij} - \phi_{oj}) \\ & - \frac{k_{2,j} l_{ij} z_{3,j}}{\rho_{2,j}} \sin(\phi_{ij} - \phi_{oj}) \\ & - \frac{\hat{a}_{3,j}}{l_{ij}} \sin(\phi_{ij} - \phi_{oj}) \tanh\left(\frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j} \epsilon_j}\right) \end{aligned} \quad (24)$$

where $l_{r,ij}(0) = 0$ and $\psi_{r,ij}(0) = \psi_{d,ij}$.

Remark 6: In (20)–(24), the variables l_{oj} in $\rho_{2,j}$ and ϕ_{oj} can be measured within the obstacle detection range $R_{s,j}$. Therefore, in the absence of detected obstacles, l_{oj} and ϕ_{oj} are defined as $l_{oj} = R_{s,j}$ and $\phi_{oj} = 0$.

Remark 7: In existing results [16], [17], [18], [19], the potential functions for collision avoidance, obstacle avoidance, and connectivity maintenance are designed separately and independently of the formation tracking error term when constructing the formation controller in the Lyapunov functions. Since the virtual control laws are included in both the time derivatives of the potential functions and the formation tracking error function, this approach may lead to conflicts among the control objectives. For instance, when the potential functions for formation tracking and obstacle avoidance have equal magnitudes but opposite directions, the robot may stop moving altogether. Thus, it is challenging to design unified virtual control laws that ensure the stability of all potential functions and the formation tracking error function. Moreover, deriving adaptive mechanisms to estimate the unknown obstacle velocity and attack signals in the Lyapunov sense poses significant challenges. To overcome these issues, we employ the nonlinear formation error function (5) and the error function (7) for obstacle avoidance as error variables in the Lyapunov-based recursive design. In (23) and (24), the guidance signals $l_{r,ij}$ and $\psi_{r,ij}$ are designed to adjust the performance trade-off between formation tracking and obstacle avoidance while preserving connectivity between the leader and follower. Subsequently, the Lyapunov-based control design procedure allows us to derive the adaptive tuning laws (i.e., (34)) for $\hat{a}_{1,j}$, $\hat{a}_{2,j}$, and $\hat{a}_{3,j}$. This represents the advantages of the proposed method when compared to existing potential-field-based formation tracking results [16], [17], [18], [19].

By Assumption 1, inserting (20)–(24) into (15)–(17) yields

$$\begin{aligned} \dot{e}_{1,j} = & -k_{1,j} e_{1,j} - \frac{a_{v,i}}{\rho_{1,j}} \cos \psi_{ij} - \frac{\hat{a}_{1,j}}{\rho_{1,j}} \tanh\left(\frac{\hat{a}_{1,j} z_{1,j}}{\rho_{1,j} \epsilon_j}\right) \\ & - \frac{k_{1,j} \rho_{1,j} z_{3,j}}{\rho_{2,j}} \cos(\phi_{ij} - \phi_{oj}) + \mathbf{e}_{4,j}^\top \Upsilon_{1,j} + f_{1,j} \end{aligned} \quad (25)$$

$$\begin{aligned} \dot{e}_{2,j} = & -k_{2,j} e_{2,j} - a_{\omega,i} + \frac{a_{v,i}}{l_{ij}} \sin \psi_{ij} \\ & - \frac{\hat{a}_{1,j}}{l_{ij}} \tanh\left(\frac{\hat{a}_{1,j} z_{2,j}}{l_{ij} \epsilon_j}\right) - \hat{a}_{2,j} \tanh\left(\frac{\hat{a}_{2,j} z_{2,j}}{\epsilon_j}\right) \\ & + \frac{k_{2,j} l_{ij} z_{3,j}}{\rho_{2,j}} \sin(\phi_{ij} - \phi_{oj}) + \mathbf{e}_{4,j}^\top \Upsilon_{2,j} + f_{2,j} \end{aligned} \quad (26)$$

$$\dot{e}_{3,j} = \begin{cases} 0, & \text{if } l_{oj}(t) \geq R_{s,j} \\ g_{1,j}, & \text{otherwise} \end{cases} \quad (27)$$

$$\begin{aligned} \bar{\mathbf{M}}_j \dot{\mathbf{e}}_{4,j} = & -\mathbf{B}_j(\mathbf{C}_j \mathbf{v}_j + \mathbf{D}_j \mathbf{v}_j + \boldsymbol{\tau}_{d,j}) - \bar{\mathbf{M}}_j \dot{\boldsymbol{\alpha}}_j \\ & - k_{4,j} \mathbf{e}_{4,j} - \widehat{\mathbf{W}}_{4,j}^\top \Phi_{4,j} - \sum_{n=1}^3 \Upsilon_{n,j} z_{n,j} \end{aligned} \quad (28)$$

where

$$\begin{aligned} f_{1,j} = & \frac{1}{\rho_{1,j}} \{ -(\bar{\alpha}_{1,j} - \alpha_{1,j}) \cos \phi_{ij} + d_j (\bar{\alpha}_{2,j} - \alpha_{2,j}) \sin \phi_{ij} \} \\ f_{2,j} = & \frac{1}{l_{ij}} \{ (\bar{\alpha}_{1,j} - \alpha_{1,j}) \sin \phi_{ij} + d_j (\bar{\alpha}_{2,j} - \alpha_{2,j}) \cos \phi_{ij} \} \\ g_{1,j} = & -k_{3,j} e_{3,j} - \frac{v_o}{\rho_{2,j}} \cos \psi_{oj} - \frac{\hat{a}_{3,j}}{\rho_{2,j}} \tanh\left(\frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j} \epsilon_j}\right) \\ & + \frac{k_{1,j} \rho_{1,j} z_{1,j}}{\rho_{2,j}} \cos(\phi_{ij} - \phi_{oj}) \\ & - \frac{k_{2,j} l_{ij} z_{2,j}}{\rho_{2,j}} \sin(\phi_{ij} - \phi_{oj}) + \mathbf{e}_{4,j}^\top \Upsilon_{3,j} + f_{3,j} \\ f_{3,j} = & \frac{1}{\rho_{2,j}} \left[\cos(\phi_{ij} - \phi_{oj}) \left\{ \bar{v}_i \cos \psi_{ij} + \lambda_{1,j} \rho_{1,j} l_{r,ij} \right. \right. \\ & + \hat{a}_{1,j} \tanh\left(\frac{\hat{a}_{1,j} z_{1,j}}{\rho_{1,j} \epsilon_j}\right) \Big\} \\ & + \sin(\phi_{ij} - \phi_{oj}) \left\{ \bar{v}_i \sin \psi_{ij} - \hat{a}_{1,j} \tanh\left(\frac{\hat{a}_{1,j} z_{2,j}}{l_{ij} \epsilon_j}\right) \right\} \\ & - l_{ij} \sin(\phi_{ij} - \phi_{oj}) \left\{ \bar{\omega}_i + \lambda_{2,j} (\psi_{r,ij} - \psi_{d,ij}) \right. \\ & + \hat{a}_{2,j} \tanh\left(\frac{\hat{a}_{2,j} z_{2,j}}{\epsilon_j}\right) \Big\} \\ & + (\bar{\alpha}_{1,j} - \alpha_{1,j}) \cos \phi_{oj} - d_j (\bar{\alpha}_{2,j} - \alpha_{2,j}) \sin \phi_{oj} \\ & + k_{1,j} \rho_{1,j} \xi_{1,j} \cos(\phi_{ij} - \phi_{oj}) \\ & \left. \left. - k_{2,j} l_{ij} \xi_{2,j} \sin(\phi_{ij} - \phi_{oj}) \right] \right] \end{aligned}$$

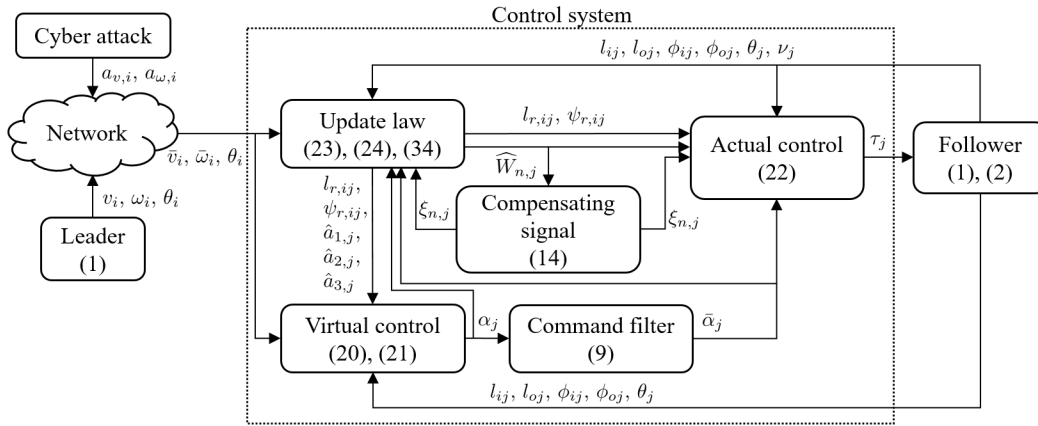


FIGURE 2. Block diagram of the proposed formation control system.

The time derivatives of (10)–(13) along with (25)–(28) are

$$\begin{aligned} \dot{z}_{1,j} = & -k_{1,j}z_{1,j} - \frac{a_{v,i}}{\rho_{1,j}} \cos \psi_{ij} - \frac{\hat{a}_{1,j}}{\rho_{1,j}} \tanh \left(\frac{\hat{a}_{1,j}z_{1,j}}{\rho_{1,j}\epsilon_j} \right) \\ & - \frac{k_{1,j}\rho_{1,j}z_{3,j}}{\rho_{2,j}} \cos(\phi_{ij} - \phi_{oj}) + \mathbf{z}_{4,j}^\top \Upsilon_{1,j} + f_{1,j} \\ & - \widehat{\mathbf{W}}_{1,j}^\top \Phi_{1,j} \end{aligned} \quad (29)$$

$$\begin{aligned} \dot{z}_{2,j} = & -k_{2,j}z_{2,j} - a_{\omega,i} - \hat{a}_{2,j} \tanh \left(\frac{\hat{a}_{2,j}z_{2,j}}{\epsilon_j} \right) \\ & + \frac{a_{v,i}}{l_{ij}} \sin \psi_{ij} - \frac{\hat{a}_{1,j}}{l_{ij}} \tanh \left(\frac{\hat{a}_{1,j}z_{2,j}}{l_{ij}\epsilon_j} \right) \\ & + \frac{k_{2,j}l_{ij}z_{3,j}}{\rho_{2,j}} \sin(\phi_{ij} - \phi_{oj}) + \mathbf{z}_{4,j}^\top \Upsilon_{2,j} + f_{2,j} \\ & - \widehat{\mathbf{W}}_{2,j}^\top \Phi_{2,j} \end{aligned} \quad (30)$$

$$\dot{z}_{3,j} = \begin{cases} -\dot{\xi}_{3,j}, & \text{if } l_{oj}(t) \geq R_{s,j} \\ g_{2,j}, & \text{otherwise} \end{cases} \quad (31)$$

$$\begin{aligned} \overline{\mathbf{M}}_j \dot{\mathbf{z}}_{4,j} = & -\mathbf{B}_j(\mathbf{C}_j \mathbf{v}_j + \mathbf{D}_j \mathbf{v}_j + \boldsymbol{\tau}_{d,j}) - \overline{\mathbf{M}}_j \dot{\boldsymbol{\alpha}}_j - k_{4,j} \mathbf{z}_{4,j} \\ & - \widehat{\mathbf{W}}_{4,j}^\top \Phi_{4,j} - \sum_{n=1}^3 \Upsilon_{n,j} z_{n,j} \end{aligned} \quad (32)$$

where

$$\begin{aligned} g_{2,j} = & -k_{3,j}z_{3,j} - \frac{v_o}{\rho_{2,j}} \cos \psi_{oj} - \frac{\hat{a}_{3,j}}{\rho_{2,j}} \tanh \left(\frac{\hat{a}_{3,j}z_{3,j}}{\rho_{2,j}\epsilon_j} \right) \\ & + \frac{k_{1,j}\rho_{1,j}z_{1,j}}{\rho_{2,j}} \cos(\phi_{ij} - \phi_{oj}) + f_{3,j} - \widehat{\mathbf{W}}_{3,j}^\top \Phi_{3,j} \\ & - \frac{k_{2,j}l_{ij}z_{2,j}}{\rho_{2,j}} \sin(\phi_{ij} - \phi_{oj}) + \mathbf{z}_{4,j}^\top \Upsilon_{3,j}. \end{aligned}$$

Define $\mathbf{f}_{4,j} = \bar{\tau}_{d,j}^2 \mathbf{z}_{4,j} / (4\beta_j) - \mathbf{B}_j(\mathbf{C}_j \mathbf{v}_j + \mathbf{D}_j \mathbf{v}_j) - \overline{\mathbf{M}}_j \dot{\boldsymbol{\alpha}}_j$ where $\bar{\tau}_{d,j}$ is a constant satisfying $\|\boldsymbol{\tau}_{d,j}\| \leq \bar{\tau}_{d,j}$ and β_j is a positive

constant. RBFNs are employed to approximate the nonlinear functions $f_{n,j}$ and $\mathbf{f}_{4,j}$ as follows:

$$\begin{aligned} f_{n,j} &= \mathbf{W}_{n,j}^\top \Phi_{n,j}(\chi_{n,j}) + \varepsilon_{n,j}, \quad n = 1, 2, 3 \\ \mathbf{f}_{4,j} &= \mathbf{W}_{4,j}^\top \Phi_{4,j}(\chi_{4,j}) + \boldsymbol{\varepsilon}_{4,j} \end{aligned} \quad (33)$$

where $\mathbf{W}_{n,j} \in \mathbb{R}^{N_{n,j}}$ and $\mathbf{W}_{4,j} \in \mathbb{R}^{N_{4,j} \times 2}$ are optimal weight vectors, $\varepsilon_{n,j}$ and $\boldsymbol{\varepsilon}_{4,j}$ are reconstruction errors, $\Phi_{m,j} = [\Phi_{m,j,1}, \dots, \Phi_{m,j,N_{m,j}}]^\top$, $m = 1, \dots, 4$ are Gaussian functions; $\Phi_{m,j,p} = \exp(-(\chi_{m,j} - \mathbf{q}_{m,j,p})^\top (\chi_{m,j} - \mathbf{q}_{m,j,p}) / (2\varphi_{m,j,p}^2))$ for $p = 1, \dots, N_{m,j}$, $\chi_{m,j}$ are inputs of RBFNs, $N_{m,j}$ denotes the number of nodes of RBFNs, and $\mathbf{q}_{m,j,p}$ and $\varphi_{m,j,p}$ are the parameters of the Gaussian functions. Note that Gaussian functions $\Phi_{m,j}$ are bounded such that $\|\Phi_{m,j}\| \leq \sqrt{N_{m,j}}$. Owing to the universal approximation property [31], [32], [33], there exist positive constants $\overline{W}_{m,j}$ and $\bar{\varepsilon}_{m,j}$, $m = 1, \dots, 4$ satisfying $\|\mathbf{W}_{n,j}\| \leq \overline{W}_{n,j}$, $n = 1, 2, 3$, $\|\mathbf{W}_{4,j}\|_F \leq \overline{W}_{4,j}$, $|\varepsilon_{n,j}| \leq \bar{\varepsilon}_{n,j}$, and $\|\boldsymbol{\varepsilon}_{4,j}\| \leq \bar{\varepsilon}_{4,j}$ where $\|\cdot\|_F$ denotes the Frobenius norm.

The estimates $\hat{a}_{n,j}$, $n = 1, 2, 3$, and $\widehat{\mathbf{W}}_{m,j}$ of the optimal weights $\mathbf{W}_{m,j}$, $m = 1, \dots, 4$, are updated by

$$\begin{aligned} \dot{\hat{a}}_{1,j} &= \gamma_{1,j} \left(\left| \frac{z_{1,j}}{\rho_{1,j}} \right| + \left| \frac{z_{2,j}}{l_{ij}} \right| \right) - \gamma_{1,j} \sigma_{1,j} \hat{a}_{1,j} \\ \dot{\hat{a}}_{2,j} &= \gamma_{2,j} |z_{2,j}| - \gamma_{2,j} \sigma_{2,j} \hat{a}_{2,j} \\ \dot{\hat{a}}_{3,j} &= \gamma_{3,j} \left| \frac{z_{3,j}}{\rho_{2,j}} \right| - \gamma_{3,j} \sigma_{3,j} \hat{a}_{3,j} \\ \dot{\widehat{\mathbf{W}}}_{1,j} &= \Gamma_{1,j} \Phi_{1,j}(\chi_{1,j}) z_{1,j} - \bar{\sigma}_{1,j} \Gamma_{1,j} \widehat{\mathbf{W}}_{1,j} \\ \dot{\widehat{\mathbf{W}}}_{2,j} &= \Gamma_{2,j} \Phi_{2,j}(\chi_{2,j}) z_{2,j} - \bar{\sigma}_{2,j} \Gamma_{2,j} \widehat{\mathbf{W}}_{2,j} \\ \dot{\widehat{\mathbf{W}}}_{3,j} &= \Gamma_{3,j} \Phi_{3,j}(\chi_{3,j}) z_{3,j} - \bar{\sigma}_{3,j} \Gamma_{3,j} \widehat{\mathbf{W}}_{3,j} \\ \dot{\widehat{\mathbf{W}}}_{4,j} &= \Gamma_{4,j} \Phi_{4,j}(\chi_{4,j}) \mathbf{z}_{4,j}^\top - \bar{\sigma}_{4,j} \Gamma_{4,j} \widehat{\mathbf{W}}_{4,j} \end{aligned} \quad (34)$$

where $\hat{a}_{1,j}(0) = \hat{a}_{2,j}(0) = \hat{a}_{3,j}(0) = 0$, $\gamma_{n,j}$ and $\sigma_{n,j}$, $n = 1, 2, 3$ are positive constants. $\bar{\sigma}_{m,j}$ and $\Gamma_{m,j}$,

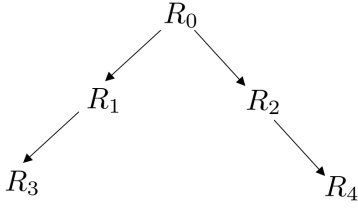


FIGURE 3. Connection between robots.

$m = 1, \dots, 4$ denote positive constants and positive-definite matrices, respectively. The inputs $\chi_{m,j}$ of RBFNs for $m = 1, \dots, 4$ are defined as $\chi_{1,j} = [\bar{\alpha}_j^\top, \alpha_j^\top, \phi_{ij}, \rho_{1,j}]^\top$, $\chi_{2,j} = [\bar{\alpha}_j^\top, \alpha_j^\top, \phi_{ij}, l_{ij}]^\top$, $\chi_{3,j} = [z_{1,j}, z_{2,j}, \bar{\alpha}_j^\top, \alpha_j^\top, \phi_{ij}, \phi_{oj}, \psi_{ij}, \rho_{1,j}, \rho_{2,j}, l_{ij}, \xi_{1,j}, \xi_{2,j}, \hat{a}_{1,j}, \hat{a}_{2,j}]^\top$, $\chi_{4,j} = [v_j^\top, \bar{\alpha}_j^\top, \alpha_j^\top, z_{4,j}^\top]^\top$, where $\alpha_j = [\alpha_{1,j}, \alpha_{2,j}]^\top$. The block diagram of the proposed formation control system is shown in Fig. 2.

The main results of this paper are given in the following theorem.

Theorem 1: Consider the leader-follower model (4) controlled by the control torque (22) with the adaptive laws (34). If Assumptions 1–3 are satisfied, then there are design parameters $k_{m,j}$, $\gamma_{n,j}$, $\Gamma_{m,j}$, $\sigma_{n,j}$, and $\bar{\sigma}_{m,j}$ that achieve the control objectives (O-i)–(O-iv) given in Section II-B where $m = 1, \dots, 4$ and $n = 1, 2, 3$. Furthermore, the formation errors can be made arbitrarily small by design parameters, which results in achieving the desired formation.

Proof: The Lyapunov function is defined as

$$V = \frac{1}{2} \left\{ \sum_{n=1}^3 z_{n,j}^2 + z_{4,j}^\top \bar{M}_j z_{4,j} + \text{tr}(\tilde{W}_{4,j}^\top \Gamma_{4,j}^{-1} \tilde{W}_{4,j}) + \sum_{n=1}^3 \left(\frac{1}{\gamma_{n,j}} \tilde{a}_{n,j}^2 + \tilde{W}_{n,j}^\top \Gamma_{n,j}^{-1} \tilde{W}_{n,j} \right) \right\} \quad (35)$$

where $\tilde{a}_{1,j} = \bar{a}_{v,i} - \hat{a}_{1,j}$, $\tilde{a}_{2,j} = \bar{a}_{\omega,i} - \hat{a}_{2,j}$, $\tilde{a}_{3,j} = \bar{v}_o - \hat{a}_{3,j}$, $\tilde{W}_{m,j} = W_{m,j} - \hat{W}_{m,j}$ for $m = 1, \dots, 4$, and $\text{tr}(\cdot)$ denotes the trace of the matrix.

Assume that the follower j detects obstacles, i.e., $l_{oj}(t) < R_{s,j}$. By Assumptions 1 and 3, the time derivative of (35) along with (29)–(32) is

$$\begin{aligned} \dot{V} \leq & -k_{1,j} z_{1,j}^2 + \bar{a}_{v,i} \left| \frac{z_{1,j}}{\rho_{1,j}} \right| - \frac{\hat{a}_{1,j} z_{1,j}}{\rho_{1,j}} \tanh \left(\frac{\hat{a}_{1,j} z_{1,j}}{\rho_{1,j} \epsilon_j} \right) \\ & - k_{2,j} z_{2,j}^2 + \bar{a}_{\omega,i} |z_{2,j}| - \hat{a}_{2,j} z_{2,j} \tanh \left(\frac{\hat{a}_{2,j} z_{2,j}}{\epsilon_j} \right) \\ & + \bar{a}_{v,i} \left| \frac{z_{2,j}}{l_{ij}} \right| - \frac{\hat{a}_{1,j} z_{2,j}}{l_{ij}} \tanh \left(\frac{\hat{a}_{1,j} z_{2,j}}{l_{ij} \epsilon_j} \right) - k_{3,j} z_{3,j}^2 \\ & + \bar{v}_o \left| \frac{z_{3,j}}{\rho_{2,j}} \right| - \frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j}} \tanh \left(\frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j} \epsilon_j} \right) \\ & + \sum_{n=1}^3 z_{n,j} (f_{n,j} - \hat{W}_{n,j}^\top \Phi_{n,j}) - k_{4,j} z_{4,j}^\top z_{4,j} \end{aligned}$$

$$\begin{aligned} & - z_{4,j}^\top (\hat{W}_{4,j}^\top \Phi_{4,j} + B_j (C_j v_j + D_j v_j + \tau_{d,j}) + \bar{M}_j \dot{\alpha}_j) \\ & - \frac{1}{\gamma_{1,j}} \tilde{a}_{1,j} \dot{\hat{a}}_{1,j} - \frac{1}{\gamma_{2,j}} \tilde{a}_{2,j} \dot{\hat{a}}_{2,j} - \frac{1}{\gamma_{3,j}} \tilde{a}_{3,j} \dot{\hat{a}}_{3,j} \\ & - \sum_{n=1}^3 \tilde{W}_{n,j}^\top \Gamma_{n,j}^{-1} \dot{\hat{W}}_{n,j} - \text{tr}(\tilde{W}_{4,j}^\top \Gamma_{4,j}^{-1} \dot{\hat{W}}_{4,j}). \quad (36) \end{aligned}$$

By Young's inequality, the following inequality holds.

$$\begin{aligned} z_{4,j}^\top B_j \tau_{d,j} & \leq \frac{1}{\sqrt{2}} \|z_{4,j}\| \|\tau_{d,j}\| \\ & \leq \frac{\bar{\tau}_{d,j}^2}{4\beta_j} z_{4,j}^\top z_{4,j} + \frac{\beta_j}{2}. \quad (37) \end{aligned}$$

Furthermore, using the inequality $|\eta| - \eta \tanh(\eta/\epsilon) \leq \kappa \epsilon$ for any $\eta \in \mathbb{R}$, $\kappa = 0.2785$, and a positive constant ϵ (see Lemma 1 in [34] for details), the following inequalities are satisfied:

$$\begin{aligned} \bar{a}_{v,i} \left| \frac{z_{1,j}}{\rho_{1,j}} \right| - \frac{\hat{a}_{1,j} z_{1,j}}{\rho_{1,j}} \tanh \left(\frac{\hat{a}_{1,j} z_{1,j}}{\rho_{1,j} \epsilon_j} \right) & \leq \tilde{a}_{1,j} \left| \frac{z_{1,j}}{\rho_{1,j}} \right| + \kappa \epsilon_j \\ \bar{a}_{\omega,i} |z_{2,j}| - \hat{a}_{2,j} z_{2,j} \tanh \left(\frac{\hat{a}_{2,j} z_{2,j}}{\epsilon_j} \right) & \leq \tilde{a}_{2,j} |z_{2,j}| + \kappa \epsilon_j \\ \bar{a}_{v,i} \left| \frac{z_{2,j}}{l_{ij}} \right| - \frac{\hat{a}_{1,j} z_{2,j}}{l_{ij}} \tanh \left(\frac{\hat{a}_{1,j} z_{2,j}}{l_{ij} \epsilon_j} \right) & \leq \tilde{a}_{1,j} \left| \frac{z_{2,j}}{l_{ij}} \right| + \kappa \epsilon_j \\ \bar{v}_o \left| \frac{z_{3,j}}{\rho_{2,j}} \right| - \frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j}} \tanh \left(\frac{\hat{a}_{3,j} z_{3,j}}{\rho_{2,j} \epsilon_j} \right) & \leq \tilde{a}_{3,j} \left| \frac{z_{3,j}}{\rho_{2,j}} \right| + \kappa \epsilon_j. \quad (38) \end{aligned}$$

From (33), (37), and (38), $\dot{V}$ in (36) is expressed as

$$\begin{aligned} \dot{V} \leq & -k_{1,j} z_{1,j}^2 - k_{2,j} z_{2,j}^2 - k_{3,j} z_{3,j}^2 - k_{4,j} z_{4,j}^\top z_{4,j} \\ & + z_{1,j} \epsilon_{1,j} + z_{2,j} \epsilon_{2,j} + z_{3,j} \epsilon_{3,j} + z_{4,j}^\top \epsilon_{4,j} \\ & + \tilde{a}_{1,j} \left(\left| \frac{z_{1,j}}{\rho_{1,j}} \right| + \left| \frac{z_{2,j}}{l_{ij}} \right| - \frac{1}{\gamma_{1,j}} \dot{\hat{a}}_{1,j} \right) \\ & + \tilde{a}_{2,j} \left(|z_{2,j}| - \frac{1}{\gamma_{2,j}} \dot{\hat{a}}_{2,j} \right) + \tilde{a}_{3,j} \left(\left| \frac{z_{3,j}}{\rho_{2,j}} \right| - \frac{1}{\gamma_{3,j}} \dot{\hat{a}}_{3,j} \right) \\ & + \sum_{n=1}^3 \tilde{W}_{n,j}^\top (\Phi_{n,j} z_{n,j} - \Gamma_{n,j}^{-1} \dot{\hat{W}}_{n,j}) \\ & + \text{tr}(\tilde{W}_{4,j}^\top (\Phi_{4,j} z_{4,j} - \Gamma_{4,j}^{-1} \dot{\hat{W}}_{4,j})) + \frac{\beta_j}{2} + 4\kappa \epsilon_j. \quad (39) \end{aligned}$$

Substituting (34) into (39) and using Young's inequality, we have

$$\begin{aligned} \dot{V} \leq & -\left(k_{1,j} - \frac{1}{2}\right) z_{1,j}^2 - \left(k_{2,j} - \frac{1}{2}\right) z_{2,j}^2 - \left(k_{3,j} - \frac{1}{2}\right) z_{3,j}^2 \\ & - \left(k_{4,j} - \frac{1}{2}\right) z_{4,j}^\top z_{4,j} - \frac{\sigma_{1,j}}{2} \tilde{a}_{1,j}^2 - \frac{\sigma_{2,j}}{2} \tilde{a}_{2,j}^2 - \frac{\sigma_{3,j}}{2} \tilde{a}_{3,j}^2 \\ & - \sum_{n=1}^3 \frac{\bar{\sigma}_{n,j}}{2} \|\tilde{W}_{n,j}\|^2 + \frac{\bar{\sigma}_{4,j}}{2} \|\tilde{W}_{4,j}\|_F^2 + c_1 \quad (40) \end{aligned}$$

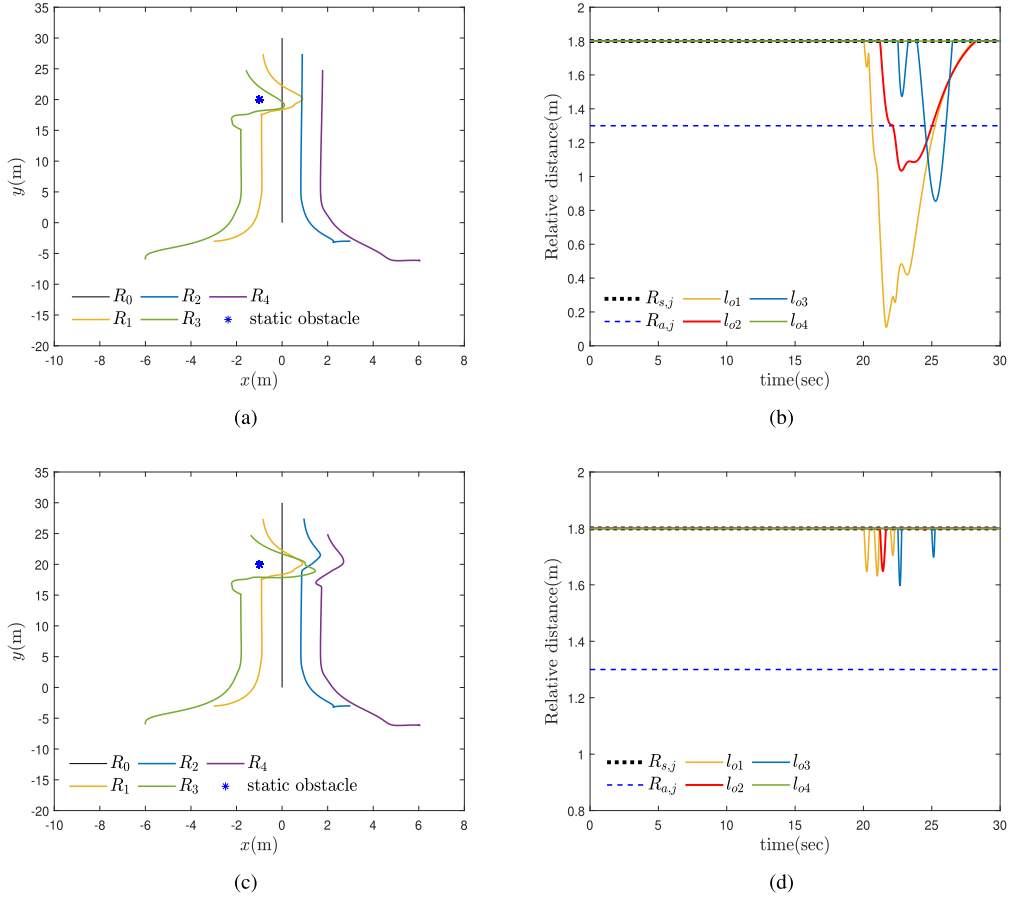


FIGURE 4. Comparison results (a) trajectory for Case 1 (b) l_{oj} for Case 1 (c) trajectory for Case 2 (d) l_{oj} for Case 2.

where $c_1 = \beta_j/2 + 4\kappa\epsilon_j + \sum_{m=1}^4(\bar{\epsilon}_{m,j}^2 + \bar{\sigma}_{m,j}\bar{W}_{m,j}^2)/2 + \sigma_{1,j}\bar{a}_{v,i}^2/2 + \sigma_{2,j}\bar{a}_{\omega,i}^2/2 + \sigma_{3,j}\bar{v}_o^2/2$.

If the design parameters are selected as $k_{m,j} = 1/2 + k_{m,j}^*$ where $k_{m,j}^*$ are positive constants and $m = 1, \dots, 4$, then (40) is expressed as

$$\dot{V} \leq -2c_0V + c_1 \quad (41)$$

where $c_0 = \min\{k_{m,j}^*, \sigma_{n,j}\gamma_{n,j}/2, \bar{\sigma}_{m,j}\Gamma_{m,j}^*/2\}$; $\Gamma_{m,j}^*$ denote maximum eigenvalues of $\Gamma_{m,j}$. From (41), $\dot{V}(t)$ is negative-definite when $V(t) > c_1/(2c_0)$. Therefore, $V(t)$ is bounded such that $V(t) \leq \max\{V(0), c_1/(2c_0)\}$, which results in $\hat{W}_{m,j}, m = 1, \dots, 4, z_{n,j}, z_{4,j}$, and $\hat{a}_{n,j}, n = 1, 2, 3$, are in $\mathcal{L}_\infty$. From the boundedness of V , $\dot{V}$ is bounded, which leads to the boundedness of $\dot{z}_{n,j}$ and $\dot{z}_{4,j}$. $\xi_{n,j} \in \mathcal{L}_\infty$ is satisfied from the boundedness of $\hat{W}_{m,j}$ and $\Phi_{m,j}$. Since $z_{n,j}, z_{4,j}$, and $\xi_{n,j}$ are bounded, $e_{n,j}, e_{4,j} \in \mathcal{L}_\infty$ is ensured. Then, from the boundedness of $z_{n,j}, z_{4,j}, \dot{z}_{n,j}, \dot{z}_{4,j}$, and $e_{n,j}, l_{r,ij} \in \mathcal{L}_\infty$ is ensured. This leads to $s_{ij} \in \mathcal{L}_\infty$. As stated in Remark 3, the control objectives (O-i), (O-ii), and (O-iv) are achieved because s_{ij} and $e_{3,j}$ are bounded. Thus, connectivity maintenance, collision avoidance, and obstacle avoidance are guaranteed even while avoiding obstacles.

To prove the control objective (O-iii), assume that there are no obstacles (i.e., $l_{oj}(t) \geq R_{s,j}$). Because $e_{3,j} = 0, z_{3,j} =$

$-\xi_{3,j}$ is satisfied from (12). Thus, we consider the following Lyapunov function candidate

$$V_o = \frac{1}{2}(\xi_{3,j}^2 + \hat{W}_{3,j}^\top \Gamma_{3,j}^{-1} \hat{W}_{3,j}). \quad (42)$$

From (14) and (34), the time derivative of (42) is derived as

$$\begin{aligned} \dot{V}_o &= \xi_{3,j}(-k_{3,j}\xi_{3,j} + \hat{W}_{3,j}^\top \Phi_{3,j}) \\ &\quad + \hat{W}_{3,j}^\top \Gamma_{3,j}^{-1}(-\Gamma_{3,j}\Phi_{3,j}\xi_{3,j} - \bar{\sigma}_{3,j}\Gamma_{3,j}\hat{W}_{3,j}) \\ &= -k_{3,j}\xi_{3,j}^2 - \bar{\sigma}_{3,j}\hat{W}_{3,j}^\top \hat{W}_{3,j}. \end{aligned} \quad (43)$$

Therefore, $\dot{V}_o$ is negative-definite. Because $\dot{\xi}_{3,j}$ and $\hat{W}_{3,j}$ are bounded, by Barbalat's Lemma [35], $\lim_{t \rightarrow \infty} \xi_{3,j}(t) = 0$ and $\lim_{t \rightarrow \infty} \hat{W}_{3,j}(t) = 0$ are ensured. If there are no obstacles, then $\dot{e}_{3,j}$ in (27) is zero by Remark 6. Thus, the convergence of $\hat{W}_{3,j}$ to zero is reasonable because the RBFN does not need to estimate the nonlinear function $f_{3,j}$. This leads to $\lim_{t \rightarrow \infty} z_{3,j}(t) = 0$. Then, $\lim_{t \rightarrow \infty} l_{r,ij}(t) = 0$ and $\lim_{t \rightarrow \infty} \psi_{r,ij}(t) = \psi_{d,ij}$ are ensured from (23) and (24). Integrating both sides of (41) yields

$$V(t) \leq (V(0) - \frac{c_1}{2c_0})e^{-2c_0t} + \frac{c_1}{2c_0}. \quad (44)$$

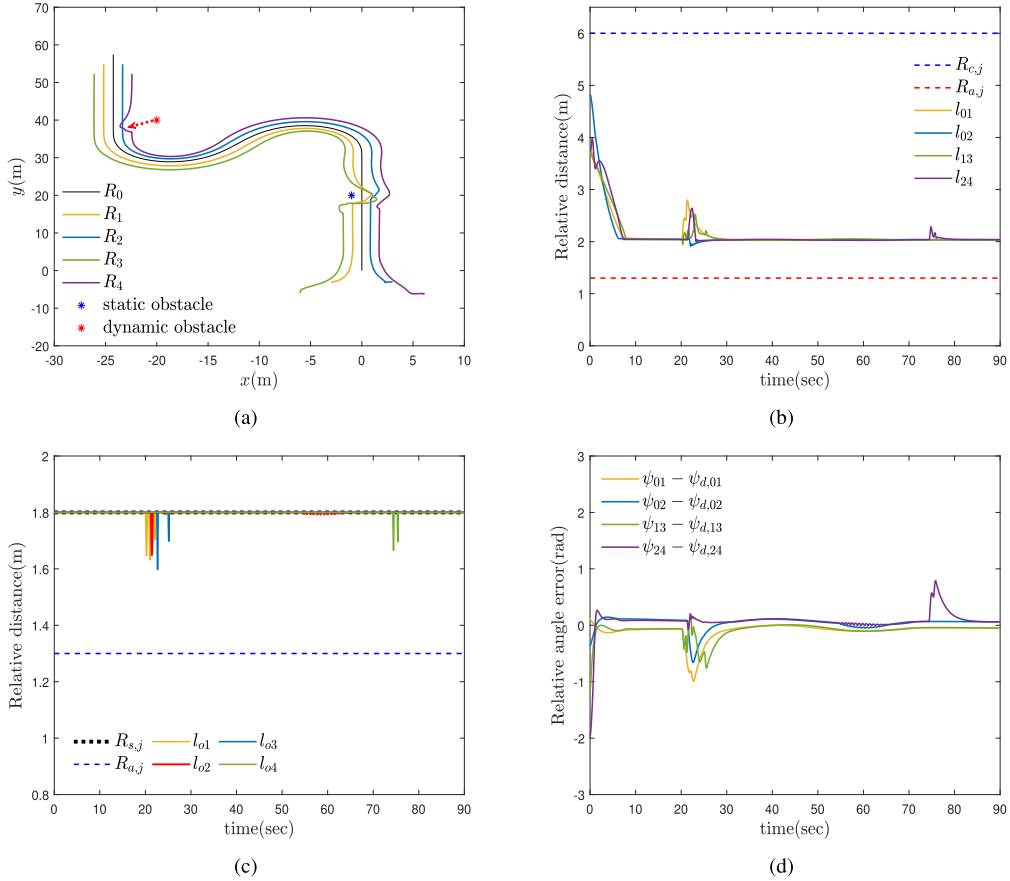


FIGURE 5. Simulation results for Case 2 (a) trajectory (b) l_{ij} (c) l_{oj} (d) $\psi_{ij} - \psi_{d,ij}$.

From (44), it is induced that $|z_{n,j}| \leq \sqrt{c_1/c_0}$, $|\xi_{n,j}| \leq (\bar{W}_{n,j} + \sqrt{c_1 \bar{\Gamma}_{n,j} N_{n,j}/c_0})/k_{n,j}$, and $\|\hat{W}_{n,j}\| \leq \bar{W}_{n,j} + \sqrt{c_1 \bar{\Gamma}_{n,j}/c_0}$ where $\bar{\Gamma}_{n,j}$ denote the minimum eigenvalues of $\Gamma_{n,j}$ and $n = 1, 2$. Therefore, from (5), (6), (10), and (11), $\lim_{t \rightarrow \infty} |s_{ij}(t)| \leq \lim_{t \rightarrow \infty} (|z_{1,j}(t)| + |\xi_{1,j}(t)|) \leq \bar{\mu}_1$ and $\lim_{t \rightarrow \infty} |\psi_{ij} - \psi_{d,ij}| \leq \lim_{t \rightarrow \infty} (|z_{2,j}| + |\xi_{2,j}|) \leq \mu_1$ where $\bar{\mu}_1 = \sqrt{c_1/c_0} + (\bar{W}_{1,j} + \sqrt{c_1 \bar{\Gamma}_{1,j} N_{1,j}/c_0})/k_{1,j}$ and $\mu_1 = \sqrt{c_1/c_0} + (\bar{W}_{2,j} + \sqrt{c_1 \bar{\Gamma}_{2,j} N_{2,j}/c_0})/k_{2,j}$. From the fact that $\lim_{t \rightarrow \infty} |s_{ij}(t)| \leq \bar{\mu}_1$, it holds that

$$\lim_{t \rightarrow \infty} |l_{ij}(t) - l_{d,ij}| \leq \mu_0, \quad (45)$$

where

$$\mu_0 = \max \left\{ \left| \frac{(l_{d,ij} - R_{a,j})(e^{-\bar{\mu}_1} - 1)}{1 + \delta_{ij} e^{-\bar{\mu}_1}} \right|, \left| \frac{(l_{d,ij} - R_{a,j})(e^{\bar{\mu}_1} - 1)}{1 + \delta_{ij} e^{\bar{\mu}_1}} \right| \right\}.$$

This completes the proof of (O-iii). Furthermore, increasing $k_{m,j}$, $\gamma_{n,j}$, and $\Gamma_{m,j}^*$ where $m = 1, \dots, 4$ and $n = 1, 2, 3$ reduces the error bounds $\bar{\mu}_1$ and μ_1 . This leads to a reduction in the error bound μ_0 . Therefore, μ_0 and μ_1 can be made arbitrarily small by design parameters, which results in

achieving the desired formation. This completes the proof of Theorem 1.

Remark 8: The guidelines for selecting design parameters to improve the performance of the control system are as follows. First, set $\sigma_{n,j}$ for $n = 1, 2, 3$ and $\bar{\sigma}_{m,j}$ for $m = 1, \dots, 4$ to small values to reduce c_1 . Second, increase $k_{m,j}$, $\gamma_{n,j}$, and $\Gamma_{m,j}$ to make c_0 larger where $m = 1, \dots, 4$ and $n = 1, 2, 3$. Then, μ_1 and $\bar{\mu}_1$ become smaller, which in turn makes μ_0 smaller.

IV. SIMULATION RESULTS

Five robots denoted by R_n for $n = 0, \dots, 4$ are considered to demonstrate the performance and theoretical results of the proposed leader-follower formation control scheme. The connection between robots is shown in Fig. 3. To generate the reference trajectory, it is assumed that the trajectory of the robot R_0 is generated by (1) with velocities as follows: $\mathbf{v}_0 = [1, 0]^T$ for $t < 30$ s, $\mathbf{v}_0 = [1, 0.2 \cos(\pi t/20)]^T$ for $30 \leq t < 70$ s, and $\mathbf{v}_0 = [1, 0]^T$ for $t \geq 70$ s. For the formation of robots, the desired distance and angle are given as $l_{d,01} = l_{d,02} = l_{d,13} = l_{d,24} = 2$, $\psi_{d,01} = \psi_{d,13} = \pi/6$, and $\psi_{d,02} = \psi_{d,24} = -\pi/6$. The parameters for connectivity maintenance and avoiding obstacles are set to $R_{c,j} = 6$, $R_{s,j} = 1.8$, and $R_{a,j} = 1.3$. All system parameters for the kinetics

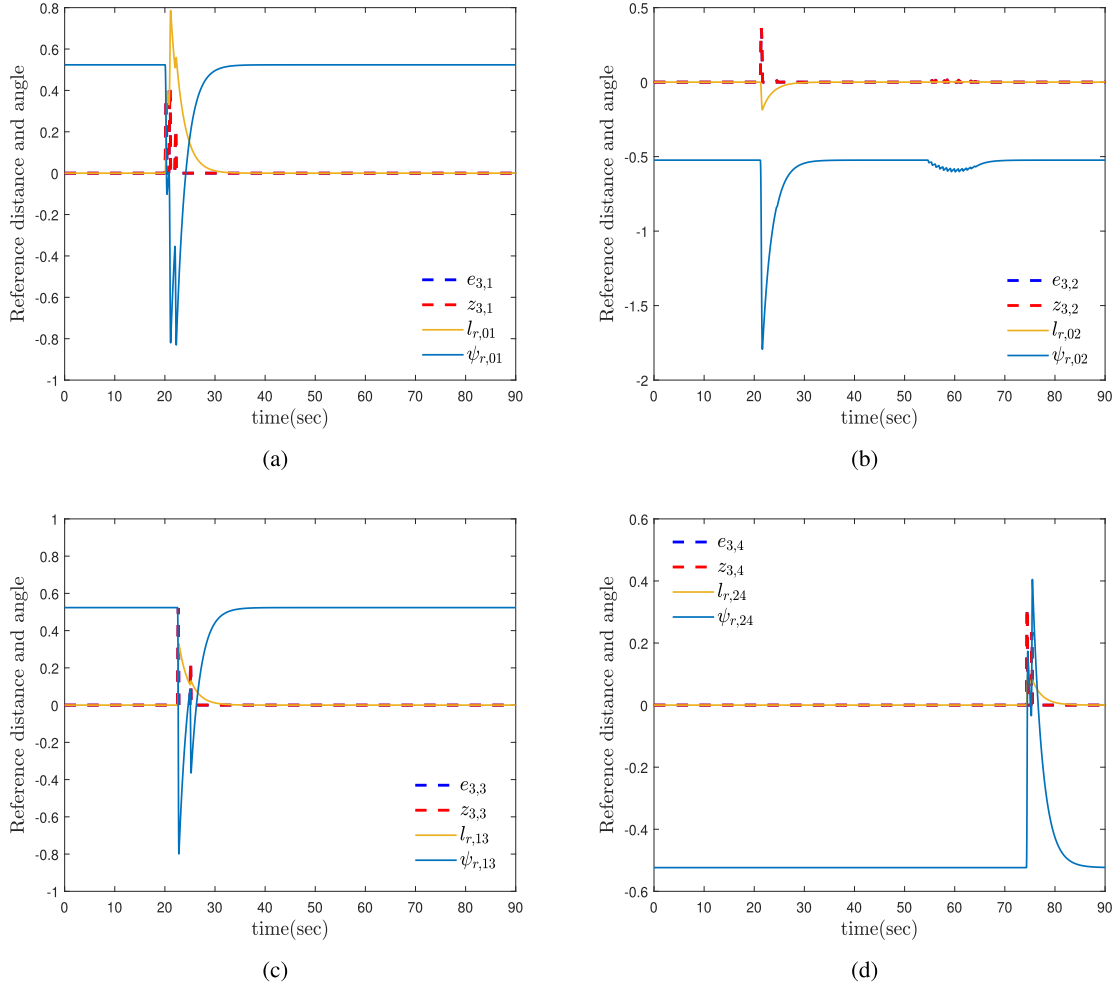


FIGURE 6. Guidance signals $l_{r,j}$ and $\psi_{r,j}$ (a) $l_{r,01}$ and $\psi_{r,01}$ (b) $l_{r,02}$ and $\psi_{r,02}$ (c) $l_{r,13}$ and $\psi_{r,13}$ (d) $l_{r,24}$ and $\psi_{r,24}$.

of mobile robots are taken from [27]. The initial postures of robots are selected as $\mathbf{q}_0 = [0, 0, \pi/2]^\top$, $\mathbf{q}_1 = [-3, -3, 0]^\top$, $\mathbf{q}_2 = [3, -3, 0]^\top$, $\mathbf{q}_3 = [-6, -6, \pi/2]^\top$, and $\mathbf{q}_4 = [6, -6, \pi/2]^\top$. The external disturbances and the deception attacks are given by $\boldsymbol{\tau}_{d,j} = [0.7 \cos(1.5t), 1.2 \sin(0.5t)]^\top$ and $[a_{v,i}, a_{\omega,i}] = [0.1 \sin(t/5), 0.1 \cos(t/5)]$, respectively. The design parameters are chosen as $k_{1,j} = k_{2,j} = 3$, $k_{3,j} = 1$, $k_{4,j} = 5$, $\gamma_{1,j} = \gamma_{2,j} = \gamma_{3,j} = 1$, $\boldsymbol{\Gamma}_{1,j} = \text{diag}[1, \dots, 1] \in \mathbb{R}^{N_{m,j} \times N_{m,j}}$, $N_{m,j} = 3$, $\sigma_{1,j} = \sigma_{2,j} = \sigma_{3,j} = 0.015$, $\bar{\sigma}_{1,j} = \bar{\sigma}_{2,j} = \bar{\sigma}_{3,j} = \bar{\sigma}_{4,j} = 0.015$, $\epsilon_j = 1$, $\lambda_{1,j} = \lambda_{2,j} = 0.5$, and $\varpi_{1,j} = \varpi_{2,j} = 100$.

To show the performance of collision avoidance and obstacle avoidance, two cases are simulated for $0 \leq t < 30$ s. In Case 1, the controllers without considering obstacle avoidance are employed for robots R_2 and R_3 . Furthermore, we assume that R_1 detects the static obstacle, but does not detect R_2 that becomes an unknown dynamic obstacle for R_1 because two robots R_1 and R_2 are not connected, as shown in Fig. 3. On the other hand, in Case 2, the controllers considering obstacle avoidance are used for all robots. In Fig. 4, the simulation results for Cases 1 and 2 are

compared where l_{oj} for $j = 1, \dots, 4$ denote the relative distances between the nearest obstacle and the robot j . For Case 1 shown in Figs. 4(a) and (b), we can observe that (i) R_1 detects and avoids the static obstacle (*) at $t = 20$ s and during the avoidance maneuver of R_1 , R_1 collides with R_2 , which is regarded as a dynamic obstacle, at $t = 20 \sim 22$ s (i.e., the relative ranges l_{o1} and l_{o2} are within the minimum avoidance range $R_{a,j}$); (ii) during the avoidance maneuver of R_1 , R_3 follows its connected robot R_1 to maintain the relative distance and angle to R_1 (see Fig. 3 for the connection between R_1 and R_3); and (iii) However, as shown in Fig. 4(b), R_3 collides with the static obstacle (*) at $t = 24.5$ s (i.e., the relative range l_{o3} is within the minimum avoidance range $R_{a,j}$). This shows that robots can collide with obstacles in the absence of obstacle detection. On the other hand, when detecting the static and dynamic obstacles in Case 2, robots R_2 and R_3 move to avoid collisions with obstacles, as shown in Figs. 4(c) and (d). Then, R_4 connected to R_2 moves to maintain the desired distance and angle between these robots. These results demonstrate that the proposed method is effective in avoiding collisions with static and dynamic

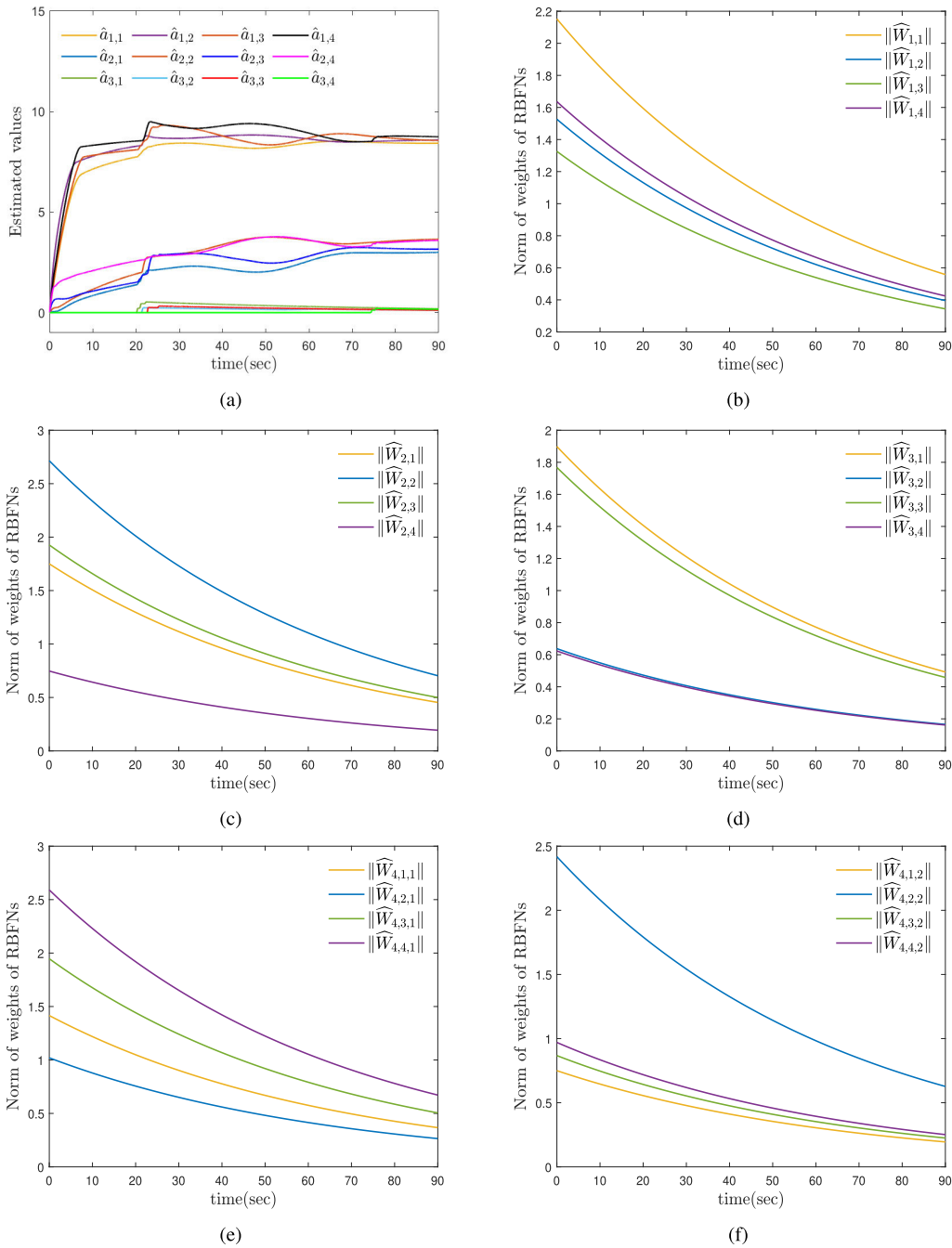


FIGURE 7. Estimation results (a) $\hat{a}_{1,j}$, $\hat{a}_{2,j}$, $\hat{a}_{3,j}$; (b) $\|\hat{W}_{1,j}\|$ (c) $\|\hat{W}_{2,j}\|$ (d) $\|\hat{W}_{3,j}\|$ (e) $\|\hat{W}_{4,j,1}\|$ (f) $\|\hat{W}_{4,j,2}\|$.

obstacles. Fig. 5 shows the formation tracking results of Case 2 for $0 \leq t < 90$ s, where a dynamic obstacle is added. The robot trajectories are displayed in Fig. 5(a). In this figure, we can see that all robots do avoidance maneuvers against static and dynamic obstacles. In Fig. 5(b), the relative distances of all robots converge to the desired distances $l_{d,ij}$ and are within the ranges $R_{a,j} < l_{ij} < R_{c,j}$. Thus, connectivity maintenance, collision avoidance, and obstacle avoidance between the connected robots are achieved, regardless of the

dynamic obstacle and the deception attack. Fig. 5(c) reveals that the robots avoid the detected obstacles, as discussed on Fig. 4. Fig. 5(d) shows that the angle tracking errors $\psi_{ij} - \psi_{d,ij}$ are bounded and converge near zero. The guidance signals $l_{r,ij}$ and $\psi_{r,ij}$ are displayed in Fig. 6. As mentioned in the proof of Theorem 1, the time responses of $l_{r,ij}$ and $\psi_{r,ij}$ are only changed when the robots avoid obstacles and are set to $l_{r,ij} = 0$ and $\psi_{r,ij} = \psi_{d,ij}$ when there are no obstacles (i.e., $e_{3,j} = z_{3,j} = 0$). Fig. 7 depicts the estimated values $\hat{a}_{n,j}$

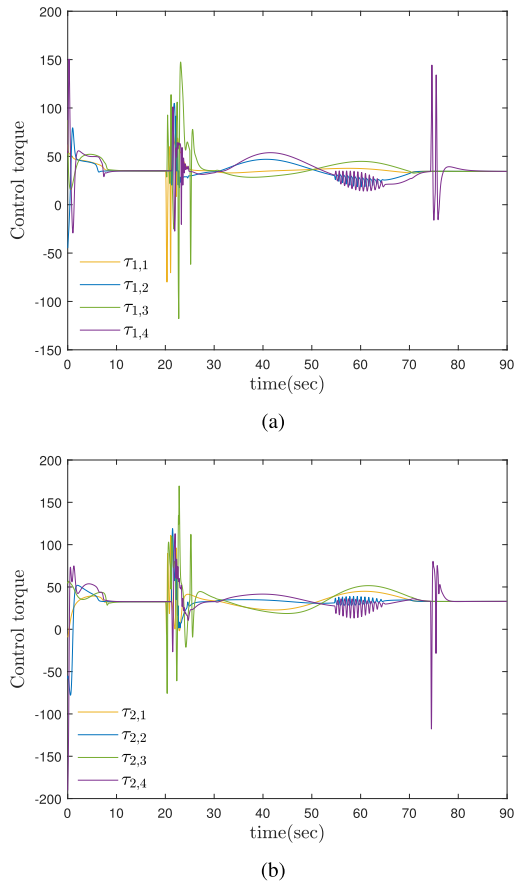


FIGURE 8. Control torques τ_j (a) $\tau_{1,j}$ (b) $\tau_{2,j}$.

for $n = 1, 2, 3$ and $\hat{W}_{m,j}$ for $m = 1, \dots, 4$. Fig. 8 shows the control torques of all robots. Thus, the simulation results confirm the satisfaction of all theoretical outcomes.

V. CONCLUSION

The adaptive collision avoidance approach against dynamic obstacles has been proposed to design a nonholonomic multi-robot formation system with unknown deception attacks. The primary contribution of this study lies in the development of the guidance signals with adaptive collision avoidance mechanism derived from the Lyapunov-based formation tracker design. Different from the existing collision avoidance strategies, the adaptive tuning mechanism for dynamic obstacle avoidance has been derived to estimate unknown velocity information of obstacles in the presence of unknown deception attacks. The resilient formation tracking controller using the guidance signals has been designed for uncertain multiple nonholonomic mobile robots. The effectiveness of the proposed approach has been demonstrated through theoretical proofs and simulation results.

The strategy to ensure obstacle and collision avoidance aims to evade the closest object among obstacles and neighboring robots. However, since this strategy focuses on the nearest object, a selection problem arises when two

objects are at the same distance. Solutions to this limitation are meaningful for future research.

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BONG SEOK PARK received the B.S., M.S., and Ph.D. degrees in electrical and electronic engineering from Yonsei University, in 2005, 2008, and 2011, respectively. Since 2015, he has been with the Division of Electrical, Electronic, and Control Engineering, Kongju National University. His research interests include nonlinear control, formation control, and the control of robots.



SUNG JIN YOO (Member, IEEE) received the B.S., M.S., and Ph.D. degrees in electrical and electronic engineering from Yonsei University, Seoul, South Korea, in 2003, 2005, and 2009, respectively. He was a Postdoctoral Researcher with the Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Illinois, from 2009 to 2010. Since 2011, he has been with the School of Electrical and Electronics Engineering, Chung-Ang University, Seoul, where he is currently a Professor. His research interests include nonlinear adaptive control, decentralized control, distributed control, fault-tolerant control, and neural networks theories, and their applications to robotic, flight, nonlinear time-delay systems, large-scale systems, and multi-agent systems. He is an Associate Editor of IEEE TRANSACTIONS ON CYBERNETICS and *International Journal of Control, Automation, and Systems*.

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